On a Semiautomatic Method of Aberrations Correction of Optical Systems

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A semiautomatic method of realizing the aberration correction in optical systems, filled to the computer National Elliott 803 B is described. The method is characterized as very universal and easy in application. As illustration serve two examples of optical systems optimized by the described method.

Introduction

The automation of the process of the aberration correction in optical systems is realized on electronic computers by different methods. It depends upon size and efficiency of the used computer, on the numerical approach and on the assumptions accepted for the programming.

The large computers (above 100 000 operations per second) enable a high degree of automation of the correction process, excluding the intervention of the lens designer during the process. It is very difficult to work out a sufficiently universal program, which could enable at the same time the optimization of the aberration characteristics of different types of optical systems. The programs of the high degree of automation are usually designed in such a way that they optimise a specified kind of optical system, only.

The small computers (below 10 000 operations per second) do not admit a high degree of automation. The process of correction in that case would last many hours, which requires a computer with great infallibility of work. Using a small computer we are compelled to diminish the degree of automation. This basic disadvantage can be compensated by the gains obtained by working out a semiautomatic version of the correction process. The main gain is the possibility of frequent interventions of the designer during the correction process, enabling him to change the conditions of the process (parameters, equations, the weight coefficients etc.) dependently on to the obtained results. In that case we get rid of a too schematic estimation of the quality of optical system, characteristic for the general case of a completely automized process. Of course, the semiautomatic process is easier to be worked out in the universal form, enabling the optimization of the different kinds of optical systems with respect to the selected aberration characteristics and the selected correction parameters. If this method distinguishes itself by practice qualities such as minimum quantity of information for the input data and if this information does not require any supplementary computations by ordinary machines, it will be a basic tool for the designer of optical systems.

In this paper a semiautomatic method of aberration correction is described, that is very close to the above named advantages. This semiautomatic method of aberration correction realizes the correction process by dividing it into its component parts. This method is based on correction tables and on the modified method of least-squares.

In the general case the correction process of the optical systems is an iteration process. The numerical approach, described in this paper, introduces the division of each iteration of this process into two following stages:

1. The computation of the increments of the aberration and transaberration quantities caused by the changes of the construction parameters;

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2. The computation of the parameters of the optical systems with adjusted aberration characteristics.

These stages are realized by two programs: the first one enabling the computation of correction tables (program TAKO), and the second one enabling the computation of the parameters of the optical system by the modified method of least-squares (program AUKO).

In this paper we confine ourselves to the optical systems with spherical surfaces. We take into account only the classical aberration characteristics, not requiring any special skew ray tracing. It is recommended to compute and verify, by separate programs, the modern quality characteristics based either on wave aberrations or on spot-diagram, depending on the nature of optical image formation. The programs of semiautomatic aberration correction were worked out for National–Elliott 803 B computer.

The computation of the increments of aberration quantities

Correction tables

As the correction table we understand a tabular arrangement of the aberrations of a nominal optical system and the same tabular arrangements of the aberration increments of the optical system for any variant with changes of system parameters. The nominal optical system is an optical system of which parameters serve as the basic for appreciation of the individual changes. There is an infinite number of optical systems more or less close to a given nominal optical system, differing by the change of one, two ... or in the extreme case of all the parameters. We treat these systems as the variants with a change.

The increments of the aberration and transaberration quantities can be obtained by the approximate or direct methods. The approximate methods are based on the differential approach applied to finite wave aberration [1], or to the transverse aberrations [2] or to the third order aberrations [3, 4]. The usefulness of these methods to the automation of correction process in optical systems seems to be not sufficiently verified in practice. Therefore, in this paper we have assumed, as it is customary in the majority of works on this subject, the direct calculations of these increments, that is by finite differences.

If we denote by $f^{(n)}$ any aberration or transaberration magnitude, ascribed to the nominal optical system, and by $f^{(r)}$ — the corresponding magnitude for the variant with the change, then the increment of the aberration or transaberration magnitude is called the difference Δf defined by the formula

$$\Delta f = f^{(r)} - f^{(n)}. \tag{1}$$

In the correction tables arranged according to parameter increments to each change of construction parameters correspond all the specified increments of the aberration and transaberration magnitudes which are of interest. In the application of this system of tables the informations on the change of the system parameters can be successively read off from the input data while the structure of these changes can be very complicated.

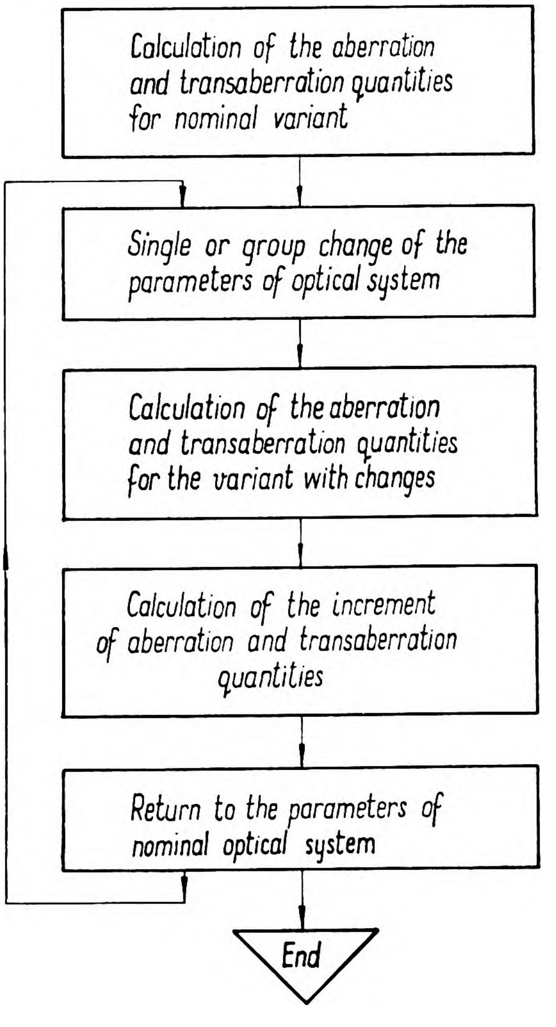

Fig. 1. The general block diagram of the program TAKO used for the calculation of correction tables

The Figure 1 represents a general scheme of the program TAKO, enabling the computation of correction tables in the arrangement after the parameter increment accepted in this paper. As it can be seen in this scheme, the computation of the aberration and trans-aberration quantities for the nominal system is necessary and should antecede all further operations, while keeping them in the memory. All the other operations are of cyclic character and are repeated for all the variants with changes. Each time to compute the changes with respect to the parameters of the nominal optical system we must return to the parameters of this nominal system.

The changes of correcting parameters

During the computation of correction tables we should be able to take into account both the variants with the individual changes of parameters and the variants with the group changes. The consideration of group changes results from many reasons.

First of all the theory of aberrations suggests certain group changes useful for the process of correction. The well known change of this kind is the change by lens bending. The main gain of this change lies in that the bending of the lens does not change its optical power, which implies the stability of axial chromatism, Petzval curvature and of the paraxial constants of the optical system.

The necessity of applying the group change of the parameters in one variant appears acutely in symmetrical optical systems if the correction process should not lose its symmetry. Instead of the individual change of the parameter we have then double change of the parameters, coupled by the symmetry. The situation in partially symmetrical systems is similar.

Another example of group changes are the asymmetric optical systems with repeated parameters, e. g. curvatures and optical glasses. In the optical system the possibility of the repeatability of these elements is its profitable feature, often appreciated by the optical designers.

The necessity of the group changes in one variant can occur in also catadioptric systems. It may happen, that a certain reflecting surface twice reflects the light rays and hence twice appears in the rays tracing.

Therefrom, it follows, that the group changes of system parameters can exhibit the structure difficult to be foreseen. These changes may pertain to the parameters of one or more kinds. But they can be contained in one scheme. In this paper a sequential scheme is applied. The succesion of changes in the sequence is to be decided in dependence on the kind of changed parameters. The sequence part pertaining to the given kind of the parameter is anteceded by the counter of the elements of one kind changes. This counter can be equal to zero. This means that the specified kind of parameters does not take part in the group change. The description of the parameters change must contain the number of the changed parameter and the magnitude of the increment with its sign.

The represented system of group changes enables the realization of any programmed group changes in the nominal optical system without any restrictions, and particularly, enables the individual changes of parameters.

Numerical methods

To work out a program for the computation of correction tables we should have: the methods of ray tracing through the optical system (paraxial, meridional and astigmatic), the methods of the computations of the geometrical aberrations on the bases of the quantities acquired from the rays tracing or the iteration methods, bound up to the fixing of the course of the principal field rays and marginal rays, designated by the dimensions of the aperture diaphragm.

The numerical methods of ray tracing for the case of electronic computers differ very much from the analogous methods used actually for the electrical machines. The methods are worked out from the point of view of high accuracy, simplicity of the scheme and great speed of computing.

The ray tracing should not always start with the first and finish with the last surface of the optical system. In the general case we are interested in the ray tracing through a part of the optical system, e. g. from the surface with the index j to the surface with the index m. The ray tracing is direct for $m > j$, with the step $s = 1$, reversed if $m < j$ with the step $s = -1$. In the case of ray tracing through a part of optical system we call it a partial ray tracing, and in the case of a ray tracing through the whole optical system — a full or complete ray tracing. An example of partial ray tracing would be the fixing of the pupils in a system with the aperture diaphragm inside, while that of complete ray tracing — the determining of aberrations or of focal length of the whole system.

The actual computations on electrical machines use for the reversed ray tracing the same computational scheme with system parameters rewritten in reversed order. This idea, applied to the case of electronic computer, would require an expensive and a somehow complicated readdressing of the parameters of selected parts of optical systems. Often repeated process of readdressing the parameters would lengthen and complicate the program. Hence the idea of the synthesis of both the direct and reversed ray tracing in one universal paraxial or meridional ray tracing.

The schemes of these ray tracings give the chance for the computations of any partial or complete, direct or reversed, ray tracings for the given refractive indices. The well known schemes of the paraxial and meridional ray tracing are modified in this paper by introducing the value of the step s, which intervenes either in arithmetic expressions or in the index expressions causing, thus, required changes of the computational scheme. This modification will be illustrated on the example of the paraxial ray tracing with the so called Lange's variables.

If we denote by ϱ_k the curvature of the k-th surface, by d_k the axial thickness between the surfaces k and $k+1$, by $n_{k,l}$ — the refractive index of the optical medium between the surfaces k and $k+1$ for the given spectral line l, then the calculation scheme of the paraxial ray tracing through one optical surface can be described by the three following formulas

$$h_k = h_{k-s} - s d_{(2k-1-s)/2} \alpha^{(*)}_{k-s}, \tag{2}$$

$$\mu_k = \frac{n_{(2k-1-s)/2,l}}{n_{(2k-1+s)/2,l}}, \tag{3}$$

$$\alpha^{(*)}_k = (1-\mu_k) h_k \varrho_k + \mu_k \alpha^{(*)}_{k-s}, \tag{4}$$

where

h — the height of incidence of the paraxial ray on the surface,

α — the angle with the optical axis of the paraxial ray.

According to Figure 2 in which the coordinates of the paraxial ray tracing are shown the symbol (*) means that for the reversed ray tracing it is a magnitude to the left of the surface.

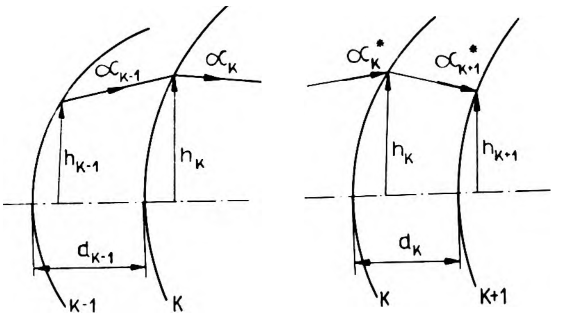

Fig. 2. Coordinates of paraxial ray tracing: a) direct (given: h_{k-1}, α_{k-1}; required: h_k, α_k), b) reversed (given: h_{k+1}, α_{k+1}; required: h_k, α^*_k)

In the case of partial ray tracing from the surface j to m we start always from the coordinates bound up to the surfaces $j-s$ (from the surface $j-1$ for the direct or from the surface $j+1$ for the reversed ray tracing).

In a similar manner the meridional scheme of ray tracing, proposed by W. J. Smith [6], was modified. The scheme of astigmatic ray tracing for the rays near to the principal field ray is based on angular variables [1, 6, 7].

The basis of the further numerical methods applied to the program of correction tables is formed by the assumed convention of extended system parameters of the optical system. According to this convention to the system parameters formally belong also the surfaces of the object and of the image, the plane of the diaphragm stop and the separations of these fictitious refracting surfaces in the optical system, which result from optical conjugation or from the location of aperture stop in the optical system. Thanks to this convention the optical system is joined in a suitable unity with its environment, thus facilitating the programming very much. Another convention pertains to the writing of the quantities liable to become ∞. We assign to these quantities, as impossible to be represented in the electronic computer, the symbol 0. It refers also to the object or exit pupil at infinity, and too to the object or exit pupil at infinity, and to the values of plane radii for the input data.

As "working conditions" we understand usually: the conjugation object-image, aperture and field of view. These conditions may be stated in a different manner. Except trivial cases, these methods need some modification and for automatic fixing of the conjugation object-image we have to establish the distance of the object from the 1-st surface of the system, transverse magnification and the distance object-image. The maximum aperture is determined by: the aperture number, object aperture and the diameter of entrance pupil.

Automatic fixing of maximum field of view depends upon the position of aperture diaphragm in the optical system and on the vignetting. In this paper there are accepted four cases of the location of aperture diaphragm in the optical system (with the exeption of the case of the aperture diaphragm being located at ∞ before the system) as well as symmetrical type of linear vignetting. The automatic fixing of the maximum field of view is determined by: the object field of view, the linear magnitude of the object and image. The program is based on the iterative Newton scheme, in which the derivatives are computed as the ratio of finite differences being the result of 0.5% changes of the coordinates.

The individual geometrical aberrations are calculated in the 4 fixed aperture and field zones, according to the definitions, which are known from the aberration theory.

The determining of aberrations and aberration increments

Among the different geometrical aberrations constituting the classical aberration characteristics are the aberrations the exact computation of which is not easy to realize, e. g. the aberration of the principal ray and marginal rays. The difficulties consist in the proper localization of the principal and marginal rays. The principal field ray is defined as the middle ray of the bungle of field rays limited either by the dimensions of the aperture diaphragm or by the vignetting lens edges.

The simplest is the case of an optical system without the vignetting over the whole field of view and with a material aperture diaphragm. The principal ray passes then through the center of the aperture diaphragm for each field zone. The case when the principal ray is in the field of view restricted by the geometrical vignetting belongs to those more complicated. Then the acting diameters of the surfaces of the optical system are usually fixed for the maximum angle of field of view and for the given symmetrical vignetting. In that manner the principal ray in the object space is synchronized in two points of the field of view of spherical aberration in the entrance pupil, that means at the edge of field and formally for the field of zero value. In the medial field zone the principal ray under the influence of the vignetting diameters of the optical system acquires an additionnal transversal shift in the aperture stop, which reaches a maximum value for a certain medial angle of field of view. But as the field decreases this shift, before all in the aplanatic systems, plays more and more minor role in the accuracy of determining the aberrations of the principal ray. The exact localization of the principal ray, taking into account the field vignetting, is complicated and would require a great loss of time. Therefore, we assume that the principal rays pass through the center of the meridional entrance pupils and consequently through the center of the aperture diaphragm for each field zone. If the principal rays pass through the paraxial entrance pupil or through the meridional pupil for the maximum field of view, the methods seem to be less perfect. In some optical systems the aberrations of principal rays do not exhibit serious differences if calculated by different methods of localization of principal rays. But especially in wide angle systems working at a magnification 1 : 1 the differences are great. The difficulties of the optimization of these systems computed by improper methods of localization of the principal ray have been emphasized in the discussion after the lecture of Wynne [8].

The problem of the localization of the principal ray is bound up to the method of the computation of the transverse chromatic aberration. And as it is not easy to find, by a simple method, the proper localization of the principal ray in the optical systems vignetting for the medial field zones, so more it is difficult for the colours. In this paper we assume, that for the fixed object point the principal colour rays pass through the center of the aperture stop, and consequently in the case of the object at finite distance they are in the object space inclined towards the axis at different angles, or in the case of the object at infinity they suffer a parallel shift.

The calculation of the meridional coma is bound up not with the dimensions of the entrance pupil, but with the dimensions of the aperture stop.

All these problems are solved by the iterative Newton method.

The magnitude of the calculated aberration increments caused by any change of parameters depends to some extent on the keeping constant some basic quantities. If we assume the invariancy of some parameter determining the optical conjugation e. g. the distance of the object from the optical system, then during the calculation of aberration increments also other parameters will change, e. g. transversal magnification or the distance object-image. To get rid of the influence of the conjugation on the magnitude of aberration increments, we should prefer the situation in which its influence is as small as possible. It happens if while computing the variants with changes we keep constant the distance of the object to the optical system. Of course, we assume that the increments of the parameters of the optical system are sufficiently small. Hence, a general recommendation would be to calculate the optical systems in the direction of the absolutely minor conjugate distance.

Similarly the increments of the aperture aberrations depend on the method of keeping constant the selected aperture parameter. In this paper the constancy of the aperture number is assumed, which means the approximate stabilization of the object aperture.

In a similar manner the increments of the field aberrations depend also upon the method of fixing the constancy of a selected parameter of field of view. For practical reasons we assume in this paper the constancy of the angles of the principal rays in the aperture diaphragm. If we change the parameters of the optical system, we transfer then the principal rays fixed in the aperture stop by means of the meridional reversed ray tracing in the object space. In this method the change of the system parameters before the aperture diaphragm results in changes of the magnitude (linear or angular) of the object, while the change of the system parameters besides the aperture diaphragm changes the magnitude of the image.

The program TAKO

The program TAKO realizes the calculation of the correction tables on electronic computer after the conception described in the foregoing chapters. It is composed of the main program and 6 following subroutines: subroutine for the paraxial, meridional and astigmatic ray tracing, subroutine for the calculation of aberration and transaberration quantities or of their increments, subroutine for printing a single aberration table and the subroutine for introducing by iteration the rays into the elements of aperture stop.

Using the program TAKO one can calculate the systems with spherical and plane surfaces, and the systems with finite focal distance only. The number of the surfaces of the optical system cannot exceed 30. The optical media are represented by three refractive indices corresponding to three colours. Before and behind the optical system the optical media different from the air are admissible. The object surface can be a spherical one. The number of variants with single or group changes of system parameters can attain any value. The program TAKO enables us to obtain great scope of informations about the aberration and transaberration characteristics of the optical system and its changes under the influence of the changes of system parameters in a form completely worked out.

The calculation of the parameters of the optical system

The least-squares method

The application of the least-squares method to the correction of the aberrations in the optical systems has attracted the attention of many authors e. g. [9–17]. This method can be applied very successfully if the number of the correction equations is greater than the number of correction parameters. This favours the general tendency among the lens designers to acquire the maximum quality of the image in the relatively simple optical systems. The idea of the method of least-squares in its clear form was for the first time presented by J. L. Legendre in 1806, and later independently by C. F. Gauss in 1809. After this method the solution of an indefinite set of equations is found from the condition that the sum of the squares of deviations of the values of several equations should be a minimum one. When being applied to the automatized correction process in the optical systems, this method requires some modifications.

The first and at the same time the most important modification of the least-squares method relies upon the taking into account the damping parameter. This modification was introduced for the first time by K. LEVENBERG [18], and later independently by two other authors [11, 13]. The idea of the damped method of least-squares consists generally in the following: enlarging the squares of deviations of correction equations we acquire a solution more suitable due to minor increments of correction parameters. The damping or Levenberg's parameter p represents the relative importance of the sum of squares of deviations for the correction equations in relation to the sum of the squares of the increments of the system parameters.

The second modification of the least-squares method relies upon the consideration of the weighting coefficients of correction equations. The correction equations are of different weight. Sometimes the lens designer is particularly interested in the fullfilling of certain correction equations. Choosing suitably the weight coefficients of correction equations one can attain a required degree of the fullfilling the corresponding correction equations.

Usually the weighting coefficients of correction equations are calculated as the quantities inversely proportional to admissible deviations of correction equations. In this paper the weight coefficients are computed as the product of the admitted weight coefficients and automatically computed weight coefficients inversely proportional to the square root of the sum of squares of equation coefficients. The weight coefficients of equations are additionally normalized by automation to unity. This approach has been shown to be very useful in practice.

The weight coefficients W_i of correction equations are the diagonal elements of the matrix, defined as

$$W = \begin{bmatrix} w_1 & 0 & \dots & 0 \\ 0 & w_2 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & w_m \end{bmatrix} \qquad (5)$$

The third modification of the least-squares method takes into account the weight coefficients of correction parameters. We have accepted in this paper the method of J. MEIRON [12], in a little different manner worked out earlier by A. GIRARD [11] and R. W. HOPKINS [19]. The weight coefficients of correction parameters represent the degree of damping of the parameter change. After J. MEIRON [14] the weight coefficients of correction parameters should be applied to equalizing the sensibility of the influence of several system parameters of correction on the aberration characteristics. In the case of approximately equalised sensibility of the correction parameters, assumed in this paper, the weighting coefficients are used to set off or damp the changes of selected correction parameters. E. g. in the correction process, in which three kinds of parameters are changed (curvatures, separations and optical glass constants), by a suitable choice of weight coefficients q_i one can set off the changes of curvatures before other kinds of parameters.

The weight coefficients q_i of correction parameters are the diagonal elements of the matrix Q of the following form

$$Q = \begin{bmatrix} q_1 & 0 & \dots & 0 \\ 0 & q_2 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & q_n \end{bmatrix} \qquad (6)$$

Some authors [11, 19] examine a diagonal matrix with the products $p \cdot q_i$ as its elements. The separation of these two quantities, as it has been done by J. MEIRON [14] seems to be more justified, since the choice of damping parameter is a different problem from the choice of weight coefficients q_i.

The S. M. SPENCER'S [20] modification of the least-squares method is very interesting. But it is difficult to realize it on small computers.

Instead of partial derivatives of correction functions with respect to the correction parameters we apply the differences of the functions using for that purpose the quantities from the correction tables.

The matrix of the differences of correction functions is as follows

$$A = \begin{bmatrix} \Delta f_{11} & \Delta f_{12} & \dots & \Delta f_{1n} \\ \Delta f_{21} & \Delta f_{22} & \dots & \Delta f_{2n} \\ \vdots & \vdots & & \vdots \\ \Delta f_{m1} & \Delta f_{m2} & \dots & \Delta f_{mn} \end{bmatrix} \quad (7)$$

where

Δf_{ji} — the difference of the correction function denoted by the index j, caused by the change of the correction parameter with the index i.

When using (7) it will be convenient to define the relative increment v_i of the parameter

$$v_i = \frac{\Delta x_i}{\Delta x_i^*}, \quad (8)$$

where

Δx_i — immediate increment of the correction parameter,

Δx_i^* — initial increment of the correction parameter (fixed while computing the correction tables).

The ensamble of the relative increments of correction parameters is being treated as the components of the vector V

$$V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \quad (9)$$

The modified method of least-squares as applied in this paper can be reduced to the following matrix equation [14], equivalent to the set of linear equations

$$(A^T W^2 A + pQI)V = A^T W b, \quad (10)$$

where

A^T — matrix transposed with respect to A,

I — unit matrix,

b — vector with the components equal to the difference between the required magnitude of the aberration and the magnitude of this aberration in the nominal variant.

The function s minimized by the modified method of least-squares is

$$s = \sum_{j=1}^{m} \Big[w_j\Big(\sum_{i=1}^{n} \Delta f_{ji} v_i - b_j\Big)\Big]^2 + p\sum_{i=1}^{n} q_i v_i^2. \quad (11)$$

The system of equations resulting from (10) has a symmetrical matrix. It can be easily solved by the method of square root [21]. If we divide the symmetrical matrix in two reciprocally transposed triangular matrices due to this method, then we must be aware of the appearance (temporarily) of complex numbers.

The parameters of the optical system resulting from the $k+1$ iteration may be obtained on the base of the parameters resulting from with the k iteration and of the calculated relative increments of correction parameters

$$x_i^{(k+1)} = \begin{cases} x_i^{(k)} + \Delta x_i^* v_i & \text{for correction parameters,} \\ x_i^{(k)} & \text{for non-correction parameters.} \end{cases} \quad (12)$$

The described approach in the modified method of least-squares was used by Russian authors, first by N. W. Ceno [16] with some reservations, later by S. A. Rodionow [17] without reserve. This proves the increasing vitality of the described approach, which begins to win a prominent place in the practice of correction process.

The limitations of the parameter changes

Beyond the linearized correction equations, fixing several aberrations and determining some magnitudes (e. g. focal length) there are supplementary relations between the correction parameters. G. G. Wynne [13] suggested a method of adjoining these relations to the ensemble of correction equations. We have linearized in this paper, systematically, the most typical relations between the correction parameters thus transforming them into equations easy to be applied for the described correction process.

The relation between two succesive increments of the curvatures $\Delta\varrho_i$ and $\Delta\varrho_{i+1}$ and the increment of their separation Δd_i belongs to the most frequently occurring. This is the case if these surfaces form a positive lens. Then we should generally admitt a certain increase of the minimum thickness $(\Delta d_{\min})_i$ for each acting height h_i.

The relation between the increments of the distances ΔS_i and ΔS_{i+1} between the surfaces and the increments of the thickness is

$$\Delta S_i - \Delta S_{i+1} + \Delta d_i = (\Delta d_{\min})_i \quad (13)$$

and

$$S_i = \frac{h_i^2 \varrho_i}{1+(1-h_i^2\varrho_i^2)^{1/2}}.$$

By differentiation one can obtain the relation between the increment of the sagitta ΔS_i and the increment of the curvature $\Delta \varrho_i$

$$\Delta S_i = \Omega_i h_i^2 \Delta \varrho_i . \tag{14}$$

The factor Ω is a function of $h\varrho$ after the formula (15) and for convenience it has been tabulated (table 1) for $h\varrho$ from 0 to 0.95 by step 0.05.

$$\Omega_i = \frac{1+\dfrac{(h_i \varrho_i)^2}{(1+l_i)l_i}}{1+l_i}, \tag{15}$$

Table 1

The factor Ω as the function of $h\varrho$

$h\varrho$	Ω	$h\varrho$	Ω
0.00	0.5000	0.50	0.6188
0.05	0.5009	0.55	0.6525
0.10	0.5038	0.60	0.6944
0.15	0.5086	0.65	0.7477
0.20	0.5155	0.70	0.8169
0.25	0.5247	0.75	0.9100
0.30	0.5365	0.80	1.0417
0.35	0.5512	0.85	1.2433
0.40	0.5693	0.90	1.5977
0.45	0.5915	0.95	2.4405
0.50	0.6188	0.99	6.2124

where

$$l_i = (1-h_i^2 \varrho_1^2)^{1/2} .$$

Substituting now (14) into (13) and keeping in mind the necessity of introducing the initial increments denoted $\Delta \varrho_i^*$, $\Delta \varrho_{i+1}^*$ and Δd_i^* respectively, we obtain the following correction equation

$$-\Omega_i h_i^2 \Delta \varrho_i^* \Delta \varrho_i + \Omega_{i+1} h_{i+1}^2 \Delta \varrho_{i+1}^* \Delta \varrho_{i+1} + \\ + \Delta d_i^* \Delta d_i \simeq (\Delta d_{\min})_i , \tag{16}$$

where $\Delta \varrho_i$, $\Delta \varrho_{i+1}$, Δd_i — relative increments.

The correction equation (16) can be applied to the thinning of the positive lenses being too thick ($\Delta d_{\min} < 0$) in the nominal variant, or to the thickening of positive lenses too thin ($\Delta d_{\min} > 0$) or to the stabilization of the minimum thickness at the edge of the lens ($\Delta d_{\min} = 0$) during the correction process.

It happens that the magnitudes of the two successive curvatures ϱ_i and ϱ_{i+1} of the optical system are brought closer together during the correction process. The equalization of these curvatures $\varrho_i + \Delta \varrho_i = \varrho_j + \Delta \varrho_j$ can be realized by the introduction of the following correction equation for the curvatures (of the same sign)

$$\Delta \varrho_j^* \Delta \varrho_j - \Delta \varrho_i^* \Delta \varrho_i = \varrho_i - \varrho_j . \tag{17}$$

To avoid the phenomenon of the pushing aside of the surfaces disadvantegeous with respect to the vignetting, one can formulate the following general correction equation assuming all the separations of the system surfaces to undergo the changes in the correction process:

$$\sum_{i=1}^{p} \Delta d_i^* \Delta d_i = l_r - \sum_{i=1}^{p} d_i , \tag{18}$$

where

l_r — required overall length of the optical system.

From the formula (18) simplified correction equations can be easily deduced being right for selected correction separations, only.

There are many plausible combinations of adjoining supplementary correction equations. Here we point out certain advantages bound up to the change of the parameters of optical glasses, used in this paper. The inspection of the Abbe's diagram (n, ν) of optical glasses shows, that the optical glasses of the type F, CF, LF, LLF (after SCHOTT) on the diagram are located along a curve with an approximate equation

$$n = a\nu^2 + b\nu + c , \tag{19}$$

where

a, b, c — constant values depending on the manner of correction.

In the case of the visual correction (lines e, F' and C') and of flint glasses of the type (29 glasses) being in use in Poland, these constants, calculated by the least-squares method, are $a = 0.0046597$, $b = -0.359917$ and $c = 8.040909$.

The relation between the relative increment of the refractive index Δn_i and the Abbe's number $\Delta \nu_i$ of the optical glass of flint type can be easily obtained from the Eq. (19) differentiating the correction equation

$$\Delta n_i^* \Delta n_i - (2a\nu_i^* + b) \Delta \nu_i^* \Delta \nu_i = 0 . \tag{20}$$

In certain parts of Abbe's diagram the glasses are placed rarely enough. Hence, there arises the necessity of reducing the parameters of fictitious glasses n_{fic}, ν_{fic} to nearest parameters of catalogue glasses n_{cat}, ν_{cat} by means of the two correction equations

$$\Delta n_i^* \Delta n_i = (n_{cat})_i - (n_{fic})_i, \quad \Delta \nu_i^* \Delta \nu_i = (\nu_{cat})_i - (\nu_{fic})_i. \tag{21}$$

The introduction of supplementary correction equations does not facilitate the correction process. The possible negative influence on the aberration characteristics of the supplementary equation must be compensated by the change of the parameters during the whole correction process.

The program AUKO

The program AUKO allows to perform on the electronic computer the calculation of the increments and of the correction parameters based on the modified least-squares method. It renders possible the calculation of the curvatures, and separations of the surfaces and parameters of optical glasses. The input data are perforated immediately from the tablegram of correction tables, completed by the additionnal informations. The maximum number of correction equations should not exceed 50, the maximum number of correction parameters — 20. The program AUKO provides the results depending on the changed damping parameter. The immediate increments of correction parameters are added to the correction parameters of the optical system. In the case of the change of optical glasses the refractive indices for different correction lines of the fictitious optical medium are calculated.

The practice of the semiautomatic method

The choice of the damping parameter

The authors using the modified least-squares method for correction process express very different opinions about the criteria of the choice of damping parameters. Thus choosing the damping parameter we choose at the same time the optical system prefered by this parameter. We mention here some of the up to date known criteria of the choice of damping parameters: criterion of the sufficient linearity of correction equations [13], criterion of maximum success in the correction of aberration characteristics [11] and the criterion of attaining of a great convergency for the whole correction process [16].

In the automized correction process the algorithm of the choice of damping parameter is an important problem. The improper algorithm of the parameter choice causes either a process with a too slow characteristics improvement or a too hazardous process. In the semiautomatic process the choice of the damping parameter can be based not on an exact algorithm but on the selection of the optical systems obtained as the result of the change of damping parameter in assumed limits with assumed frequency. The task of the designer consists in performing a selection based on the specific for each optical system principles which are frequently variable during the correction process. In this approach the final merit function intervenes rarely, but more frequently the structure of the aberrations is studied by observing the advances in the correction of the aberrations difficult to be corrected, which as soon as possible should be reduced to admissible magnitudes.

The damping parameter in input data for program AUKO can be changed in two ways: either according to the geometrical progression (rough procedure) or exactly by arithmetic progression (exact procedure). In this paper the damping parameter was changed in the interval $10^{-9} \leq p \leq 10$.

For the preliminary appreciation of the obtained optical systems without the verification of the aberration characteristics of the optical systems the interpolation index was introduced. I have calculated it after the formula

$$I = \sum_{i=1}^{n} v_i^2. \tag{22}$$

According to the interpolation index the obtained optical systems can be classified in two categories: 1 — the optical systems rejected and not verified as for their too strong extrapolation or too weak interpolation and 2 — the optical systems, of which exact characteristics undergo a verification.

The changes of the parameters of optical glasses

For the calculations after the program TAKO any optical glass is specified by three refractive indices e. g. for visual correction n_e, $n_{F'}$, $n_{C'}$.

The change of the parameter of glasses is easier with other parameters. We assume for example that these parameters are: refractive index of the medial line n_e, Abbe's number ν_e, partial relative dispersion $P_{F'}$. These last parameters of optical glass are bound up to the three refractive indices by the formulas

$$n_e = n_e,$$
$$\nu_e = \frac{n_e - 1}{n_{F'} - n_{C'}}, \tag{23}$$
$$P_{F'} = \frac{n_{F'} - n_e}{n_{F'} - n_{C'}}.$$

glasses attainable in Poland (86 glasses). If the correction is visual, then $a = 0.542862$, $b = -0.000561937$.

The substitution of the formula (25) in the formulae (24) diminishes the number of the parameters determining the optical glass to two, n_e and ν_e, and consequently

$$n_e = n_e,$$
$$n_{F'} = (1+b)n_e + \frac{(n_e-1)a}{\nu_e} - b, \tag{26}$$
$$n_{C'} = (1+b)n_e - \frac{(n_e-1)(1-a)}{\nu_e} - b.$$

The two-parameter system of the change of optical glasses applied in this paper is based on the change of the parameters n_e and ν_e by the increments Δn_e and $\Delta \nu_e$. In that manner the changes of the glasses can be observed on the Abbe's diagram.

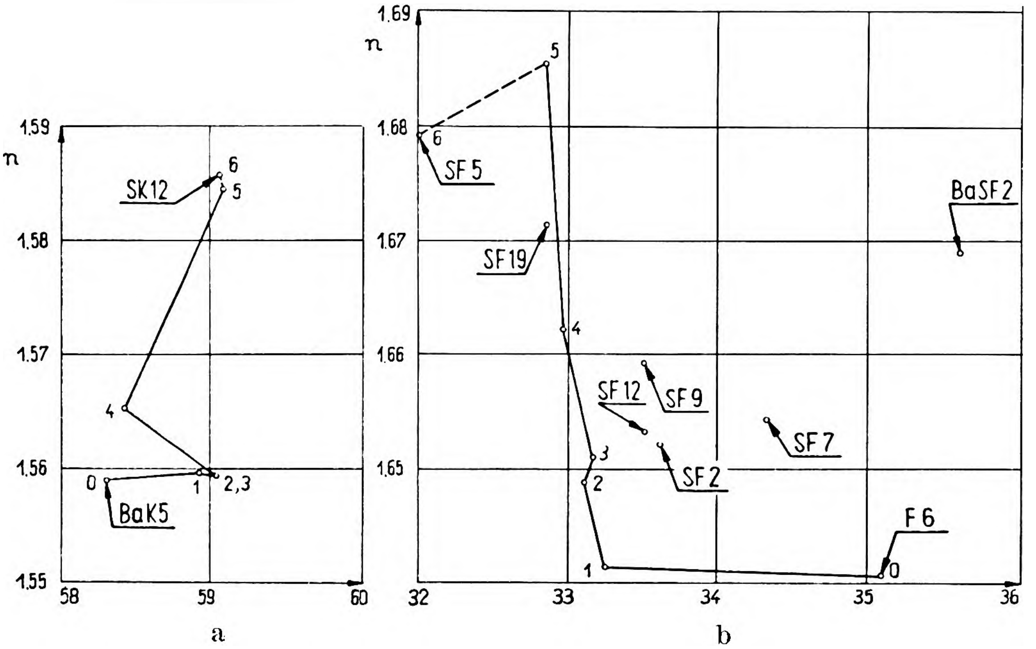

Fig. 3. The diagrams of the zigzac line linking the fictitious glasses calculated during the correction process of the doubled cemented lens $f100/2.8$; a) for the crown, b) for the flint

The inverse relation is defined as follows

$$n_e = n_e,$$
$$n_{F'} = n_e + \frac{n_e - 1}{\nu_e} P_{F'}, \tag{24}$$
$$n_{C'} = n_e - \frac{n_e - 1}{\nu_e}(1 - P_{F'}).$$

The partial relative dispersion $P_{F'}$ is not an independent parameter in the case of classical optical glasses. For these glasses there is a linear relationship between $P_{F'}$ and ν_e represented by the formula

$$P_{F'} = a + b\nu_e, \tag{25}$$

where

a, b — are constants for all classical glasses.

The constants a and b were calculated by the least-squares method for all the classical glasses attainable in Poland (86 glasses).

By differentiation of the formulas (26) we obtain the relation between the increments of the refractive indices Δn_e, $\Delta n_{F'}$ and $\Delta n_{C'}$ and the increments of the parameters Δn_e and $\Delta \nu_e$.

The increment Δn_e bears the consequence of the change of refractive indices

$$\Delta n_e = \Delta n_e$$
$$\Delta n_{F'} = \left(1 + b + \frac{a}{\nu_e}\right)\Delta n_e, \tag{27}$$
$$\Delta n_{C'} = \left(1 + b - \frac{1-a}{\nu_e}\right)\Delta n_e.$$

On the other side the increment $\Delta\nu_e$ carries on the following increments

$$\Delta n_e = 0,$$

$$\Delta n_{F'} = -\frac{(n_e-1)a}{\nu_e^2}\Delta\nu_e, \qquad (28)$$

$$\Delta n_{C'} = \frac{(n_e-1)(1-a)}{\nu_e^2}\Delta\nu_e.$$

The formulas (26, 27, 28) are realized automatically by the program AUKO during the correction process. These formulas given as an example for the visual correction can be easily modified so as to be right for the photographic correction.

When changing the optical glass parameters we assume that the whole area occupied by the glasses on the Abbe's diagram is covered in a continuous manner by an infinite number of fictitious optical glasses. After the calculation of the parameters of fictitious glass during the correction process it is not necessary to round off the parameters of the glass calculated for the real glass (from the catalogue), because very often we observe then the secondary deterioration of the characteristics of the optical system. The rounding off of the parameters of the fictitious glass is indispensable towards the end of the correction process. It is desirable to hit at the real glass with possible small deviation, in which the formulas (21) should be helpful. The change of glasses in the succeeding iterative steps may be visualized on the Abbe's diagram as a zigzac line marked through the points corresponding to the parameters of the successive fictitious glasses (Fig.3).

Examples of the optimization of aberration characteristics

Doubled cemented lens

System data: visual correction for the lines e, F', C' focal lenth $f = 100$ mm, aperture 1 : 2.8, object at infinity, glass type — crown at the front. The system parameters for the nominal system (iteration 0) were assumed as the glass combination BaK 5 and F6 (after Schott catalogue) and for economy reasons two identical rays and plane surface.

The correction process was conducted in changed conditions. The correction equations concerned the aberration characteristics of the aperture bundle and supplementary correction equations such as: the equation for the focal length, the equation for the minimum thickness at the edge of positive lens (16) and the equation of the change of the glass of flint type (20). The thickness of the negative lens was not changed. The table 2 contains the system parameters of the double cemented lens $f100/2.8$ acquired as the result of the successive iterations of the correction process. The table 2 proves that the system parameters during the correction process suffered radical changes.

The table 3 illustrates the aberrations of this lens for the same successive iterations. The figures in the table 3 permit to get an orientation on the advances in the optimization of aberration characteristics.

The weight coefficients of the correction parameters and of the correction equations were admitted during the correction process so

Table 2

Parameters of the optical system of the doubled cemented lens $f100/2.8$ acquired during the successive interations of the correction process

Kind of parameter	No. iteration 0	1	2	3	4	5	6
r_1	47.732	58.809	60.983	63.034	63.782	65.246	65.938
r_2	−47.732	−43.543	−42.599	−42.422	−42.984	−44.916	−43.626
r_3	∞	−275.21	−215.17	−190.81	−188.58	−201.75	−202.62
d_1	8.7	10.24	10.02	10.11	10.21	10.2	10.2
d_2	3.2	3.2	3.2	3.2	3.2	3.2	3.2
n_1	1.55897	1.55990	1.559382	1.559144	1.565301	1.584566	1.58547
ν_1	58.2868	58.9243	59.02178	59.0232	58.4094	59.0804	59.0787
n_2	1.64062	1.64131	1.648843	1.651133	1.66218	1.685565	1.67764
ν_2	35.1217	33.2497	33.1225	33.1604	32.9824	32.8549	32.0094

Table 3

The selected geometrical aberrations of the doubled cemented lens $f100/2.8$ acquired in the result of successive iterations of the correction process

Name of aberration	No. iteration						
	0	1	2	3	4	5	6
Spherical longitudinal aberration for maximum aperture zone (mm)	−0.9207	−0.1446	+0.0915	+0.0480	+0.0765	−0.0351	−0.0197
Spherical longitudinal aberration for aperture zone 0.707 h_{max} (mm)	−0.6111	−0.2616	−0.1640	−0.1829	−0.1633	−0.1888	−0.1891
Departure from the sine condition for maximum aperture zone (%)	−2.2894	−0.5199	−0.1136	−0.01057	+0.0466	−0.0233	+0.0287
Departure from the sine condition for aperture zone 0.707 h_{max} (%)	−1.2501	−0.4089	−0.2254	−0.1734	−0.1407	−0.1507	−0.1321
Sferochromatic aberration for maximum aperture zone (mm)	−0.0126	+0.2186	+0.2229	+0.1808	+0.1384	+0.1677	+0.3484
Sferochromatic aberration for the aperture zone 0.707 h_{max} (mm)	−0.2034	−0.0150	−0.0139	−0.0487	−0.0824	−0.0388	+0.1116
Chromatic axial aberration for paraxial space (mm)	−0.3493	−0.1901	−0.1905	−0.2203	−0.2486	−0.1974	−0.0686

as to realize the tactics of the damping in the different manner the changes of the glass parameters and the thickness of the positive lens, and to emphasis the aberrations conected with the higher aperture zones.

The initial increments of correction parameters were diminished by the successive iterations of the correction process.

The rounding off of the fictitious optical glasses took place after the 5-th iteration. The fictitious crown without difficulty could be aproximated by the crown SK12 with little deviation. The rounding off to the catalogue parameters of the fictitious crown required a fairly great step, with the result of the deterioration of the final spherochromatic aberration (table 3) iteration 5 and 6.

The Figure 3 provides the diagrams of the zigzac line linking the fictitious glasses calculated during the corrections process of the doubled cemented lens $f100/2.8$.

Projection lens

System data: visual correction for the lines e, F', C' focal length $f = 70$ mm, aperture 1 : 1.6, object at infinity, the object angular field of view $w = 10.46°$ and linear coefficient of vignetting $\eta = 0.8$ (without vignetting $\eta = 1$). The system parameters for the initial system were modified after a lens described by A. Cox [22]. The optical scheme of the projection lens $f70/1.6$ is on the Figure 4. The correction process was conducted assuming constant glass parameters and some surfaces separations. The correction parameters constituted 10 curvatures (without the curvature $\varrho_9 = 0$) and some

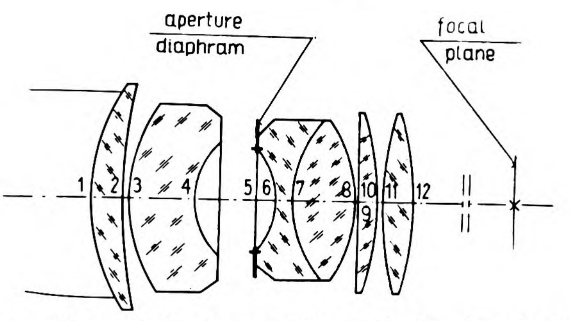

Fig. 4. The optical scheme of the projection lens $f70/1.6$

surfaces separations depending on the iteration. The number of correction parameters oscillated from 11 to 13, the number of correction equations — from 18 to 19.

The correction process was continued by 4 iterations. During the correction the curvatures ϱ_{11} i ϱ_{12} were kept close together. In the last iteration these parameters with equalized values appeared as a group change.

In the table 4 are presented the magnitudes of the selected geometrical aberrations of the projection lens $f70/1.6$. This example illustrates the semiautomatic correction of the aberrations of an optical system slightly declined from the state near to the optimum.

Table 4

The selected geometrical aberrations of the projection lens $f100/1.6$ acquired in the result of successive iterations of the correction process

Name of aberration	No. iteration 0	1	2	3	4
Spherical longitudinal aberration for maximum aperture zone (mm)	−1.1526	0.3967	−0.0255	0.0195	0.0339
Spherical longitudinal aberration for aperture zone 0.707 h_{max} (mm)	−0.5975	0.0408	−0.1564	−0.1482	−0.1537
Departure from the sine condition for maximum aperture zone (%)	−1.3078	0.3580	−0.0191	0.1309	0.1934
Departure from the sine condition for aperture zone 0.707 h_{max} (%)	−0.6920	0.0142	−0.1699	−0.1196	−0.1076
Sferochromatic aberration for the aperture zone 0.866 h_{max} (mm)	−0.0850	0.1007	0.0816	0.1074	0.0754
Chromatic axial aberration for paraxial space (mm)	−0.2840	−0.1831	−0.1980	−0.1883	−0.1998
Longitudinal sagittal curvature for maximum field angle (mm)	0.3997	−0.1047	−0.03949	−0.05595	−0.08744
Longitudinal sagittal curvature for maximum field angle (mm)	0.0939	−0.0764	−0.06278	−0.06805	−0.08798
Astigmatism for maximum field angle (mm)	0.3059	−0.0283	0.02329	0.01210	0.000542
Astigmatism for field zone 0.707 tan w_{max} (mm)	0.1718	−0.00779	0.01747	0.01094	0.004264
Meridional coma for aperture bundle 0.866 h and zone 0.707 tan w_{max} (%)	−1.3526	−0.5927	−0.4484	−0.1351	0.06549
Meridional coma for aperture bundle 0.707 and maximum field zone (%)	−1.2051	−0.6367	−0.5485	−0.3662	−0.2387
Distorsion for maximum field angle (%)	−1.2448	−1.2609	−1.2436	−1.2349	−1.2176
Mean longitudinal field curvature for maximum field angle (mm)	0.2468	−0.09054	−0.05113	−0.0620	−0.08771
Lateral chromatic aberration for maximum field angle (mm)	0.01244	−0.005898	−0.001404	−0.003045	0.0000526
Lateral chromatic aberration for field zone 0.707 tan w_{max} (mm)	0.00785	−0.004659	−0.001622	−0.002844	−0.0006592

Conclusions

A semiautomatic method of aberration correction in optical systems was elaborated to be applied for optimization of different kinds of optical systems, which can work in diverse "working conditions" with any fields of view or apertures specified in several ways.

The advantage consists in solving in a manner as accurate as possible the several parts of the correction process such as getting rid of the faults and inaccuraties of numerical methods. Therefore the main care rested upon the exact determination of: aberrations of principal rays for one and more colour, field marginal rays aberrations, aberrations increments and changes of parameters of optical glasses.

Another advantage of the method lies in facilitating and simplifying the work connected with the process as a whole. The automation of "working conditions" of the optical system, the facility of introducing any suitably combined changes of parameters serve to that purpose. Without any additional trouble a great lot of information on the aberration and trans-aberration characteristics of the optical system can be abtained and the aberration changes under the influence of any changes of the correction parameters.

The possible interventions of lens designer during the correction process enable an elastic adaptation of correction process to the concrete optical system being optimized. Thanks to the division of the process into its component parts in each iteration we are able to change equations, parameters, magnitudes of wanted aberrations, weight coefficients and criteria of appreciation. In that manner a variable tactics of conducting the process can be realized. This tactic results from the concrete aberration properties of the optimized optical system and is stated by the lens designer on the basis of experience. It is a rule that the automatic correction processes exhibit a fixed tactic which can be only slightly changed by the entrance data.

The modified least-squares method applied to the semiautomatic process is free of criticized faults [23]. In that work by means of special supplementary correction equations the parameter changes can be limited or specified so as to make possible the physical reality of the optical system.

О полуавтоматическом методе корригирования абберации в оптических системах

Разработан полуавтоматический метод корригирования абберации в оптических системах с применением компутера „Национал Эльот 803-В". Преимуществом этого метода являются его универсальность и несложность применения. Для иллюстрации приводятся примеры двух оптических систем, оптимальные значения которых были получены этим методом.

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