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## ON THE STATE OF BUDGETARY BALANCE OVER TIME VIA THE ONE-WAY CLASSIFICATION MODEL

A study on the state of budgetary balance over time in an economy has been conducted. The planned revenues and expected expenditures contained in the budget statements over the years are used as economic instruments for the study. The one-way classification model in statistical theory is used as the theoretical underpinning to describe the budget equation. The economic implications of the signs of the fixed effects in the model are stated.

Keywords: *budget, expenditure, fixed effects, one-way classification model, zero-sum constraint*

### 1. Introduction

This study is aimed at analyzing the state of budgetary balance in an economy using the one-way classification model. This aim is achieved by creating a statistical platform to test the significance of the state of budgetary balance over time. The study considers a scenario where the budget is a means of influencing economic activities. The budget, which is a statement of the expected revenues and planned expenditures, consists of two sides: the revenue and the expenditure. The revenue side of the budget in Nigeria comprises all the expected receipts, which include both oil and non-oil revenues. Of these two, the oil revenue forms the major part of the revenue to the Federation Account [9], and the state's, in turn, share from the Federation Account based on the revenue allocation formula [4, 10]. The expenditure side of the budget is normally divided into two categories: recurrent expenditures and capital expenditures. Details on the categorization of expenditures are found in [3]. Andersen et al. [2] provided

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a rationale for state governments to avoid lengthy delays in the budget process, so as to improve fiscal governance. Kameda [7] analyzed the effect of budget deficits and government debt on real long-term interest rates and concluded that the projected deficit is more important than the current deficit and that budget deficits have larger effects than government debt. The relationship between external balance and government budget balance has been studied [1]. Algieri [1] showed that there are no systematic linkages between budget and current account deficits or between budget deficit and trade balance deficits. Falco-Gimeno and Jurado [5] focused on the political explanations for budget deficits. More specifically, the role of the opposition in the passage of a budget was considered. In a study conducted by Ramsey and Hackbart [11], it was reported that budget reform has a reasonable impact on planning and output effectiveness. What is more, the economic drive of a government is seen from the budget. For instance, when the government reduces recurrent expenditures such as personnel costs, and limits capital intensive projects, then the government drives towards achieving a surplus. We use the fixed effects of the one-way classification model [8] as a proxy for the economic drive over time. We assume that the sum of the fixed effects is zero. This assumption rhymes with the zero-sum constraint in the experimental design literature [12]. This study would be of interest to public finance analysts, researchers and those saddled with the responsibility of budget formulation and economic planning.

## 2. Method

This section contains the theoretical framework for the study. We study the pattern of budgets over time. By so doing, we gain some insights into the state of the budgetary balance. These insights form the basis of the propositions and comments on the economic implications in the last two subsections.

### 2.1. Description of the model

We divide the budget into three components: revenue, recurrent expenditure and capital expenditure. We index these components of the budget by  $i$ , where  $i = 1$  denotes the revenue,  $i = 2$  represents recurrent expenditure and  $i = 3$  stands for capital expenditure. Let  $y_{it} \geq 0$  be the budgeted amount for the index  $i$  at period  $t$ , and  $BB_t = y_{1t} - (y_{2t} + y_{3t})$  the state of the budgetary balance at period  $t$ . If

$$y_{1t} = y_{2t} + y_{3t} \quad (1)$$

then the budget is balanced for the fiscal period  $t$  and  $BB_t = 0$ . On the other hand,

$$y_{1t} < y_{2t} + y_{3t} \quad (2)$$

and

$$y_{1t} > y_{2t} + y_{3t} \quad (3)$$

imply a deficit and a surplus in the budget, where  $BB_t < 0$  and  $BB_t > 0$ , respectively. In practice, Eq. (1) may not hold [7]. Instead, the budget may be balanced over time, say a period of length  $T$ . When this is the case, then

$$\sum_{t=1}^T y_{1t} = \sum_{t=1}^T y_{2t} + \sum_{t=1}^T y_{3t} . \quad (4)$$

We model the budgeted amount  $y_{it} \geq 0$  over time using the one-way classification model of the form

$$y_{it} = \eta + \tau_i + \varepsilon_{it} \quad (5)$$

where  $\eta$  is the grand mean of the budgeted amount,  $\tau_i$  is the fixed effect of the amount budgeted for component  $i$ , and  $\varepsilon_{it}$  is the error term. The inclusion of the error term is to account for distortions to the budget owing to fiscal activities which are not captured earlier, such as grants, flexible exchange rates, funds outsourced to respond to emergencies, the bailing out of ailing industries, etc. Assembling all the components of the budget into a matrix-vector, we get

$$\mathbf{Y} = [\mathbf{e} \ \mathbf{X}] \boldsymbol{\beta} + \boldsymbol{\varepsilon} \quad (6)$$

where  $\mathbf{Y} = [y_{11} \ y_{21} \ y_{31} \ y_{12} \ y_{22} \ y_{32} \ \dots \ y_{3T}]'$ ,  $\mathbf{e}$  is a  $3T \times 1$  vector of ones,

$$\mathbf{X}' = \left[ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \dots, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right]$$

is the transpose of the  $3T \times 3$  matrix of regression vectors, wherein the vectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ are each repeated } T \text{ times,}$$

$$\boldsymbol{\beta} = [\eta \quad \tau_1 \quad \tau_2 \quad \tau_3]', \quad \text{and} \quad \boldsymbol{\varepsilon} = [\varepsilon_{11} \quad \varepsilon_{21} \quad \varepsilon_{31} \quad \varepsilon_{12} \quad \varepsilon_{22} \quad \varepsilon_{32} \quad \dots \quad \varepsilon_{3T}]'$$

The model in Eq. (6) is over-parameterized as there are four unknown parameters,  $\eta$ ,  $\tau_i$ ,  $i=1, 2, 3$ , whereas only three components of the budget are considered. Consequently, the vector  $\boldsymbol{\beta}$ , as well as the standard errors, cannot be estimated by the method of least squares, as the product  $[\mathbf{e} \mathbf{X}]'[\mathbf{e} \mathbf{X}]$  is singular. The common approach to circumvent this is to impose a zero-sum constraint on the model [12]. That is

$$\tau_1 + \tau_2 + \tau_3 = 0 \tag{7}$$

The trivial solution to Eq. (7) is that  $\tau_1 = \tau_2 = \tau_3 = 0$ . This is the solution when there is no significant difference in the amount budgeted over time. Thus the null hypothesis of interest is  $H_0 : \tau_1 = \tau_2 = \tau_3 = 0$  against the alternative hypothesis  $H_1$  that at least one  $\tau_i \neq 0$ . To study the state of budgetary balance, we set  $\tau_1 = -\tau_2 - \tau_3$ . We therefore simplify the matrix of regression vectors as follows:

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tau_1 + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tau_2 + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tau_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} (-\tau_2 - \tau_3) + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tau_2 + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tau_3 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \tau_2 + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \tau_3$$

In this light, the matrix of regression vectors becomes a  $3T \times 2$  matrix and its transpose is of the form

$$\widehat{\mathbf{X}}' = \left[ \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \dots, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right]$$

The parameter vector becomes  $\boldsymbol{\theta} = [\eta \quad \tau_2 \quad \tau_3]'$ . Applying the least squares method, we get

$$\hat{\boldsymbol{\theta}} = ([\mathbf{e} \widehat{\mathbf{X}}]'[\mathbf{e} \widehat{\mathbf{X}}])^{-1}[\mathbf{e} \widehat{\mathbf{X}}]'\mathbf{Y} \tag{8}$$

where the hat on  $\theta$  denotes estimate. Further simplifications lead to

$$\begin{bmatrix} \hat{\eta} \\ \hat{\tau}_2 \\ \hat{\tau}_3 \end{bmatrix} = \frac{1}{3T} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} \sum_{i=1}^3 \sum_{j=1}^T y_{ij} \\ \sum_{t=1}^T (y_{2t} - y_{1t}) \\ \sum_{t=1}^T (y_{3t} - y_{1t}) \end{bmatrix}$$

Therefore,

$$\hat{\eta} = \frac{1}{3T} \sum_{i=1}^3 \sum_{t=1}^T y_{it} \quad (9)$$

$$\hat{\tau}_2 = \frac{1}{3T} \sum_{t=1}^T (2y_{2t} - y_{1t} - y_{3t}) \quad (10)$$

and

$$\hat{\tau}_3 = \frac{1}{3T} \sum_{t=1}^T (2y_{3t} - y_{1t} - y_{2t}) \quad (11)$$

Using the zero-sum constraint (7), we get

$$\hat{\tau}_1 = \frac{1}{3T} \sum_{t=1}^T (2y_{1t} - y_{2t} - y_{3t}) \quad (12)$$

The test of the significance of deviations from the null hypothesis is achieved using the  $F$ -statistic

$$F = \frac{\left( \hat{\theta}' [\mathbf{e} \ \widehat{\mathbf{X}}]' [\mathbf{e} \ \widehat{\mathbf{X}}] \hat{\theta} - \frac{\mathbf{e}' \mathbf{Y} \mathbf{Y}' \mathbf{e}}{3T} \right)}{\frac{2}{\mathbf{Y} - [\mathbf{e} \ \widehat{\mathbf{X}}] \hat{\theta}' (\mathbf{Y} - [\mathbf{e} \ \widehat{\mathbf{X}}] \hat{\theta})} \frac{\hat{\theta}}{3T - 3}}$$

## 2.2. Propositions

**Proposition 1.** The fixed effect  $\hat{\tau}_1$  summarizes the state of budgetary balance over time.

**Proof.** If the budget is balanced over time, then we obtain from Eq. (12) that  $\hat{\tau}_1 = \frac{1}{3T} \sum_{t=1}^T y_{1t}$ . In the same vein,  $\hat{\tau}_1 > \frac{1}{3T} \sum_{t=1}^T y_{1t}$  and  $\hat{\tau}_1 < \frac{1}{3T} \sum_{t=1}^T y_{1t}$  indicate a surplus and a deficit in the budget over time, respectively.

**Proposition 2.** Whenever  $\hat{\tau}_1 < 0$ , then the budgets over time are in deficit.

**Proof.** Since  $\sum_{t=1}^T y_{1t} > 0$ , it follows from the proof of Proposition 1 that  $\hat{\tau}_1 < 0$  only when the budgets are in deficit over time.

**Proposition 3.** The estimates  $\hat{\tau}_2$  and  $\hat{\tau}_3$  can simultaneously be negative regardless of the state of budgetary balance, whereas  $\hat{\tau}_2 > 0$  and  $\hat{\tau}_3 > 0$  hold only when the budgets over time are in deficit.

**Proof.** Suppose  $\hat{\tau}_2 < 0$  and  $\hat{\tau}_3 < 0$ . Then

$$2 \sum_{t=1}^T y_{2t} - \sum_{t=1}^T y_{1t} - \sum_{t=1}^T y_{3t} < 0 \quad \text{and} \quad 2 \sum_{t=1}^T y_{3t} - \sum_{t=1}^T y_{1t} - \sum_{t=1}^T y_{2t} < 0$$

Thus

$$\sum_{t=1}^T y_{2t} + \sum_{t=1}^T y_{3t} - 2 \sum_{t=1}^T y_{1t} < 0$$

So that

$$\sum_{t=1}^T y_{1t} > \sum_{t=1}^T y_{2t} + \sum_{t=1}^T y_{3t} - \sum_{t=1}^T y_{1t}$$

When the budget is either in balance or deficit, we have that

$$\sum_{t=1}^T y_{1t} > \sum_{t=1}^T y_{2t} + \sum_{t=1}^T y_{3t} - \sum_{t=1}^T y_{1t} \geq 0$$

This implies that  $\sum_{t=1}^T y_{1t} > 0$ , which is true. Nonetheless, when the budget is in surplus, it follows from

$$\sum_{t=1}^T y_{2t} + \sum_{t=1}^T y_{3t} - 2\sum_{t=1}^T y_{1t} < 0$$

that

$$\sum_{t=1}^T y_{1t} + \left( \sum_{t=1}^T y_{1t} - \sum_{t=1}^T y_{2t} - \sum_{t=1}^T y_{3t} \right) > 0$$

Thus  $\hat{\tau}_2 < 0$  and  $\hat{\tau}_3 < 0$  can hold regardless of the state of budgetary balance. Now suppose  $\hat{\tau}_2 > 0$  and  $\hat{\tau}_3 > 0$ . Then

$$\left( \sum_{t=1}^T y_{2t} + \sum_{t=1}^T y_{3t} - \sum_{t=1}^T y_{1t} \right) - \sum_{t=1}^T y_{1t} > 0, \quad \text{or} \quad \sum_{t=1}^T y_{1t} + \left( \sum_{t=1}^T y_{1t} - \sum_{t=1}^T y_{2t} - \sum_{t=1}^T y_{3t} \right) < 0$$

Since

$$\sum_{t=1}^T y_{1t} > 0, \quad \sum_{t=1}^T y_{1t} + \left( \sum_{t=1}^T y_{1t} - \sum_{t=1}^T y_{2t} - \sum_{t=1}^T y_{3t} \right) < 0$$

is true only when

$$\sum_{t=1}^T y_{1t} - \sum_{t=1}^T y_{2t} - \sum_{t=1}^T y_{3t} < 0$$

**Proposition 4.** When  $\hat{\tau}_2 > 0$  and  $\hat{\tau}_3 < 0$ , then the recurrent expenditures exceed the capital expenditures over time and vice versa.

**Proof.** Suppose  $\hat{\tau}_2 > 0$  and  $\hat{\tau}_3 < 0$ . Then

$$2\sum_{t=1}^T y_{2t} - \sum_{t=1}^T y_{1t} - \sum_{t=1}^T y_{3t} > 0 \quad \text{and} \quad -2\sum_{t=1}^T y_{3t} + \sum_{t=1}^T y_{1t} + \sum_{t=1}^T y_{2t} > 0.$$

Therefore

$$\sum_{t=1}^T y_{2t} > \sum_{t=1}^T y_{3t}$$

Similarly, when  $\hat{\tau}_2 < 0$  and  $\hat{\tau}_3 > 0$ , then

$$-2 \sum_{t=1}^T y_{2t} + \sum_{t=1}^T y_{1t} + \sum_{t=1}^T y_{3t} > 0 \quad \text{and} \quad 2 \sum_{t=1}^T y_{3t} - \sum_{t=1}^T y_{1t} - \sum_{t=1}^T y_{2t} > 0.$$

Thus

$$\sum_{t=1}^T y_{3t} > \sum_{t=1}^T y_{2t}$$

### 2.3. The economic link

We state the economic implications of the signs of the estimated fixed effects whenever the null hypothesis is rejected. As postulated earlier,  $\hat{\tau}_1 < 0$  implies that the budgets over time are in deficit. On the other hand,  $\hat{\tau}_2 < 0$  and  $\hat{\tau}_3 < 0$  indicate that the economic policies employed limit the expenditures over time.  $\hat{\tau}_2 > 0$  and  $\hat{\tau}_3 < 0$  represent the situation where there is an increase in the recurrent expenditures, such as the wage bill, while limits are placed on capital intensive projects. Moreover,  $\hat{\tau}_2 < 0$  and  $\hat{\tau}_3 > 0$  denote the situation where there is a reduction in the recurrent expenditures (e.g. a cut in wages), whereas more capital intensive projects are financed over time. Finally, we comment on the link between the estimated fixed effects and counter-cyclical fiscal policies based on the state of an economy. An economy may experience a period of either economic boom or recession. During a period of recession, it is suggested that the budget should be in deficit. This is because a budget deficit is expansionary. Accordingly,  $\hat{\tau}_1 < 0$  and  $\hat{\tau}_2 + \hat{\tau}_3 > 0$  are consistent with a counter-cyclical fiscal policy during a recession. Conversely, a budget surplus is recommended during a period of economic boom. In this light, we conclude that  $\hat{\tau}_1 > \frac{1}{3T} \sum_{t=1}^T y_{1t}$  and  $\hat{\tau}_2 + \hat{\tau}_3 < 0$  are consistent with such a fiscal policy during an economic boom.





From Table 1, the state of budgetary balance for each year is in deficit, except for 2014, since

$$BB_1 = y_{11} - (y_{21} + y_{31}) = -0.43, \quad BB_2 = y_{12} - (y_{22} + y_{32}) = -6.02,$$

$$BB_3 = y_{13} - (y_{23} + y_{33}) = -0.03, \quad BB_4 = y_{14} - (y_{24} + y_{34}) = -0.01,$$

$$BB_5 = y_{15} - (y_{25} + y_{35}) = 0.38, \quad BB_6 = y_{16} - (y_{26} + y_{36}) = -0.08.$$

It is not surprising that the budgets are mostly in deficit over time, as the policy thrusts of the government require more spending. However, the mere computation of the state of budgetary balance for each year does not tell how significant this thrust is over time. In order to determine the significance of the state of the budgets over time, we employ the method described in Section 2. We carry out our analysis in Microsoft Excel. The results of the analysis are given in Table 2.

Table 2. Summary output

Regression statistics				
Multiple R	0.837455			
R square	0.701331			
Adjusted R square	0.661509			
Standard error	26.26746			
Observations	18			
ANOVA				
	<i>Df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>
Regression	2	24303.06	12151.53	17.6114
Residual	15	10349.69	689.9793	
Total	17	34652.75		
Coefficients				
Coefficient	Value	Standard error	<i>t</i> -Stat	<i>P</i> -value
$\hat{\eta}$	100.1572	6.191299	16.17709	$6.65 \times 10^{-11}$
$\hat{\tau}_2$	-38.3056	8.755819	-4.37487	0.000544
$\hat{\tau}_3$	-11.2572	8.755819	-1.28568	0.218048

The results indicate that over 66% of the variations in the budgeted amounts are explained by this one-way classification model and that the estimated fixed effects are jointly significant at the 0.1% level. The *t*-Stat values show that the fixed effect associated with capital expenditure,  $\hat{\tau}_3$ , is not significant at the 5% level, whereas  $\hat{\tau}_2$  and  $\hat{\eta}$  are, even at the 0.1% level. The fixed effect  $\hat{\tau}_1$  is estimated as  $\hat{\tau}_1 = 49.56$ . Furthermore, the budget is in deficit over time as  $\hat{\tau}_1 = 49.56 < \frac{1}{18} \sum_{t=1}^6 y_{1t} = 49.91$ . This rhymes with the earlier results on the state of the budgets. We conclude that the budget of the state government is in deficit over the six-year period and that restraints are

imposed on the expenditures, as  $\hat{\tau}_2 < 0$  and  $\hat{\tau}_3 < 0$ . However, this conclusion should be treated with care, as the model does not consider budget performance.

#### 4. Concluding remarks

In this study, an attempt has been made to describe the state of budgetary balance over time using the one-way classification model. Our approach is novel, as it is not based on a simple balancing of the revenue and the expenditure within a period of time. The length of a period may be a fiscal cycle or the tenure of a government of a political entity. Some aspects of public finance relating to budgets are illustrated by the estimated fixed effects in the model. The study provides an insight on whether or not budgets over time achieve a counter-cyclical fiscal policy based on the estimated fixed effects obtained using data on budgeted amounts over a period and the test of the hypothesis  $H_0 : \tau_1 = \tau_2 = \tau_3 = 0$ . The major achievements of the study are the formulation of a statistical model to describe government budgets over time in an economy and the use of the model to adjudge the state of the budgetary balance over a period of time. The application of the model to real-life data validates its usefulness.

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