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The kurtosis parameter of partially coherent controllable dark hollow beams in free space

JINHONG LI, WEIWEI WANG, YUNZHONG LAI*

Department of Physics, Taiyuan University of Science and Technology,
Taiyuan 030024, China

*Corresponding author: sculijh@qq.com

Based on the extended Huygens–Fresnel principle and the definition of second-order moments of the Wigner distribution function, the analytical expression for the kurtosis parameter of partially coherent controllable dark hollow beams in free space is derived, and used to study the influence of beam parameters on the kurtosis parameter. It is shown that the kurtosis parameter increases with increasing the wavelength and the waist width, or decreasing the scaling factor, however, the effect of the beam order and the spatial correlation length on the kurtosis parameter depends on the propagation distance. The results can be interpreted physically.

Keywords: kurtosis parameter, partially coherent controllable dark hollow beams, free space.

1. Introduction

Optical beams with zero central intensity are called dark hollow beams (DHBs). In the recent years, DHBs and their propagation characteristics have attracted much attention because of their increasing applications in the field of atom optics [1–7]. DENG *et al.* investigated the far field intensity distribution of DHBs based on the scalar diffraction theory [2]. EYYUBOGLU reported the power in the bucket and the degree of coherence for higher order partially coherent DHBs in atmospheric turbulence [4]. Based on the generalized Rayleigh–Sommerfeld diffraction integrals, YUAN *et al.* obtained the nonparaxial propagation formulas of a vector partially coherent DHB for the elements of the cross-spectral density matrix in free space [6]. MEI and ZHAO introduced a new mathematical model called controllable dark hollow beams (CDHBs) to describe DHBs, and thereby also derived the analytical propagation formula for CDHBs through a paraxial optical system [8]. The propagation properties of fully coherent and partially coherent CDHBs propagating through free space [9], atmospheric turbulence [10], circular aperture [11] and hard-edged aperture [12] have also been extensively studied.

The kurtosis parameter written in terms of the fourth- and second-irradiance moments can be used to describe the degree of sharpness (or flatness) of any beam, so it is an important parameter to evaluate the beam propagation [13–21]. The irradiance profile of beams is classified as platykurtic, mesokurtic or leptokurtic, according to the kurtosis parameter K being less, equal or larger than 3 [13]. As yet the kurtosis parameter of Gaussian beams, flat-topped beams, Bessel–Gauss beams, Lorentz–Gauss beams, partially coherent cosh-Gaussian beam, Ince–Gauss beams, annular beams, *etc*., have been derived [13–21]. More recently, Eyyuboglu analyzed the kurtosis parameter of Ince–Gauss beams propagating in turbulent atmosphere [20]. The kurtosis parameter of decentered annular beams was dealt with numerically by XIAOQING LI and XIAOLING JI who found that the kurtosis parameter depends on a turbulence parameter [21]. However, as far as we know, the research on the kurtosis parameter of partially coherent CDHBs has not been reported elsewhere. In this paper, the kurtosis parameter of partially coherent CDHBs in free space will be studied. Some interesting results are also given and discussed.

2. Theoretical formulation

The field at the source plane $z = 0$ of fully coherent CDHBs is expressed as [8]

$$E_N(\mathbf{s}, 0) = \sum_{n=1}^{N} \frac{(-1)^{n-1}}{N} \binom{N}{n} \left[\exp\left(-\frac{n\mathbf{s}^2}{w_0^2}\right) - \exp\left(-\frac{n\mathbf{s}^2}{p^2 w_0^2}\right) \right] \tag{1}$$

where $\mathbf{s} = (s_x, s_y)$ is the two-dimensional position vector at the plane $z = 0$, $\binom{N}{n}$ denotes the binomial coefficient, N is the beam order of the CDHBs, w_0 denotes the waist width of the Gaussian part, p $(0 < p < 1)$ is the scaling factor for controlling the dark size of the CDHBs. When $N > 1$ and $p \to 0$, Eq. (1) reduces to the field of flat topped beams at the source plane $z = 0$. When $N = 1$ and $p \to 0$, Eq. (1) becomes the field of the Gaussian beam at the source plane $z = 0$.

Introducing the Schell-model correlator, we can express the cross-spectral density function [22] of partially coherent CDHBs at the source plane $z = 0$ as

$$W(\mathbf{s}_1, \mathbf{s}_2, 0) = \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{(-1)^{n+m}}{N^2} \binom{N}{n} \binom{N}{m} \left[\exp\left(-\frac{n\mathbf{s}_1^2}{w_0^2}\right) - \exp\left(-\frac{n\mathbf{s}_1^2}{p^2 w_0^2}\right) \right] \times$$
$$\times \left[\exp\left(-\frac{m\mathbf{s}_2^2}{w_0^2}\right) - \exp\left(-\frac{m\mathbf{s}_2^2}{p^2 w_0^2}\right) \right] \exp\left[-\frac{(\mathbf{s}_1 - \mathbf{s}_2)^2}{2\sigma_0^2}\right] \tag{2}$$

where $\mathbf{s}_i = (s_{ix}, s_{iy})$ $(i = 1, 2)$ specifies the two-dimensional vector in the plane $z = 0$, σ_0 denotes the spatial correlation length, for $\sigma_0 \to \infty$, Eq. (2) reduces to the cross-spectral density function of fully coherent CDHBs.

In accordance with the extended Huygens–Fresnel principle [22], the cross-spectral density function of partially coherent CDHBs propagating through free space is given by

$$W(\boldsymbol{\rho}, \boldsymbol{\rho}_d, z) = \left(\frac{k}{2\pi z}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W(\mathbf{s}, \mathbf{s}_d, 0) \exp\left[\frac{ik}{z}(\boldsymbol{\rho} - \mathbf{s})(\boldsymbol{\rho}_d - \mathbf{s}_d)\right] \mathrm{d}^2\mathbf{s}\, \mathrm{d}^2\mathbf{s}_d \quad (3)$$

where z is the propagation distance, k is the wave number related to the wavelength λ by $k = 2\pi/\lambda$. In Eq. (3) we have used the following sum and difference vector notation:

$$\boldsymbol{\rho} = \frac{\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2}{2}, \quad \boldsymbol{\rho}_d = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2, \quad \mathbf{s} = \frac{\mathbf{s}_1 + \mathbf{s}_2}{2}, \quad \mathbf{s}_d = \mathbf{s}_1 - \mathbf{s}_2 \quad (4)$$

where $\boldsymbol{\rho}_i = (\rho_{ix}, \rho_{iy})$ is the position vector in the z plane.

After some operations as shown in [5, 23], Eq. (3) can be expressed in the following alternative form

$$W(\boldsymbol{\rho}, \boldsymbol{\rho}_d, z) = \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W\left(\mathbf{s}', \boldsymbol{\rho}_d + \frac{z}{k}\boldsymbol{\kappa}_d, 0\right) \times$$

$$\times \exp(-i\boldsymbol{\rho} \cdot \boldsymbol{\kappa}_d + i\mathbf{s}' \cdot \boldsymbol{\kappa}_d)\mathrm{d}^2\mathbf{s}'\mathrm{d}^2\boldsymbol{\kappa}_d \quad (5)$$

where $\boldsymbol{\kappa}_i = (\kappa_{dx}, \kappa_{dy})$ is the two-dimensional position vector in spatial frequency domain. For the partially coherent CDHBs, using Eq. (2), we can write the cross-spectral density function as

$$W\left(\mathbf{s}', \boldsymbol{\rho}_d + \frac{z}{k}\boldsymbol{\kappa}_d, 0\right) = \sum_{n=1}^{N}\sum_{m=1}^{N} \frac{(-1)^{n+m}}{N^2}\binom{N}{n}\binom{N}{m} \times$$

$$\times \left\{ \exp\left[A_1 \mathbf{s}'^2 - A_2 \mathbf{s}'\left(\boldsymbol{\rho}_d + \frac{z}{k}\boldsymbol{\kappa}_d\right) - Q_A\left(\boldsymbol{\rho}_d + \frac{z}{k}\boldsymbol{\kappa}_d\right)^2\right] + \right.$$

$$- \exp\left[B_1 \mathbf{s}'^2 - B_2 \mathbf{s}'\left(\boldsymbol{\rho}_d + \frac{z}{k}\boldsymbol{\kappa}_d\right) - Q_B\left(\boldsymbol{\rho}_d + \frac{z}{k}\boldsymbol{\kappa}_d\right)^2\right] +$$

$$- \exp\left[C_1 \mathbf{s}'^2 - C_2 \mathbf{s}'\left(\boldsymbol{\rho}_d + \frac{z}{k}\boldsymbol{\kappa}_d\right) - Q_C\left(\boldsymbol{\rho}_d + \frac{z}{k}\boldsymbol{\kappa}_d\right)^2\right] +$$

$$\left. + \exp\left[D_1 \mathbf{s}'^2 - D_2 \mathbf{s}'\left(\boldsymbol{\rho}_d + \frac{z}{k}\boldsymbol{\kappa}_d\right) - Q_D\left(\boldsymbol{\rho}_d + \frac{z}{k}\boldsymbol{\kappa}_d\right)^2\right]\right\} \quad (6)$$

where

$$A_1 = \frac{1}{w_0^2}(n + m) \quad (7a)$$

$$B_1 = \frac{1}{w_0^2}\left(n + \frac{m}{p^2}\right) \tag{7b}$$

$$C_1 = \frac{1}{w_0^2}\left(\frac{n}{p^2} + m\right) \tag{7c}$$

$$D_1 = \frac{1}{w_0^2}\left(\frac{n}{p^2} + \frac{m}{p^2}\right) \tag{7d}$$

$$Q_A = \frac{A_1}{4} + \frac{1}{2\sigma_0^2} \tag{7e}$$

due to the symmetry, A_2, B_2, C_2 and D_2 can be obtained by replacing "+" in A_1, B_1, C_1 and D_1 with "–", respectively. Q_B, Q_C and Q_D can be obtained by replacing A_1 in Eq. (7e), with B_1, C_1 and D_1, respectively.

By means of the cross-spectral density function, the Wigner distribution function (WDF) of partially coherent CDHBs propagating through free space can be expressed as [23]

$$h(\boldsymbol{\rho}, \boldsymbol{\theta}, z) = \left(\frac{k}{2\pi}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W(\boldsymbol{\rho}, \boldsymbol{\rho}_d, z)\exp(-ik\boldsymbol{\theta}\cdot\boldsymbol{\rho}_d)\mathrm{d}^2\boldsymbol{\rho}_d \tag{8}$$

where vector $\boldsymbol{\theta} = (\theta_x, \theta_y)$, $k\theta_x$ and $k\theta_y$ are the wave vector components along the x-axis and y-axis, respectively.

The substitution from Eqs. (5), (6) and (7) into Eq. (8) yields

$$\begin{aligned} h(\boldsymbol{\rho}, \boldsymbol{\theta}, z) = {} & \frac{k^2}{16\pi^3}\sum_{n=1}^{N}\sum_{m=1}^{N}\frac{(-1)^{n+m}}{N^2}\binom{N}{n}\binom{N}{m}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{d}\boldsymbol{\rho}_d^2\mathrm{d}\boldsymbol{\kappa}_d^2 \times \\ & \times\left[\frac{\exp(-F_A\boldsymbol{\rho}_d^2 - G_A\boldsymbol{\kappa}_d^2 + H_A\boldsymbol{\rho}_d\cdot\boldsymbol{\kappa}_d - ik\boldsymbol{\theta}\cdot\boldsymbol{\rho}_d - i\boldsymbol{\rho}\cdot\boldsymbol{\kappa}_d)}{A_1} + \right. \\ & - \frac{\exp(-F_B\boldsymbol{\rho}_d^2 - G_B\boldsymbol{\kappa}_d^2 + H_B\boldsymbol{\rho}_d\cdot\boldsymbol{\kappa}_d - ik\boldsymbol{\theta}\cdot\boldsymbol{\rho}_d - i\boldsymbol{\rho}\cdot\boldsymbol{\kappa}_d)}{B_1} + \\ & - \frac{\exp(-F_C\boldsymbol{\rho}_d^2 - G_C\boldsymbol{\kappa}_d^2 + H_C\boldsymbol{\rho}_d\cdot\boldsymbol{\kappa}_d - ik\boldsymbol{\theta}\cdot\boldsymbol{\rho}_d - i\boldsymbol{\rho}\cdot\boldsymbol{\kappa}_d)}{C_1} + \\ & \left. + \frac{\exp(-F_D\boldsymbol{\rho}_d^2 - G_D\boldsymbol{\kappa}_d^2 + H_D\boldsymbol{\rho}_d\cdot\boldsymbol{\kappa}_d - ik\boldsymbol{\theta}\cdot\boldsymbol{\rho}_d - i\boldsymbol{\rho}\cdot\boldsymbol{\kappa}_d)}{D_1}\right] \end{aligned} \tag{9}$$

where

$$F_A = Q_A - \frac{A_2^2}{4A_1} \tag{10a}$$

$$G_A = \frac{Q_A z^2}{k^2} + \frac{1}{4A_1} - \frac{A_2^2 z^2}{4A_1 k^2} + \frac{iA_2 z}{2kA_1} \tag{10b}$$

$$H_A = \frac{A_2^2 z}{2A_1 k} - \frac{2zQ_A}{k} - \frac{iA_2}{2A_1} \tag{10c}$$

In consideration of the symmetry, F_B (F_C, F_D), G_B (G_C, G_D) and H_B (H_C, H_D) can be obtained by replacing A_1, A_2 and Q_A in F_A, G_A and H_A, with B_1 (C_1, D_1), B_2 (C_2, D_2) and Q_B (Q_C, Q_D), respectively.

The kurtosis parameter can describe the sharpness of any beam and is defined as the ratio of the fourth-order moment to the square of the second-order moment. Based on the WDF, the kurtosis parameter K is defined as

$$K = \frac{\langle \rho_x^4 \rangle}{\langle \rho_x^2 \rangle^2} \tag{11}$$

where

$$\langle \rho_x^4 \rangle = \frac{1}{P} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho_x^4 h(\boldsymbol{\rho}, \boldsymbol{\theta}, z) \mathrm{d}^2\rho\, \mathrm{d}^2\theta \tag{12a}$$

$$\langle \rho_x^2 \rangle = \frac{1}{P} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho_x^2 h(\boldsymbol{\rho}, \boldsymbol{\theta}, z) \mathrm{d}^2\rho\, \mathrm{d}^2\theta \tag{12b}$$

$$P = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\boldsymbol{\rho}, \boldsymbol{\theta}, z) \mathrm{d}^2\rho\, \mathrm{d}^2\theta \tag{12c}$$

On substituting from Eq. (9) into Eq. (12), one obtains

$$\langle \rho_x^4 \rangle = \frac{12\pi}{P} \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{(-1)^{n+m}}{N^2} \binom{N}{n} \binom{N}{m} \left(\frac{G_A^2}{A_1} - \frac{G_B^2}{B_1} - \frac{G_C^2}{C_1} + \frac{G_D^2}{D_1} \right) \tag{13a}$$

$$\langle \rho_x^2 \rangle = \frac{12\pi}{P} \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{(-1)^{n+m}}{N^2} \binom{N}{n} \binom{N}{m} \left(\frac{G_A}{A_1} - \frac{G_B}{B_1} - \frac{G_C}{C_1} + \frac{G_D}{D_1} \right) \tag{13b}$$

$$P = \pi \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{(-1)^{n+m}}{N^2} \binom{N}{n} \binom{N}{m} \left(\frac{1}{A_1} - \frac{1}{B_1} - \frac{1}{C_1} + \frac{1}{D_1} \right) \tag{13c}$$

On substituting from Eq. (13) into Eq. (11), we obtain the expression

$$K = 3 \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{\left(\frac{G_A^2}{A_1} - \frac{G_B^2}{B_1} - \frac{G_C^2}{C_1} + \frac{G_D^2}{D_1} \right) \left(\frac{1}{A_1} - \frac{1}{B_1} - \frac{1}{C_1} + \frac{1}{D_1} \right)}{\left(\frac{G_A}{A_1} - \frac{G_B}{B_1} - \frac{G_C}{C_1} + \frac{G_D}{D_1} \right)^2} \tag{14}$$

Equation (14) is the analytical expression for the kurtosis parameter of partially coherent CDHBs, which depends on the scaling factor p, beam order N, waist width w_0, wavelength λ and spatial correlation length σ_0. For $N > 1$ and $p \to 0$, Eq. (14) will reduce to the expression for the kurtosis parameter of flat topped beams in free space. For $N = 1$, $p \to 0$, and $\sigma_0 \to \infty$, Eq. (14) becomes the expression for the kurtosis parameter of fully coherent Gaussian beams in free space, which can be expressed as

$$K = 3 \tag{15}$$

which is the well-known result.

For the source plane $z = 0$, the analytical expression of the kurtosis parameter of partially coherent CDHBs is expressed as

$$\begin{aligned} K = 3 \sum_{n=1}^{N} \sum_{m=1}^{N} & \left[\frac{1}{(n+m)^3} - \frac{1}{\left(n + \frac{m}{p^2}\right)^3} - \frac{1}{\left(\frac{n}{p^2} + m\right)^3} + \left(\frac{p^2}{n+m}\right)^3 \right] \times \\ & \times \left[\frac{1}{n+m} - \frac{1}{n + \frac{m}{p^2}} - \frac{1}{\frac{n}{p^2} + m} + \frac{p^2}{n+m} \right] \times \\ & \times \left[\frac{1}{(n+m)^2} - \frac{1}{\left(n + \frac{m}{p^2}\right)^2} - \frac{1}{\left(\frac{n}{p^2} + m\right)^2} + \left(\frac{p^2}{n+m}\right)^2 \right]^{-2} \end{aligned} \tag{16}$$

From Eq. (16) we see that the kurtosis parameter of partially coherent CDHBs at the source plane $z = 0$ depends on the scaling factor p and beam order N, but is independent of the waist width w_0, wavelength λ and spatial correlation length σ_0. Equations (14) and (16) indicate that the waist width w_0, wavelength λ and spatial correlation length σ_0 have an effect on the kurtosis parameter only upon propagation.

3. Numerical calculations and analyses

Figures 1**a** and 1**b** give the kurtosis parameter K of partially coherent CDHBs *versus* the propagation distance z for the different scaling factor p and beam order N. The calculation parameters are $\sigma_0 = 1$ cm, $w_0 = 2$ cm, $\lambda = 632.8$ nm, $N = 4$, $p = 0.1, 0.2, 0.9$ (Fig. 1**a**) and $p = 0.9$, $N = 1, 5, 10$ (Fig. 1**b**). As can be seen, the kurtosis parameter K of partially coherent CDHBs depends on the scaling factor p, the beam order N and the propagation distance z. From Fig. 1**a** it is seen that the kurtosis parameter increases with increasing the propagation distance z and decreasing the scaling factor p, *i.e.*, the bigger the propagation distance z is and the smaller the scaling factor p is, the bigger the kurtosis parameter K is. From Fig. 1**b** we see that, when the propagation distance $z < 2$ km, the kurtosis parameter K increases with decreasing the beam order N; when the propagation distance $z > 3$ km, the kurtosis parameter K increases with increasing the beam order N. Therefore, the effect of the beam order N on the kurtosis parameter K depends on the propagation distance z.

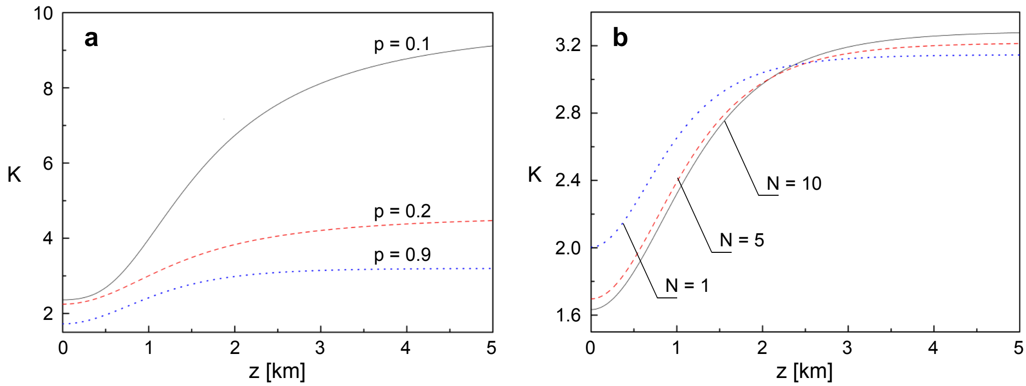

Fig. 1. The kurtosis parameter of partially coherent CDHBs *versus* the propagation distance z for the different scaling factor p (**a**) and beam order N (**b**).

The kurtosis parameter of partially coherent CDHBs propagating through free space *versus* the propagation distance z for the different waist width w_0, wavelength λ and spatial correlation length σ_0 are depicted in Figs. 2**a**, 2**b** and 2**c**, respectively. The calculation parameters are $N = 10$, $p = 0.9$, and $w_0 = 1, 2, 4$ cm (Fig. 2**a**), $\lambda = 632.8$, 1064, 1550 nm (Fig. 2**b**), and $\sigma_0 = 1$ cm, 2 cm, ∞ (Fig. 2**c**), the other calculation parameters are the same as in Fig. 1. From Figs. 2**a**–2**c**, we can find that the kurtosis parameter K depends on the waist width w_0, the wavelength λ and the spatial correlation length σ_0 of partially coherent CDHBs propagating through free space ($z > 0$). However, the val-

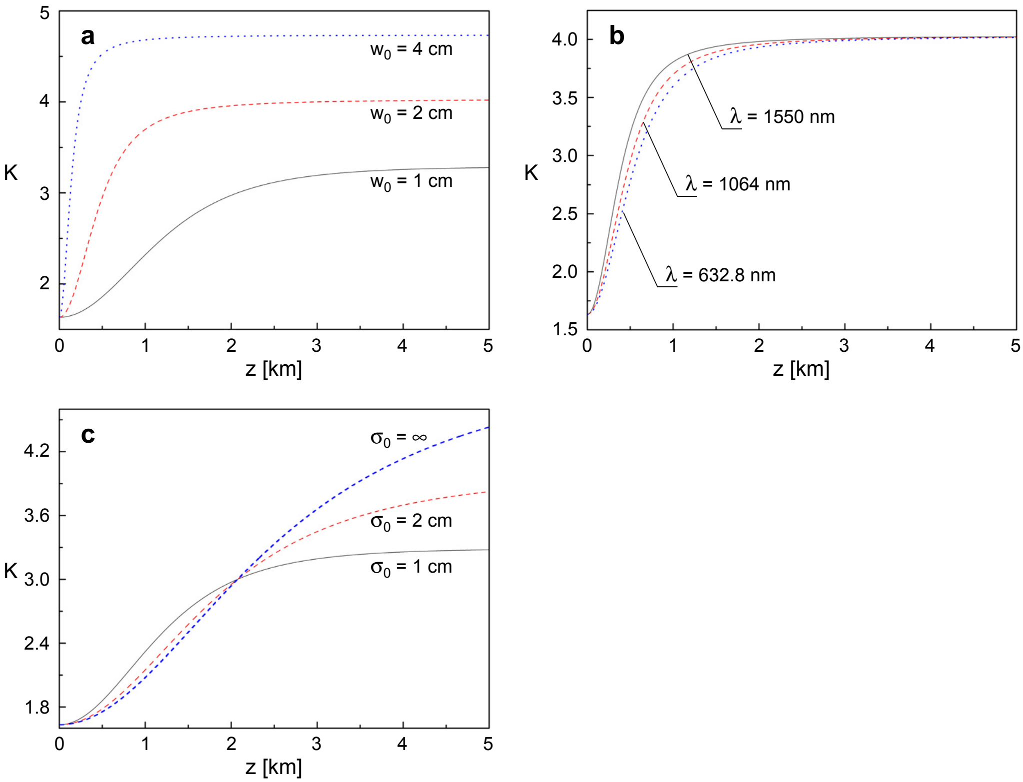

Fig. 2. The kurtosis parameter of partially coherent CDHBs *versus* the propagation distance z for the different waist width w_0 (**a**), wavelength λ (**b**) and spatial correlation length σ_0 (**c**).

ue of kurtosis parameters are equal at the source plane $z = 0$ for the different waist width w_0, wavelength λ or the spatial correlation length σ_0. The result can be interpreted reasonably from Eq. (16), that is, the kurtosis parameter is independent of the waist width w_0, wavelength λ and spatial correlation length σ_0 at the source plane $z = 0$. It can be seen from Fig. 2**a** that the kurtosis parameter K increases with increasing the waist width w_0, *i.e.*, the bigger the waist width w_0 is, the bigger the kurtosis parameter K is. Figure 2**b** indicates that the kurtosis parameter K increases with increasing the wavelength λ, and the kurtosis parameter K tends to be uniform for the different wavelength λ with increasing propagation distance. The results can be explained physically. From Eq. (14) we see that the kurtosis parameter K is proportional to $z\lambda$ ($K \propto z\lambda$), and z plays a dominant role compared with λ when the propagation distance is long. From Fig. 2**c** we see that the three curves cross at point (2073, 3), when the propagation distance $z < 2073$ m, the kurtosis parameter $K < 3$; when the propagation distance $z > 2073$ m, the kurtosis parameter $K > 3$. In terms of [9], the former irradiance profile is platykurtic and the latter is leptokurtic of partially coherent CDHBs for different spatial correlation length σ_0. Moreover, Fig. 2**c** indicates that the effect

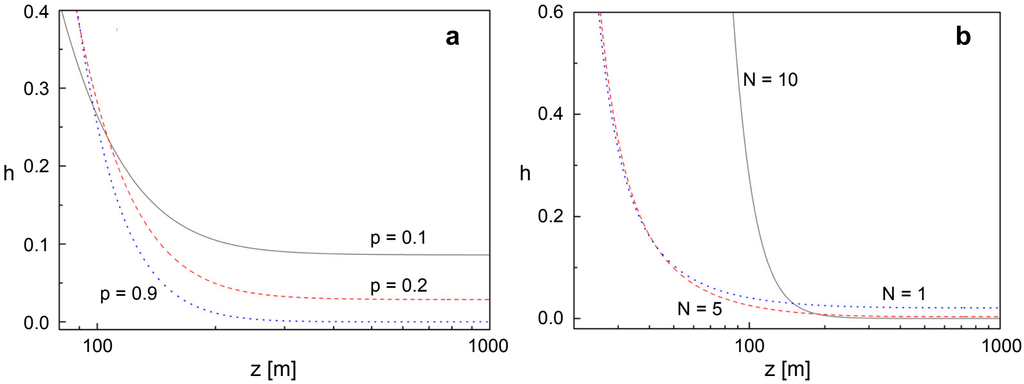

Fig. 3. The WDF of partially coherent CDHBs *versus* the propagation distance z for the different scaling factor p (**a**) and beam order N (**b**).

of the spatial correlation length on the kurtosis parameter K depends on the propagation distance z.

Figures 3**a** and 3**b** give the Wigner distribution function (WDF) h of partially coherent CDHBs *versus* the propagation distance z for the different scaling factor p (Fig. 3**a**) and beam order N (Fig. 3**b**). The calculation parameters are $\rho_x = 3$ cm, $\rho_y = 0$, $\theta_x = \theta_y = 0$, the other calculation parameters are the same as in Fig. 1. As can be seen, the effect of the scaling factor and the beam order on the WDF h depends on the propagation distance z.

4. Conclusion

In this paper, based on the extended Huygens–Fresnel principle and the definition of second-order moments of the WDF, the analytical expression for the kurtosis parameter of partially coherent CDHBs propagating through free space is derived, and used to study the influence of beam parameters on the kurtosis parameter. It is shown that the kurtosis parameter K of partially coherent CDHBs depends on the scaling factor p, the beam order N, the waist width w_0, the wavelength λ and the spatial correlation length σ_0. With the increment of the wavelength λ and the waist width w_0, as well as the decrement of the scaling factor p, the kurtosis parameter K will increase for the partially coherent CDHBs propagating through free space, *i.e.*, the bigger the wavelength λ and the waist width w_0 are, as well as the smaller the scaling factor p is, the bigger the kurtosis parameter K is. However, the effect of the beam order N and the spatial correlation length σ_0 on the kurtosis parameter K depends on the propagation distance z. The theoretical analysis presented in this paper can also be used for guiding, focusing, and trapping ultracold atoms, even Bose–Einstein condensates.

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