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A new clock synchronization scheme based on the second-order coherence of thermal light

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A novel thermal light based scheme utilizing the second-order coherence is proposed for ultra-high accuracy synchronization of two distant clocks. By using two-photon absorption, which involves an almost simultaneous absorption of two photons within a maximum delay given by the Heisenberg principle, the second-order coherence at a femtosecond scale for a real thermal source has been observed (BOITIER F. *et al.*, Nature Physics **5**(4), 2009, p. 267). We show that the clock synchronization accuracy can achieve, in principle, a femtosecond range. And the scheme can be realized with currently available technology, at least for a proof of principle.

Keywords: thermal light, clock synchronization, second-order coherence.

1. Introduction

Long-range high-precision synchronization technology is very important for maintaining the accuracy of standard time (UTC) that is used by countries in the world. There are two classical methods of time synchronization of spatially separated clocks: Eddington slow clock transport [1], and Einstein synchronization [2] by exchange of light signals. In practical applications today, the main methods for time synchronization are Global Positioning System (GPS) common view comparison, two-way satellite time transfer (TWSTT), *etc*. The accuracy provided by GPS common view comparison is on the order of nanosecond, whereas the accuracy of TWSTT could achieve a picosecond range. However, as the performance of a high-precision atomic clock [3–5] keeps being improved, the demands for more accurate synchronization technology are urgent. Recently, quantum entanglement and squeezing have been exploited in clock synchronization [6–8], interferometry [9, 10], frequency measurements [11], *etc*. It has been shown that quantum mechanical methods can provide higher accuracy than classical methods. However, quantum entanglement suffers from several vulnerable features, such as low efficiency, decoherence, *etc.*, which make it not very suitable for long-range synchronization and metrology.

In this paper, we further study the thermal light based synchronization scheme, which was first proposed in Reference [12]. Two setups of this scheme are thoroughly analyzed and compared with each other, and critical performance parameters are discussed in detail. By using the second-order coherence of thermal light [13] and two-photon absorption (TPA) technique, the synchronization accuracy of two distant clocks could be, in principle, at a femtosecond range. There are three key features for our scheme. First, no information is needed on the geometric distances between clocks. Second, no exchanging of the arrival times of the signals is needed. Third, compared to entangled photon pairs, it is easy to implement a high power thermal light source, so the synchronization can be carried out over large distances. The setup is realizable since the essential elements of the scheme, such as two-photon absorption, signal time delay techniques have already been demonstrated.

This paper is organized as follows. In Section 2, the second-order coherence function of thermal light and TPA detection are introduced firstly, and then the clock synchronization scheme is described. In Section 3, the analysis of two setups of the scheme is given by solving the equations of motion of the system and important parameters are also addressed. Finally, the conclusions are drawn in Section 4.

2. Clock synchronization scheme

2.1. Measuring the second-order coherence for real thermal light

Considering two space time points (r_1, t_1) and (r_2, t_2), according to the unifying quantum theory of optical coherence developed by GLAUBER [14], and using a quantum approach of the photodetection process, the second-order coherence function can be expressed as

$$G^{(2)}(r_1, t_1, r_2, t_2) = \langle E^{(-)}(r_1, t_1)E^{(-)}(r_2, t_2)E^{(+)}(r_2, t_2)E^{(+)}(r_1, t_1)\rangle \quad (1)$$

where $E^{(-)}(r_i, t_i)$ and $E^{(+)}(r_i, t_i)$ correspond to the negative frequency and the positive frequency field operators of the detection events at time points (r_i, t_i), for $i = 1, 2$. If we focus on the temporal coherence feature, for a multimode thermal field with a Gaussian spectrum, the second-order coherence function is given by

$$G^{(2)}(\tau) = \langle E^{(-)}(t)E^{(+)}(t)\rangle\langle E^{(-)}(t+\tau)E^{(+)}(t+\tau)\rangle\left[1 + \exp\left(\frac{-\tau^2}{\tau_c^2}\right)\right] \quad (2)$$

where the coherence time τ_c is in the femtosecond range for a real thermal source.

The measurement of the second-order coherence G in time domain can be actually done via Hanbury-Brown and Twiss (HBT) interferometry [15], which was firstly developed by HANBURY-BROWN and TWISS in 1956. The HBT interferometry is generally implemented by splitting the light through a beam splitter, and measuring the intensity correlation between the two beams using coincidence measurement. However, the second-order coherence for real thermal sources could not be observed with two separate detectors due to the limited bandwidth.

Then it has been suggested by Mollow that TPA effect is very suitable to study optical correlation at ultra-short timescales [16]. In the photon picture, this effect can be understood in the following way. When photons with the frequency ω pass onto a semiconductor medium which does not have an energy level at $\Delta E = \hbar\omega$ above the ground level, but has one at $\Delta E = 2\hbar\omega$, then the medium may absorb two photons to attain the excited state if and only if the maximum delay between these two photons is smaller than the Heisenberg lifetime $\hbar/\Delta E$. Normally this Heisenberg limit is a few femtoseconds in the optical range, and it means that although the direct arrival time of photons still cannot be detected with femtosecond resolution, the coincidence measurement can be done at the femtosecond timescale. Recently Fabre and his collaborators successfully observed photon bunching in the femtosecond range for real thermal sources [17]. Using similar TPA detectors in HBT interferometry, clock synchronization between two distant clocks could be implemented with ultra-high accuracy.

2.2. Scheme description

The experimental setup of the synchronization scheme based on the second-order coherence of thermal light is sketched in Fig. 1. The goal is to synchronize clock B with clock A.

Alice uses a halogen lamp exhibiting blackbody characteristics to generate a highly incoherent thermal light beam and divide it equally by a 50:50 beam splitter. The beams then travel up to Bob's position and are reflected back to Alice's. The TPA photo-

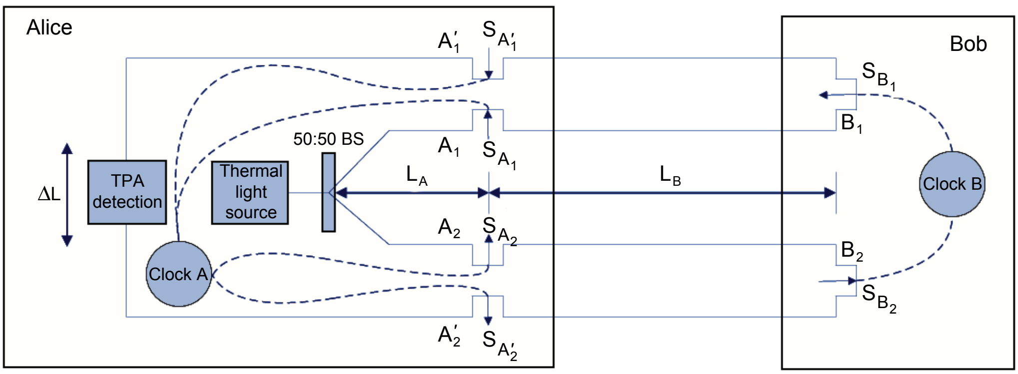

Fig. 1. The experimental setup for the clock synchronization scheme. The thermal light source produces the thermal light beam. The TPA detection is used for measuring the degree of second-order coherence as a function of path length difference ΔL. Time varying delays S_{A_1}, $S_{A'_1}$, S_{A_2}, $S_{A'_2}$, S_{B_1}, S_{B_2} are used for importing Alice's and Bob's time information. L_A and L_B are distances from the source to position A and from position A to position B.

counter [17] is co-located with the thermal light source. Alice measures the degree of the second-order coherence of the beams as a function of two optical path length differences ΔL. With fine alignment and optimization, Alice then obtains the curve shown in Reference [17] with a very narrow peak at $\Delta L = 0$.

After that, time varying delays S_{A_1}, $S_{A'_1}$, S_{A_2}, $S_{A'_2}$, S_{B_1}, S_{B_2} are introduced in order to synchronize Alice's and Bob's clock. These delays could be implemented by moving mirrors as shown in Fig. 1. Notice that three delays S_{A_1}, $S_{A'_1}$ and S_{B_1} increase in time, while the other three S_{A_2}, $S_{A'_2}$ and S_{B_2} decrease. Consider that Alice starts the delays at A_1, A'_1, A_2, A'_2 with constant speed ν at the time t^A showing on her clock, and Bob applies his delays at B_1 and B_2 with constant speed 2ν at the time t^B showing on his clock. Assuming Alice's clock and Bob's clock are stationary and $t^A = t^B$, with an ideal reference clock showing time t_r, t^A and t^B can be expressed as

$$t^A = \Delta t^A + t_r^A = t^B = \Delta t^B + t_r^B \tag{3}$$

Here Δt^A is the clock correction that relates Alice's clock and the ideal reference clock, Δt^B is the clock correction that relates Bob's clock and the ideal reference clock. Hence, the offset between Alice's clock and Bob's clock is

$$\tau = \Delta t^A - \Delta t^B = t_r^B - t_r^A \tag{4}$$

and the delays mentioned above in the frame of the reference clock t_r are

$$S_{A_1} = S_{A'_1} = \nu(t - t_r^A) \tag{5a}$$

$$S_{A_2} = S_{A'_2} = -\nu(t - t_r^A) \tag{5b}$$

$$S_{B_1} = -2\nu(t - t_r^B) \tag{5c}$$

$$S_{B_2} = 2\nu(t - t_r^B) \tag{5d}$$

Because of these delays, the final path length difference is influenced by the clock offset τ.

3. Analysis of the scheme

3.1. Motion equations

The detailed analysis is as follows. Assume that the path length from the source to the position A_1 (A_2) is L_A; the path length from A_1 (A_2) to B_1 (B_2) and from A'_1 (A'_2) to B_1 (B_2) is L_B. When mirrors begin to move, at the time the thermal light beam in the upper path reaches the position A_1, the moving mirror at A_1 increases the path length by

$$S_{A_1} = v\frac{L_A}{c - t_r^A} \tag{6}$$

This implies that the path length between the source and the position B_1 has been changed to $L_A + S_{A_1} + L_B$. When the light beam reaches the position B_1, the moving mirror at B_1 has decreased the path length by

$$S_{B_1} = -2v\frac{L_A + S_{A_1} + L_B}{c - t_r^B} \tag{7}$$

Similarly, when the light beam reaches the position A'_1, the moving mirror at A'_1 has increased the path length by

$$S_{A'_1} = v\frac{L_A + 2L_B + S_{A_1} + S_{B_1}}{c - t_r^A} \tag{8}$$

Processing in alike manner for the under path length changes at the positions A_2, B_2 and A'_2,

$$S_{A_2} = -v\frac{L_A}{c - t_r^A} \tag{9}$$

$$S_{B_2} = 2v\frac{L_A + S_{A_2} + L_B}{c - t_r^B} \tag{10}$$

$$S_{A'_2} = -v\frac{L_A + 2L_B + S_{A_2} + S_{B_2}}{c - t_r^A} \tag{11}$$

and calculating all the contributions together, the entire path length difference induced by the mirrors is given by

$$\begin{aligned}\Delta L_\tau &= (S_{A_1} + S_{B_1} + S_{A'_1}) - (S_{A_2} + S_{B_2} + S_{A'_2}) = \\ &= 4v(t_r^B - t_r^A) + \frac{4v^3 t_r^A}{c^2} - \frac{4v^3 L_A}{c^3}\end{aligned} \tag{12}$$

Given the limit $v \ll c$, then

$$\Delta L_\tau = 4v(t_r^B - t_r^A) = 4v\tau \tag{13}$$

By observing the shift of the peak position in the degree of second-order coherence curve, Alice could measure the offset $\tau = \Delta L_\tau / 4\nu$ between her clock and Bob's clock.

From Figure 1 we can see that our interferometry encodes the time difference information on both paths. It would be enough to encode just one optical path whereas leave the other optical path unchanged for a synchronization purpose. In this case, the overall setup will be simplified since only half of the moving mirrors are used. However, as clarified below, the use of only one optical path may lead to a restriction in the working range of the scheme. Supposing the moving mirrors for the upper path start to move while the moving mirrors for the under path remain stationary, the under path length changes would be

$$S_{A_2} = S_{B_2} = S_{A_2'} = 0 \tag{14}$$

According to Eqs. (6)–(8), (12) and (14), the entire path length difference induced by only the upper path mirrors is given by

$$\begin{aligned} \Delta L_\tau &= (S_{A_1} + S_{B_1} + S_{A_1'}) - (S_{A_2} + S_{B_2} + S_{A_2'}) = \\ &= 2\nu(t_r^B - t_r^A) + \frac{\nu^2 t_r^A}{c} + \frac{2\nu^2 t_r^B}{c} + \frac{2\nu^3 t_r^A}{c^2} - \frac{3\nu^2 L_A}{c^2} - \frac{2\nu^3 L_A}{c^3} - \frac{2\nu^2 L_B}{c^2} \end{aligned} \tag{15}$$

From the last term in Eq. (15), we can find that the final path difference is not only related to the time difference but it is also related to the distance L_B between the two clocks. Even given the limit $\nu \ll c$, the last term in Eq. (15) cannot be easily omitted with a large distance L_B. In order to neglect this term, the following equation should be satisfied

$$\frac{2\nu^2 L_B}{c^2} \ll 2\nu(t_r^B - t_r^A) \tag{16}$$

For instance, if the time difference between the two clocks is initially $t_r^B - t_r^A =$ $= 1$ ns, using $\nu = 100$ m/s, than we can derive that $L_B \ll 900$ km. The result shows clearly that the simplified scheme is suitable for short-range (from 0.5 to 5 km) and medium-range (from 5 to 50 km) clock synchronization. But for long-range clock synchronization applications, such as intra-satellite clock synchronization, encoding time difference information onto both paths would be a better choice.

3.2. Accuracy and update rate

Generally, there are two important parameters in clock synchronization applications: accuracy characterizing the deviation from the true value, and update rate characterizing the total time needed for each synchronization cycle.

The accuracy of determining the offset is given by [18]

$$\Delta\tau = \sqrt{\Delta\tau_{\text{TPA}}^2 + \Delta\tau_v^2} \tag{17}$$

The first term is the intrinsic accuracy in the peak position measurement. It is related to the full-width at half-maximum (FWHM) of the second-order coherence function. According to Eq. (2), the FWHM of the second-order coherence function for real thermal light is calculated as

$$t_{\text{FWHM}} = \sqrt{\ln(2)}\,\tau_c \approx 0.8\,\tau_c \tag{18}$$

Because of the intensity and phase stochastic fluctuations, the coherence time τ_c of real thermal light, such as solar light and incandescent light, is in the femtosecond scale. By using TPA detection, the second-order coherence function for a halogen lamp exhibiting blackbody characteristics is observed [17], and the estimated FWHM of $G^{(2)}$ is about 50 fs. Thus the measurement of the time offset τ can reach, in principle, the accuracy of 50 fs. The second term is related to the accuracy of the speed v. Since $\Delta\tau_v = \tau\Delta v/v$, given an initial offset $\tau = 1$ ns and $\Delta v/v < 10^{-3}$, the total error in the determination of τ is at the femtosecond range.

The update rate is related to the integration time of coincidence measurement of the second-order coherence function $G^{(2)}$. The shorter the integration time is, the faster the update rate is. However, because of the background noise, reducing the integration time would lead to a lower signal-to-noise ratio (SNR). Normally, in order to clearly distinguish the coincidence peak position, the SNR should be no less than 1. For a certain integration time t_I, given the TPA photocount rate to be R and the background noise count rate to be r, the SNR can be calculated as follows. By subtracting the background noise counts from the total TPA counts, the signal count rate is calculated as $R_s = R - r$. Since the TPA photocounts and the background noise counts both obey the Poissonian distribution [17], according to the statistical theory, the mean square error of the measurement fluctuation is expressed as

$$\sigma = \sqrt{Rt_I + rt_I} = \sqrt{R_s t_I + 2rt_I} \tag{19}$$

and the SNR is given by

$$\text{SNR} = \frac{Rt_I - rt_I}{\sqrt{Rt_I + rt_I}} = \frac{R_s t_I}{\sqrt{R_s t_I + 2rt_I}} \tag{20}$$

Thus the integration time t_I can be derived as

$$t_I = \frac{\text{SNR}^2}{R_s^2}(R_s + 2r) \tag{21}$$

According to Eq. (21), supposing the background noise count rate to be $r = 1\times10^3$ c/s, if we want the integration time $t_I = 500$ ms with the SNR = 1, then the signal count rate needs only to be $R_s = 64.3$ c/s.

4. Conclusions

In summary, a simple scheme is proposed to synchronize two distant clocks at the femtosecond range using a thermal light source. It can be readily implemented since each experimental element in the scheme has been experimentally demonstrated during the past several years. The thermal light source is much easier to obtain and more powerful, so the synchronization scheme can be done over larger distances. Another advantage of this scheme is that no time of arrival measurement needs to be exchanged because of the TPA technique. Besides, the distance between Alice and Bob plays no role in the scheme, hence there is no need to gain it.

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References

[1] EDDINGTON A.S., *The Mathematical Theory of Relativity*, Second Edition, Chelsea Publishing Company, New York, 1924.

[2] LANDAU L.D., LIFSHITZ E.M., *Classical Theory of Fields*, Fourth Revised English Edition, Pergamon Press, New York, 1975.

[3] JIANG Y.Y., LUDLOW A.D., LEMKE N.D., FOX R.W., SHERMAN J.A., MA L.-S., OATES C.W., *Making optical atomic clocks more stable with 10^{-16}-level laser stabilization*, Nature Photonics **5**(3), 2011, pp. 158–161.

[4] MIKHAILOV E.E, HORROM T., BELCHER N., NOVIKOVA I., *Performance of a prototype atomic clock based on lin/lin coherent population trapping resonances in Rb atomic vapor*, Journal of the Optical Society of America B **27**(3), 2010, pp. 417–422.

[5] BOBER M., ZACHOROWSKI J., GAWLIK W., *Designing Zeeman slower for strontium atoms – towards optical atomic clock*, Optica Applicata **40**(3), 2010, pp. 547–555.

[6] VALENCIA A., SCARCELLI G., YANHUA SHIH, *Distant clock synchronization using entangled photon pairs*, Applied Physics Letters **85**(13), 2004, pp. 2655–2657.

[7] JOSZA R., ABRAMS D.S., DOWLING J.P., WILLIAMS C.P., *Quantum clock synchronization based on shared prior entanglement*, Physical Review Letters **85**(9), 2000, pp. 2010–2013.

[8] GIOVANNETTI V., LLOYD S., MACCONE L., *Quantum-enhanced positioning and clock synchronization*, Nature **412**(6845), 2001, pp. 417–419.

[9] HOLLAND M.J., BURNETT K., *Interferometric detection of optical phase shifts at the Heisenberg limit*, Physical Review Letters **71**(9), 1993, pp. 1355–1358.

[10] DOWLING J.P., *Correlated input-port, matter-wave interferometer: Quantum-noise limits to the atom--laser gyroscope*, Physical Review A **57**(6), 1998, pp. 4736–4746.

[11] BOLLINGER J.J., ITANO W.M., WINELAND D.J., HEINZEN D.J., *Optimal frequency measurements with maximally correlated states*, Physical Review A **54**(6), 1996, pp. R4649–R4652.

[12] JUN ZHU, YUAN LU, PENG HUANG, JINGNENG LIU, GUIHUA ZENG, *Clock synchronization through second order coherence of thermal light*, AIP Conference Proceedings, Vol. 1363, 2010, pp. 117–120.

[13] SCARL D.B., *Measurements of photon correlations in partially coherent light*, Physical Review **175**(5), 1968, pp. 1661–1668.
[14] GLAUBER R.J., *Coherent and incoherent states of the radiation field*, Physical Review **131**(6), 1963, pp. 2766–2788.
[15] HANBURY-BROWN R., TWISS R.Q., *Correlation between photons in two coherent beams of light*, Nature **177**(4497), 1956, pp. 27–29.
[16] MOLLOW B.R., *Two-photon absorption and field correlation functions*, Physical Review **175**(5), 1968, pp. 1555–1563.
[17] BOITIER F., GODARD A., ROSENCHER E., FABRE C., *Measuring photon bunching at ultrashort timescale by two-photon absorption in semiconductors*, Nature Physics **5**(4), 2009, pp. 267–270.
[18] GIOVANNETTI V., LLOYD S., MACCONE L., WONG F.N.C., *Clock synchronization with dispersion cancellation*, Physical Review Letters **87**(11), 2001, pp. 117902–117905.

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