Evaluation of error in the performance of power corrected ophthalmic lenses

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In this work, we describe a new method for assessing the refractive correction error in power corrected ophthalmic lenses. The ideal power distribution map corresponding to the lens design to be measured is initially obtained. Then, using some known techniques, the real power distribution map of the lens under test is obtained. Finally, the difference between the two distributions is computed, obtaining a distribution map of the refractive correction error of the lens under analysis. In the paper, we describe the implementation of the proposed method using the Hartmann test to analyze a corrected power ophthalmic lens. The technique is analyzed, and its effects on measurement resolution discussed. This method can be useful to verify the designing errors in the lab or to verify manufacturing errors of ophthalmic lenses during mass production

Keywords: ophthalmic lenses, refractive correction error, Hartmann test.

1. Introduction

The ophthalmic lenses are thin meniscus type lenses, placed in a frame, located approximately 14 mm in front of the eye [1]. The purpose of the ophthalmic lens is to form a virtual image of the observed object, from an appropriate distance, in order to focus it on the retina while the eye is in relaxation, compensating with this the eye refractive defect. To compensate for a farsighted eye, a positive lens is used, and to compensate a myopic eye, a negative lens is used. In the case of astigmatism, spherical-cylindrical lenses are used.

Since a long time ago, there have been different criteria for the designing of ophthalmic lenses [2], all of them with advantages and disadvantages. However, the approach used at present is based on obtaining designs of corrected lenses by power

or astigmatism [1, 2]. The power corrected lenses, also called zero error power lenses, are those in which the user, regardless of the observation angle θ, always will see the same power through the lenses. The astigmatism corrected ophthalmic lenses, also called zero error astigmatism lenses, are designed so, regardless of the observation angle θ, the user always see the same astigmatism through the lenses.

The procedures for the designing of these type of lenses are fully documented elsewhere and range from the use of the Tscherning ellipses [1–6] to the Petzval curvatures trace, both sagittal and tangential [1, 4].

When an ophthalmic lens does not fulfill properly the purpose for which it was prescribed, then it creates a refractive correction error. Depending on the error magnitude, the user of the ophthalmic lens may experience from nearly imperceptible minor discomforts to greater nuisances that may even aggravate his visual problem. Therefore, it is important to be able to measure and evaluate the lens performance.

Most of the methods used to obtain the dioptric power of ophthalmic lenses, such as the methods proposed by MALACARA-DOBLADO [7], QUIROGA [8], CANABAL *et al*. [9], CANABAL and ALONSO [10], GNANVO *et al*. [11]. POZO and RUBIÑO [12] use a large pupil collimated light beam to illuminate the lens under test. However, all these methods take the ophthalmic lens such as an independent optical element, without considering the real viewing condition proposed by TRUJILLO-SCHIAFFINO and MALACARA-HERNÁNDEZ [13] and CHUANQING ZHOU *et al*. [14].

It is easy to demonstrate by a ray tracing program that the large pupil used in the traditional methods [7–11] introduces spherical aberrations in the periphery of the lens that the eye really does not see when the user looks through the lens due to the small pupil used in the real viewing conditions [13, 14].

The evaluation method presented in this paper represents a fast, simple and innovative alternative to the traditional method of analysis used nowadays. We focus on the assessment of the refractive correction error of power corrected ophthalmic lenses, considering that a similar procedure can be used in evaluating astigmatism corrected lenses.

2. Basic concepts

Traditionally in optometry and ophthalmology, the refractive correction error P_{error} of an ophthalmic lens is the difference between the lens power prescribed also called vertex power P_v and the lens real average power P_m,

$$P_{error} = P_v - P_m \tag{1}$$

The lens real average power is the arithmetic average between the tangential power P_t and the sagittal power P_s, both measured on the lens with an observation angle θ, so that

$$P_m(\theta) = \frac{P_t(\theta) + P_s(\theta)}{2} \tag{2}$$

Then, for any observation angle θ we have

$$P_{\text{error}}(\theta) = P_v - \frac{P_t(\theta) + P_s(\theta)}{2} \tag{3}$$

However, when a fast assessment of the entire lens is needed, the measurement of both powers, tangential and sagittal, to obtain the average power at each sampling point using Eq. (2), results in a slow and tedious work.

3. Proposed method

In the specific case of a power corrected ophthalmic lens, the user should see the same power regardless of the observation angle, therefore the plane should actually be a spherical surface centered on the eye rotation center, as shown in Fig. 1.
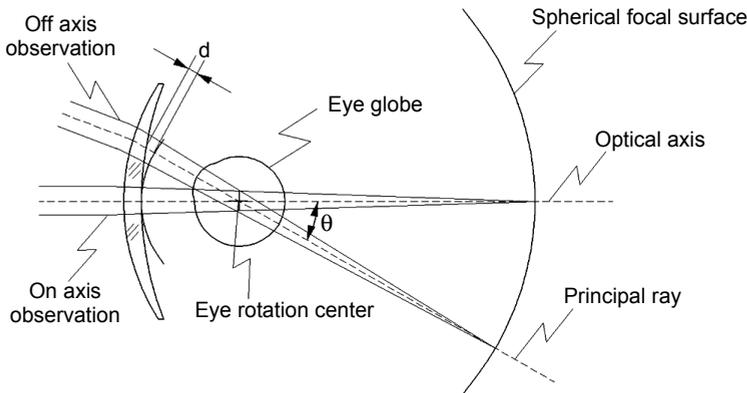

Fig. 1. Power corrected ophthalmic lens performance.

Then, starting from this fact, the ideal power distribution for any observation angle $P_{\text{ideal}}(\theta)$ that the ophthalmic lens must reach a spherical focal surface can be computed with the expression

$$P_{\text{ideal}}(\theta) = \frac{1}{f_v + d(\theta)} \tag{4}$$

where f_v is the vertex focal distance expressed in meters, *i.e.*, the inverse of the prescribed dioptric power, and $d(\theta)$. As shown in Fig. 1, the distance $d(\theta)$ is a variable distance that depends on the observation angle θ, having a value of zero on the optical axis and increasing it when increasing the angle. This distance can be calculated using a trigonometric analysis correlating parameters such as: observation angle, radius of curvature in the back surface of the lens, separation distance between the cornea and the vertex of the lens, curvature center of the back surface lens and rotation center of the eye, obtaining the expression proposed by TRUJILLO-SCHIAFFINO and MALACARA-HERNÁNDEZ [13],

$$d(\theta) = r_b \left[\sqrt{1 - \left(1 - \frac{d_v}{r_b}\right)^2 \sin^2(\theta)} - \left(1 - \frac{d_v}{r_b}\right)\cos(\theta) \right] - d_v \tag{5}$$

where r_b is the lens back surface radius of curvature, d_v is the distance from the lens vertex to the cornea, and θ is the observation angle, as shown in Fig. 2.
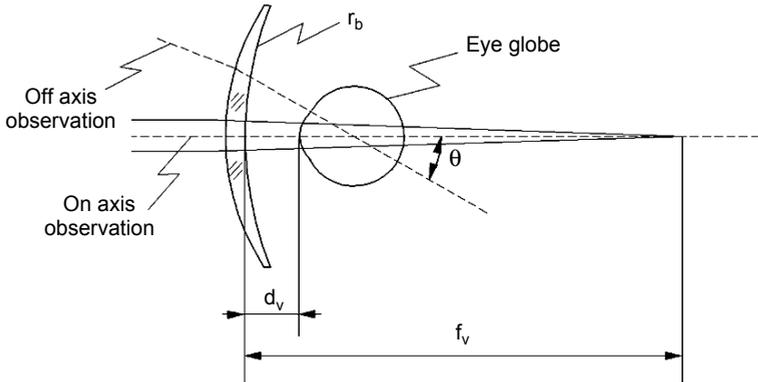

Fig. 2. Parameters used to compute d.

It is important to remark that the expression presented in Eq. (5) is valid only when the second surface of the lens is spherical. However, in the case of some modern ophthalmic lenses where the second surface is aspherical, if the asphericity is small, this expression can be applied without introducing serious error.
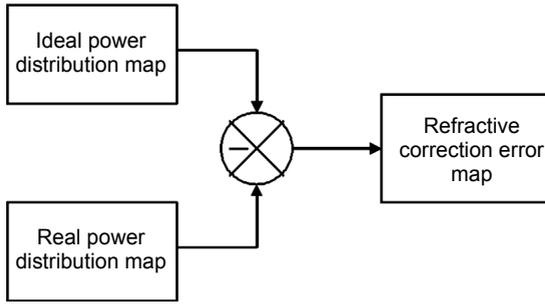
Substituting Eq. (5) into Eq. (4), we have the expression to compute the ideal power distribution $P_{\text{ideal}}(\theta)$ for any power corrected ophthalmic lens design

$$P_{\text{ideal}}(\theta) = \cfrac{1}{f_v + r_b \left[\sqrt{1 - \left(1 - \dfrac{d_v}{r_b}\right)^2 \sin^2(\theta)} - \left(1 - \dfrac{d_v}{r_b}\right)\cos(\theta) \right] - d_v} \tag{6}$$

If the ideal power for any observation angle, $P_{\text{ideal}}(\theta)$, represents the ideal performance that a power corrected lens must have, then any difference between the real power $P_{\text{real}}(\theta)$ measured in the lens and the ideal power that the lens should have, is a refractive correction error of such lens.

Then, in order to obtain a refractive correction error distribution map in the entire lens, we just need to compute the difference between the ideal power distribution and the real power distribution of the lens such that

$$P_{\text{error}}(\theta) = P_{\text{ideal}}(\theta) - P_{\text{real}}(\theta) \tag{7}$$

◀ Fig. 3. Proposed method block diagram.

Figure 3 shows a block diagram describing the proposed method, which consists of three steps.

First, we compute the ideal power distribution map matching the lens design under evaluation. Then, we obtain the ophthalmic lens real power distribution map. For this step, we can use the Hartmann test [15], Moiré deflectometry [16], null proof [17], or any other test used for this purpose.

Finally, we subtract the real map obtained in practice from the computed real map, obtaining the lens refractive correction error map.

4. Computing ideal power distribution $P_{\text{ideal}}(\theta)$

The ideal power distribution computation is based on the design parameters of the lens to be assessed. As a test specimen, a 3 D (diopters) without power correction ophthalmic lens was used, which design parameters were: lens diameter $D = 60$ mm, front radius of curvature $r_f = 68.95$ mm (7.96 D), back radius of curvature $r_b = 111.13$ mm (-4.70 D) and thickness $t = 4.6$ mm. The lens is made of ophthalmic glass with a refractive index $n = 1.523$.

Assuming an average separation distance between the vertex lens and the eye cornea $d_v = 14$ mm, with a lens back surface radius of curvature of $r_b = 111.13$ mm,
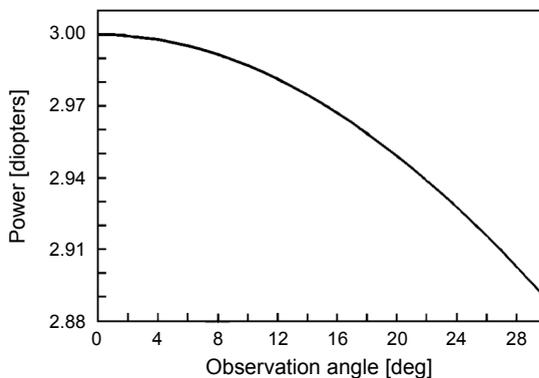

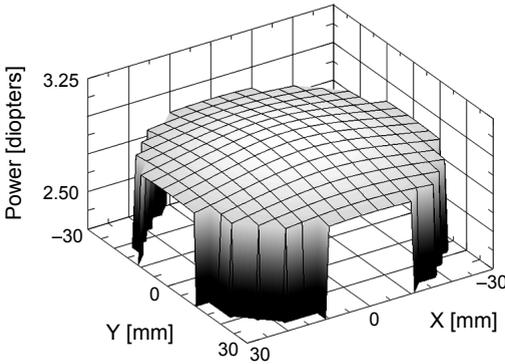Fig. 4. Ideal power calculated for the lens under test.

Fig. 5. Ideal power distribution map obtained for the lens under test.

for a 3 diopters ophthalmic lens with $f_v = 0.333$ m, the evaluations were made for observation angles range $0° \leq \theta \leq 30°$. For this lens, we computed the power radial distribution using the expression of Eq. (6), obtaining the plot shown in Fig. 4, where there is a –0.11 D maximum variation with 3 D in the center, and 2.89 D at the edge of the lens.

By rotation symmetry and transforming the observation angle θ to (x, y) coordinates on the lens, we plotted the ideal power distribution map shown in Fig. 5.

5. Measuring real power distribution $P_{\mathrm{real}}(\theta)$

In an attempt to experimentally measure the lens real power distribution, we used the assembly shown in Fig. 6 for the Hartmann test.

A helium–neon laser with the output power of 10 mW and with the wavelength $\lambda = 632.8$ nm was used. The beam diameter was 0.68 mm, a spatial filter composed of
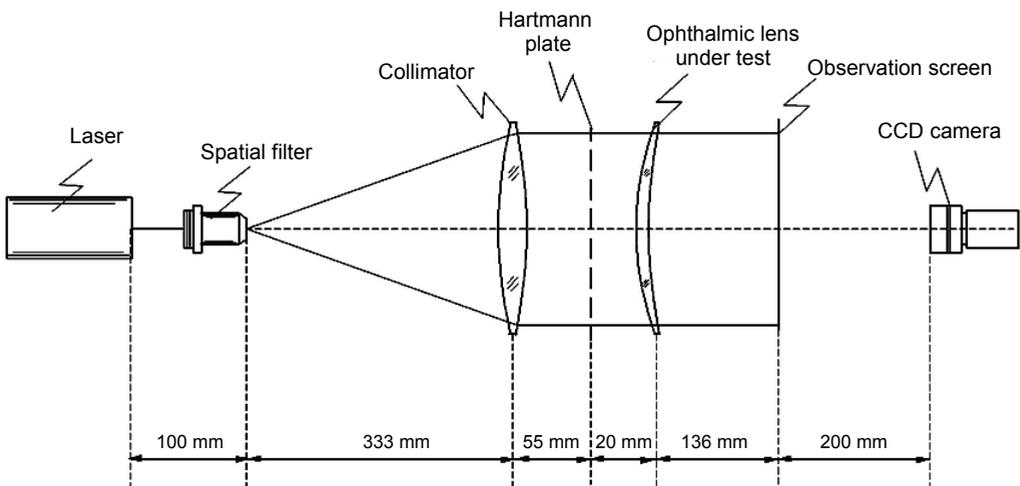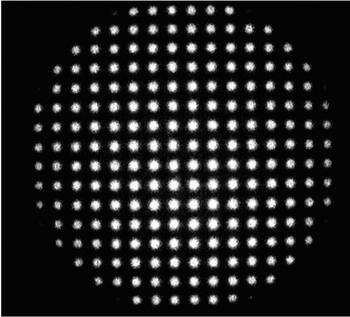
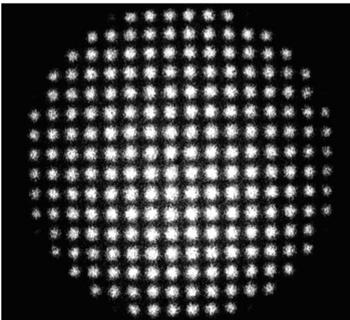

Fig. 6. Experimental assembly used.

a microscope objective of 8 mm to expand the laser beam and a 15 µm pinhole to filter it, a collimating lens with focal length $f = 333$ mm, a Hartmann plate with a 5.08 mm equidistantly separated holes square array, and an observation screen and a CCD camera with 768×494 pixels resolution were used.

Before placing the ophthalmic lens in the assembly, a Hartmann diagram (Hartmanngram) was obtained in order to use it as a reference. Figure 7 shows the obtained reference Hartmann pattern, which was captured with 11.453 pixels/mm resolution.

◀ Fig. 7. Reference Hartmann pattern obtained without the ophthalmic lens.

Then, with the ophthalmic lens placed in position, a new Hartmanngram was obtained, which deforms according to the lens refractive characteristics. Figure 8 shows the distorted Hartmann pattern, which was captured with 20.055 pixels/mm resolution.

◀ Fig. 8. Measuring Hartmann pattern obtained with the ophthalmic lens.

The Vision Assistant software version 8.0 was used for the image processing of these Hartmann patterns.

The method proposed by SALAS-PEIMBERT *et al*. [18] was used to compute the lens real power, which computes the dioptric power for each point of the Hartmann pattern using the expression

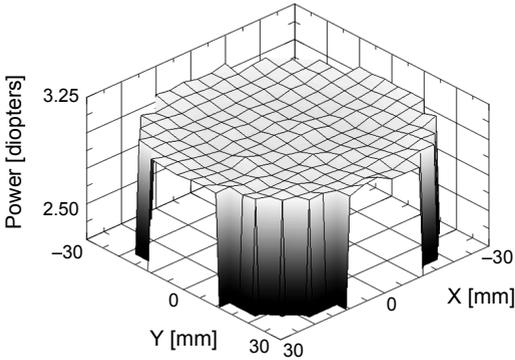$$P_{\text{real}} = \frac{\sqrt{\Delta_x^2 + \Delta_y^2}}{r\sqrt{x^2 + y^2}} \tag{8}$$

Fig. 9. Real power distribution map obtained experimentally.

where Δ_x and Δ_y are the transverse shifts in X and Y direction axes, respectively, of a point centroid in the Hartmann pattern, x and y are the centroid original coordinates of the same point in the reference Hartmann pattern, and r is the separation distance between the lens under test and the observation screen. Using this expression, we computed the values for plotting the power distribution map shown in Fig. 9, where there is a 0.08 D maximum variation with an average of 3 D in the center, and an average of 3.08 D at the edge of the lens.

It is important to note that this experimental method of measurement is valid when the ophthalmic lens can be considered as a thin lens, *i.e.*, about ±4.00 D, because the lens is measured with light rays falling perpendicular to it. For higher power lenses, where ophthalmic lens should be considered as a thick lens, it is convenient to use some other method in which the light rays fall on the lens with the observation angle θ.

6. Obtained results

Finally, by subtracting the real power distribution map from the ideal power distribution map, the lens refractive correction error distribution map is obtained.
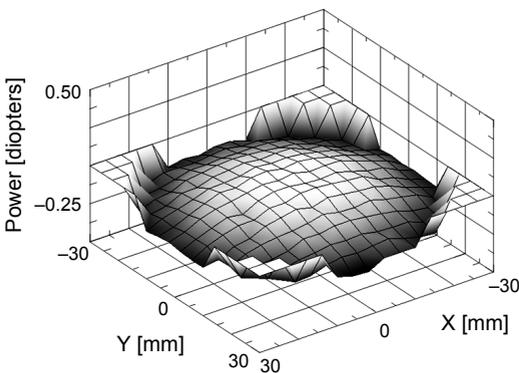

Fig. 10. Refractive correction error map obtained for the lens under test.

Figure 10 shows the map of the refractive correction error distribution obtained for the lens used as a test specimen, where we can see clearly that the error is greater on the lens periphery with an average value of 0.19 D. This is because the lens used for testing is not a power corrected lens.

Obviously the performance of this ophthalmic lens does not meet the standard ANSI Z80.1-2010 referent to prescription ophthalmic lens tolerance recommendations specifying a tolerance on sphere power of ±0.13 D for ophthalmic lenses from –6.50 to +6.50 D.

7. Conclusions

Due to the simplicity of the involved computations, the proposed method can be very useful in the mass production of ophthalmic lenses, where it is necessary to test the lenses performance in the shortest time possible. Based on that, the performance of the lens to be manufactured is already characterized, the errors introduced during the lens manufacturing process can be determined implementing this method.

The proposed method can also be useful at the laboratory to verify the possible errors in the designing of ophthalmic lenses, comparing the desired ideal performance with the real performance obtained for a given design.

When we consider the lens-eye model, we place the stop of the system, with a diameter of about 6 mm, at the center of rotation of the eye globe, about 28 mm from the back surface of the ophthalmic lens. Generally, the shift in stop positions generates astigmatism and other off-axis aberrations, however, in this case, the relative small size of the stop (6 mm or less) introduces very small aberrations.

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