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ADVECTION–DISPERSION POLLUTANT AND DISSOLVED OXYGEN TRANSPORT AS A PART OF SEWAGE BIODEGRADATION MODEL

The need to simulate the conditions of transport of the pollutants and the dissolved oxygen in the flux of sewage arises in the modelling of sewage biodegradation process in gravitational sewer system. In the properly kept sewers, sewage flow velocity is generally uniform, which allows the model describing the pollutant transport to be simplified. The application of further simplifying assumptions, justified by the conditions inside the modelled object, allows us to get the credible results with the acceptable efforts. These simplifications can be considered, depending on the costs of data for model calibration measurement and the numerical calculation costs. The present work is focused on finding the most convenient description of transport and transformation of dissolved and suspended substances by the advection–dispersion formulas, being one-dimensional description of sewage flow. The literature review concerning various treatment of longitudinal dispersion coefficient was presented. The influence of flow conditions in the modelled system on the dispersion coefficient was also shown. Finally, the matrix description of pollutant fraction transport and biodegradation in gravitational sanitation conduit based on advection–dispersion equation, along with the example of numerical procedure, was described.

1. INTRODUCTION

The need to simulate the conditions of transport of the polluting substances and the dissolved oxygen in the flux of sewage occurs in modelling of sewage biodegradation process in gravitational sewer system.

Dissolved pollutant fractions or suspension moving together with wastewater, flowing with the sewer can spread both along and perpendicularly to the flow direction. If the pollutants fractions disperse perpendicularly to the flow direction they will be carried to the points of the stream cross section with different mass velocities (JAMES 1978). As an example we may consider a discharge of a sewage portion char-

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acterized by the different parameters than the wastewater actually transported. The sewage velocity is generally smaller close to the walls and sewer bottom due to the friction forces which cause incoming sewage to move towards centre of the conduit where the flow velocities are greater. Spreading of pollutant in sewer pipes with high velocity differences in cross section (conduits with high roughness – bad technical condition or sediment beds on the bottom) is increased along the flow direction, this is the most important in the initial period of the sewage mixing. However, such situation is not often met, because during the sewer systems exploitation it is better to keep the conduits in the proper technical condition. In the properly kept sewers sewage flow velocity can be generally uniform which allows to simplify the model describing the pollution transport in the sewage.

The application of further simplifying assumptions, justified by the conditions inside the modelled flow, allows to get the credible results with the acceptable efforts both for calculations, from the numerical point of view, and field or laboratory measurement for model calibration. The coupled flow and biodegradation processes description based on advection–dispersion equation enables to create the matrix model of pollutants fraction transport and biodegradation processes in gravitational sanitation conduit. The computer program along with the exemplary numerical procedure were created as a result.

2. PRELIMINARY ASSUMPTIONS

The most often theoretical and modelling considerations are mainly focused on passive pollutants, which mean those, which have no influence on the shape of the flow velocity distribution. It has the essential influence on the model description because it allows to separate the water flow dynamics (treated as a liquid dissolving phase of multiphase mixture like sewage) from the transport of pollutants. It is assumed here that changes of additives concentration does not change the physical properties of liquid phase and does not influence the dynamics of its flow (ROMAN 1986; SZYMKIEWICZ 2000).

Often in simulation of pollutants transfer the situation occurs when in the certain moment the dissolved substance is introduced to the analyzed area locally in the form of concentrated stream. As an example one can think of the discharge of not purified sewage directly to the receiver, and after the pollutants are dispersed due to running transportation processes (SAWICKI 2003).

In the case of gravitational sewer system the situation is modestly different, because the pollutants concentrated in sewage are introduced to the sewer conduits, where the pollutants concentration is already on a high level. In such a case no rapid concentration changes are observed and the processes run more steadily. It is often possible in this situation to distinguish two possibilities in a Cartesian system of coordinates:

• Pollutants concentration in the initial moment is constant along two spatial coordinates and the changes along the third spatial variable.

• Pollutants concentration in the initial moment is constant along the one spatial coordinate and dependent along another both.

Situation described as a first occurs when the pollutants flowing to considered volume of sewage conduit are quickly mixed in its cross section. Such situation allows simplification of the three-dimensional spatial modelling with one-dimensional model of pollutants concentration, which clearly and quite precisely describes the processes running in the sewer.

The second described situation can take place when the mixing of incoming pollutants will appear along the only one dimension of the conduit – usually the depth. It is rather rare situation, which can happen in case of sewage containing oil and its derivatives hardly mixing with water forming the stable layer on the surface. In such a situation it is possible to apply the two-dimensional model instead of the three-dimensional one.

The conditions presented above allow not only to formal simplification of the calculation schemes. They also contain the practical values and can be applied as useful models of the real situations (SAWICKI 2003). In this paper there will be analyzed and applied the first of, the above mentioned, possibilities of flow parameters description. It was chosen because of the proper projection of the most cases occurring during gravitational sewage flow in the sewer trunks.

The transport of dissolved substances is traditionally described by the advection-dispersion formulas, being one-dimensional description of sewage flow. These formulas require data supplied by hydrodynamic Saint-Venant equations.

Major assumptions which should be considered for advection-dispersion model implementation for sewage conduits are the following:

• pollutants are totally mixed in the cross section of the channel which offers the homogenous conditions of their transformation – decline or appearing modelled with uniformly spatially distributed sinks or sources,

• according to Fick diffusion law dispersional flux is proportional to the concentration gradient.

The diameter of the sewers is small in comparison to their length. Thus, vertical mixing runs relatively quickly during sewage flow, causing the vertical homogeneity of concentration distribution. It justifies visible formulas simplification by decreasing the problem dimensionality. Therefore, the vertical variation can be eliminated assuming that mean vertical effect can be described by Fick law and diffusion analogy.

The general form of advection–dispersion transport equation of passive substances for the variable cross section of the canal can be described in the following way (SZYMKIEWICZ 2000; ZOPPOU 2001):

$$\frac{\partial}{\partial t}(AC)+\frac{\partial}{\partial x}(AUC)-\frac{\partial}{\partial x}\left(AD^D\frac{\partial C}{\partial x}\right)+A\delta=0\,, \tag{1}$$

where:

$C(x,t)=\int_A c\,dA/A$ – cross section mean pollution concentration,

$U(x,t)=\int_A u\,dA/A$ – cross section mean flow velocity,

A – surface of the active cross section,

D^D – coefficient of longitudinal mass dispersion,

δ – source part determining the intensity of generation or pollution decline in sewage.

Assuming no side inflows occur it is possible to write:

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial x}-\frac{1}{A}\frac{\partial}{\partial x}\left(D^D A\frac{\partial C}{\partial x}\right)+\delta=0\,. \tag{2}$$

Another simplification, after assuming the dispersion coefficient D^D is a constant, leads to the following form:

$$\frac{\partial C}{\partial t}+\left(U-\frac{D^D}{A}\frac{\partial A}{\partial x}\right)\frac{\partial C}{\partial x}-D^D\frac{\partial^2 C}{\partial x^2}+\delta=0\,. \tag{3}$$

Often applied possibility is to use the properly averaged velocity and concentration fields. The suitable quantitative description of the real velocity field influence on concentration distribution should be found together with advection description by the average velocity $U(x, t)$. Such effect, called dispersion, takes place in case of longitudinal pollution concentration distribution variability along the main flow direction. It should be underlined that such understood dispersion is not a physical phenomenon comparing to advection or diffusion. It is the result of the flow velocity and pollutants concentration averaging in the active cross section of the sewer.

Dispersion is used when the averaged pollutants concentration distribution should be determined. It is the compromise providing the technically accepted level of precision of obtained results, reducing the formal difficulties. It is mainly justified by the geometric and hydraulic properties of sewer flow. Because of the sewage mixing in the sewers, caused by the minimal required velocity of sewage for channel self-purification, conditions in the modelled object can be as constant concentration of each component of the pollution fraction. The consecutive concentration values can be interpreted as the homogeneous "slices" of substances contained in sewage (SAWICKI 2003).

3. DISPERSION COEFFICIENTS DETERMINATION

The quantitative estimation of the dispersion influencing the pollutants fractions introduced to the sewer requires the assessment of the turbulent diffusion intensity and the dispersing phase mass velocity. The measurement of the mass velocity is possible to conduct by the direct methods and is not a serious problem, but the turbulent diffusion is a complicated process for the quantitative determination (HUISMAN et al. 2000; KASHEFIPOUR and FALCONER 2002; KRUKOWSKI, 2002).

Longitudinal dispersion

The example of longitudinal dispersion in turbulent flow may be represented by a circular conduit transporting the sewage volume of highly increased concentration. During the substance transport from the place of introduction, along the flow direction, the pollutants, initially entering the stream in the form of a compact zone, are consequently dispersed. Dispersion is often defined as the sum of the influence of the radial velocity increase, extending the initial volume of pollutants to the length and the radial turbulent diffusion transporting the pollutants from the radius with the maximal velocity to the radius with the average velocity. The flow velocity close to the walls is smaller than the average and is greater then the velocity value close to the stream axis (JAMES 1978).

The most important result of the quantitative analysis of described case is the fact that D^D is inversely proportional to the radial turbulent diffusion coefficient which means that the lateral diffusion is greater when the the longitudinal diffusion is smaller.

Dispersion is small (ADAMSKI 2002) when:

$$\frac{D^D}{UL} < 0.025$$

and dispersion is described as large when:

$$\frac{D^D}{UL} > 0.2\,,$$

where:

D^D – dispersion coefficient,

U – average velocity,

L – the distance between the beginning of considered flow system and the measurement point.

The formula in the form based on the criterial Reynolds number is applied to determine the dispersion coefficient:

$$D^D = 10^{-6}\,Re^{0.875}\,. \tag{4}$$

The dispersion coefficient is calculated from the formula (4) for flows with water table depending on the conduit shape and condition. The resultant coefficient varies between $D^D = 0.00316\ \mathrm{m^2s^{-1}}$ and $D^D = 0.0237\ \mathrm{m^2s^{-1}}$.

According to Taylor, the value of dynamic/shear velocity for the open channels, mounting a few percent of the average velocity: $U/U_* \approx 25$ (SAWICKI 2003), can be written as (KRUKOWSKI, 2002):

$$U_* = \sqrt{g}\,\frac{U}{C_{CH}}, \tag{5}$$

where:

U – average velocity in the active cross-section ($\mathrm{m\ s^{-1}}$),

g – gravitational acceleration ($\mathrm{m\ s^{-2}}$),

C_{CH} – Chezy coefficient,

The shear velocity in the gravitational sewers calculations is often described in the following way (HUISMAN et al. 2000; HUISMAN, 2001):

$$U_* = \sqrt{g\ R_h\ S}, \tag{6}$$

where S – friction slope determined for example from Darcy–Weisbach, or Manning formula

The table 1 contains equations describing the axial dispersion coefficients according to the date of publishing.

The presented formulas were estimated for application in description of pollution transport and dispersion along the open channels – rivers.

The determination of the axial dispersion in gravitational sewer conduits is possible with use of the general formula in the following form (HUISMAN et al. 2000):

$$D^D = \frac{x_F^2 U'^2}{B_m} I, \tag{7}$$

where:

x_F – characteristic, lateral dimension of flow for geometry of modelled sewer equal 0.5 B (m),

B – wastewater table width in the conduit,

U' – deviation from the mean flow velocity in the conduit cross-section, assumed 0.2 U ($\mathrm{m\ s^{-1}}$),

B_m – average mixing coefficient in the cross-section of the conduit ($\mathrm{m^2s^{-1}}$),

I – dimensionless coefficient considering oscillations of velocity and turbulent mixing in cross section, for turbulent flow assumed 0.06 (–),

Table 1

Empirical dependences determining the axial dispersion coefficient basing on: (JØRGENSEN 1983; HUISMAN et al. 2000; KASHEFIPOUR and FALCONER 2002; KRUKOWSKI 2002; SAWICKI 2003)

No.	Authors	Dependences
1	Taylor (1956)	$D^D = 10.1\, H\, U_*$
2	Elder (1959)	$D^D = 5.93\, H\, U_*$
3	Parker (1961)	$D^D = 20.2\, R_h\, U_*$
4	Mc Quivey and Keefer (1974)	$D^D = 0.058\,\frac{HU}{I}$
5	Fisher (1975)	$D^D = 0.011\,\frac{U^2B^2}{HU_*}$
6	Liu (1977)	$D^D = \beta\,\frac{U^2B^2}{HU_*}, \quad \beta = 0.18\left(\frac{U_*}{U}\right)^{1.5}$
7	Magazin et al. (1988)	$\frac{D^D}{R_hU} = 7586\, P^{-1.632}, \quad P = 0.4\,\frac{U}{U_*}$
8	Iwasa i Aya (1991)	$\frac{D^D}{HU_*} = 2.0\left(\frac{B}{H}\right)^{1.5}$
9	Seo and Cheong (1998)	$\frac{D^D}{HU_*} = 5.915\left(\frac{B}{H}\right)^{0.62}\left(\frac{U}{U}\right)^{1.428}$
10	Thackston	$D^D = 7.25\, HU_*\left(\frac{U}{U_*}\right)^{0.25}$
11	Patterson and Glony	$D^D = 0.8\exp(0.34\, U\sqrt{A})$
12	Parker	$D^D = 20.2\, R_hU\,\frac{\sqrt{g}}{C_{CH}}$
13	Lin	$D^D = 0.18\left(\frac{U_*}{U}\right)\frac{Q^2}{U_*\, R_h^3}$

Basing on the field measurements conducted in the gravitational sewer conduit with the dimension of 0.9 m and the length of 2015 m it was proposed to transform the (7) formula into the following dependence (HUISMAN et al. 2000):

$$D^D = 0.003\frac{B^2U^2}{H\,U_*}, \tag{8}$$

where H – mean sewer hydraulic depth (m).

The dispersion coefficient described with the above formula is similar to the one presented in table 1, position 5 – Fisher's proposal. The only difference is the value of empirical coefficient equals to 0.011.

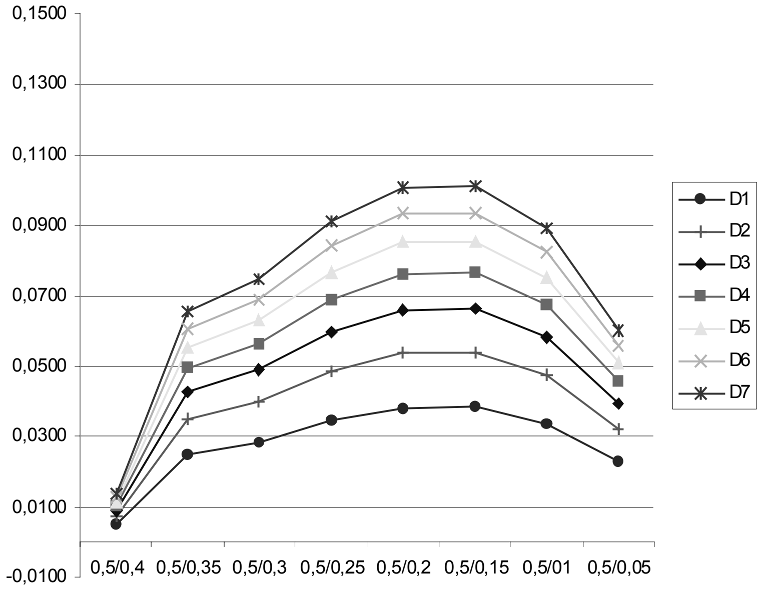

Fig. 1. Dispersion coefficient in function of conduit filling for different slopes, *s* from 0.001 to 0.007 (old concrete)

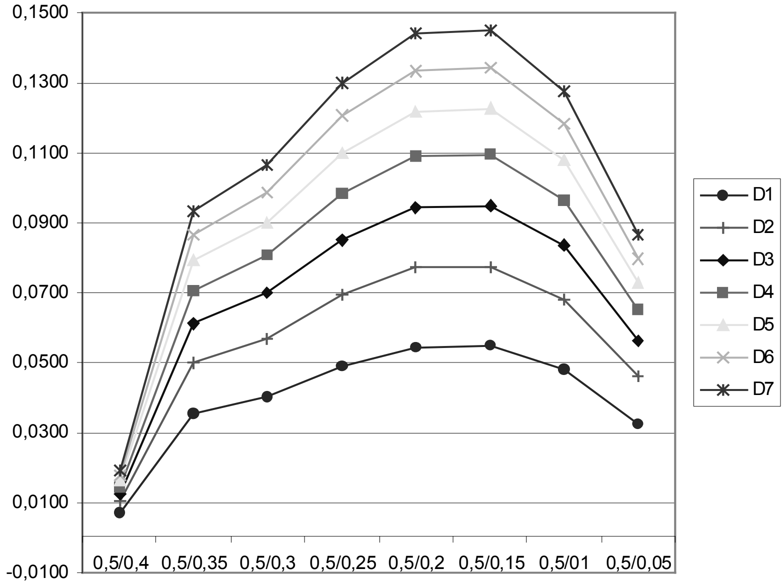

Fig. 2. Dispersion coefficient in function of conduit filling for different slopes, *s* from 0.001 to 0.007 (smooth concrete)

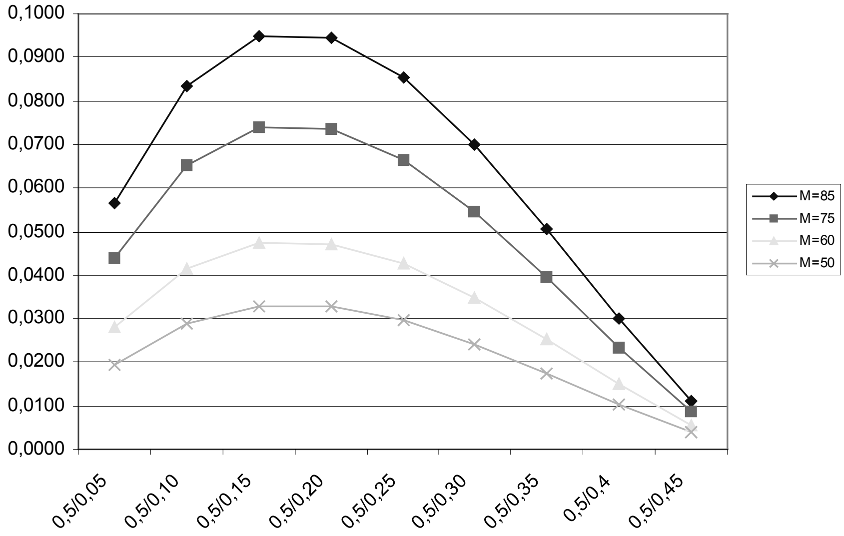

Fig. 3. Dispersion coefficient in function of conduit filling for different roughness coefficient *M*, and slope $s = 0.003$

Figure 1 shows the graphical presentations of dispersion coefficient changes in function of canal filling height inside the pipe made of an old concrete. The modelled canal is of a circular cross section shape, 0.5 m diameter. Its filling changes in range 0.4 m to 0.05 m, with the step of 5 cm. The successive curves present the dispersion coefficient D for the bottom inclination increasing for the following simulations by 0.001 in range from 0.001 for $D1$ curve to 0.007 $D7$ curve. The maximum values of dispersion coefficient for all inclinations were observed for filling in range from 0.15 to 0.2 m.

Figure 2 shows the similar situation to the one presented on figure 1 but the inner side walls of the sanitation pipe of the same shape and diameter are made of the smooth concrete. It causes the increased values of sewage flow inducing the increase of dispersion coefficient values. The shape of curves on figure 2 is similar to the earlier presented their shape is less flatten.

Figure 3 shows changes of the dispersion coefficient in function of conduit filling for the canal of geometrical parameters as mentioned above. The following curves represents the Manning-Strickler coefficient of roughness: *M* 50 represents the channel of an extremely poor technical condition or made of broken stones, *M* 60 – built of bricks, *M* 75 – made of concrete and *M* 85 made of smooth concrete. The conducted calculations show that, similarly to the earlier cases, the maximum values of dispersion coefficient was noted for the canal filling from 0.15 m to 0.2 m. The lowers values of dispersion coefficient were observed for the highest filling of the studied canal.

During the analysis of the particular curves one may note that the values of dispersion coefficient are the most diversified in the data group connected to the pipe made of smooth concrete. The lowest diversity was observed in the canal made of material described by the highest inner side wall roughness.

4. MATRIX-VECTOR DESCRIPTION OF THE FRACTIONS OF THE POLLUTANTS BIODEGRADATION IN THE FLOWING SEWAGE

The density flux of modelled components can be described as:

$$q = \overline{\rho} \bullet \hat{U} - \hat{D}\frac{\partial \overline{\rho}}{\partial x}, \tag{9}$$

where:

$\overline{\rho} \bullet \hat{U}$ – multiplication of the vector and the matrix,

$\overline{\rho}$ – concentration vector,

$\hat{D}$ – dispersion (matrix of the dispersion coefficients of the fractions),

$\hat{U}$ – velocity (matrix of the considered fractions velocities).

Each of the partial fraction flows with different velocity (different is velocity of dissolved, suspended or trailed fractions);

$$\overline{\rho} = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_n \end{bmatrix}, \quad \hat{U} = \begin{bmatrix} U^{11} & & & \\ & U^{22} & 0 & \\ & & \ddots & \\ & & & \ddots & \\ & 0 & & U^{NN} \end{bmatrix}, \quad \hat{D} = \begin{bmatrix} D^{11} & & & \\ & D^{22} & 0 & \\ & & \ddots & \\ & & & \ddots & \\ & 0 & & D^{NN} \end{bmatrix}. \tag{10}$$

The microbial transformation intensity of pollutant fractions is represented by the following formula:

$$\delta = \frac{d\overline{\rho}}{dt} = \hat{A}\overline{\rho}, \tag{11}$$

where:

$\hat{A}$ – integrated process matrix,

δ – source-drain element in advection-dispersion formula.

The total change of the fraction concentration, due to its flow and biochemical transformations can be written as:

$$\frac{\partial \overline{\rho}}{\partial t} = -\mathrm{div}\, q + \frac{d\overline{\rho}}{dt}. \tag{12}$$

Thus, the motion equation can be formed in the following way:

$$\frac{\partial \overline{\rho}}{\partial t} = \hat{A}\overline{\rho} - \operatorname{div} q\,, \tag{13}$$

$$\frac{\partial \overline{\rho}}{\partial t} = \hat{A}\overline{\rho} - \operatorname{div}\left(\overline{\rho} \bullet \hat{U} - \hat{D}\frac{\partial \overline{\rho}}{\partial x} \right), \tag{14}$$

$$\frac{\partial \overline{\rho}}{\partial t} = \hat{A}\overline{\rho} - \frac{\partial}{\partial x}(\overline{\rho} \bullet U) + \frac{\partial}{\partial x}\left(\hat{D}\frac{\partial \overline{\rho}}{\partial x} \right), \tag{15}$$

where:

$\hat{A}\overline{\rho}$ – source element,

$\frac{\partial}{\partial x}(\overline{\rho} \bullet U)$ – advection element with mean flow,

U – average cross-sectional velocity,

$\frac{\partial}{\partial x}\left(\hat{D}\frac{\partial \overline{\rho}}{\partial x} \right)$ – dispersion element.

5. SOLVER AND RESULTS

The presented formula is a set of the partial differential equations written in the vector form. To make the calculations possible it should be provided with accordingly defined boundary and initial conditions.

As a main numerical module it was applied modified PDECOL system (van der LINDE et al. 1997; WANG et al. 2004, 2004a; HU et al. 2006). Adapted numerical system solves nonlinear partial differential equations of the second order for multidimensional fields.

The field is represented by $u(k)$ function being the solution of the realized task (vector of particular modelled components changes in time).

The solver calculates numerical tasks for the system of partial differential equations described in the following form:

$$\frac{\partial u}{\partial t} = f\left(t, x, u, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2} \right), \tag{16}$$

where:

$\frac{\partial u}{\partial t}$ – vector of partial derivative solutions u in time t,

$\frac{\partial u}{\partial x}$ – first order spatial derivative of solutions,

$\frac{\partial^2 u}{\partial x^2}$ – second order spatial derivative of solutions,

t – time variable,

x – spatial variable.

Depending on the realized equation the component of vector u is a concentration distribution along the channel of relevant fraction of pollutant, biomass or oxygen, for which the initial condition is the concentration of considered component in the solutions vector u. The flow velocity, stream cross section, and dispersion coefficients of fractions are calculated in parallel according to the conduit parameters.

6. DISCUSSION AND CONCLUSIONS

The numerical model based on the matrix (10) uses the kinetic and stoichiometric parameters of sewage bio-degradation, considering also the saprobe biomass existing in the sewer system.

The biomass is being treated as the biological process factor of transformation occurring inside the sanitation conduit and also as the forerunner of the active sludge in wastewater treatment plant.

The presented model was also equipped in the module of hydrodynamic calculations of gravitational sanitation pipe based on Saint-Venant equations.

The results of hydrodynamic calculations provide the data necessary to the simulation of dynamic biochemical processes described by the advection–dispersion equation with the source element. The source element contains the integrated matrix of biochemical processes, describing the processes of atrophy and generation of components considered by the model of sewage biodegradation in the gravitational sanitation conduit in aerobic conditions.

The dispersion coefficients used in the equations may be calculated by the formulas based on similar dependences used in description of transport in natural watercourses The results of calculations show that the dispersion coefficient in the conduit of circular cross section reaches its maximum value for about 25% of maximum filling, regardless of pipe inclination or its inner side walls roughness.

The presented model may be useful to determine the dynamic of changes in pollutants load entering the wastewater treatment plants through the system of sanitation conduits. This model may be also useful in predicting the influence of storm spillways on the wastewater receiver.

The simultaneous modelling of wastewater flow and biochemical processes enables more precise determination of heterotrophic organisms development, both in suspended biomass and in sewer biofilm placed on the inner side of pipe.

Thus, the determination of pollutants transformation rate (grade) based on the shape, length, and conduit bottom inclination is possible. This feature allows an easy adaptation of the presented numerical model to the simulation of biodegradation processes occurring in the wastewater of different physical and chemical properties, in sanitation conduits of known geometry.

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