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QUANTITATIVE EVALUATION OF VETO POWER

The decisiveness index and the loose protectionism index for a single player have been introduced, starting from the decisiveness and the loose protectionism indices for a collective decision-making mechanism defined by Carreras. Attention was mainly focused on the latter index, being proposed as a quantitative measure of the power of veto of each agent. According to this index, a veto player has veto power equal to one, while each other player has a fractional power according to her/his likelihood of blocking a given proposal. Such an index coincides with the expected payoff at the Bayesian equilibrium of a suitable Bayesian game, which illustrates the non-cooperative point of view of a decision-making mechanism.

Keywords: *veto power, indices, quantitative measure, Bayesian game*

1. Introduction

The United Nations Security Council (UNSC) represents the most typical example of an assembly where some actors are endowed with veto power. It is composed of 5 permanent members (a protecting philosophy designed by the five winning countries of World War II, China, France, Russian Federation, United Kingdom and the United States, during the postwar period) and 10 non-permanent members for two-year terms starting on January 1st, with five replaced each year. Each Council member has one vote and decisions on procedural matters are made by an affirmative vote of at least 9 of the 15 members. Also, decisions on substantive matters require 9 votes but the concurring votes of all 5 permanent members are needed. This rule, called great power

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unanimity, is often simply referred to as veto power and the 5 permanent members are called veto players*.

Among others, the functions and powers of the UNSC under the Charter are to maintain international peace and security in accordance with the principles and purposes of the United Nations, which is the primary responsibility but also to investigate any dispute or situation which might lead to international friction, to take military action against an aggressor, etc. The UNSC alone has the power to take decisions which member states are compelled to carry out under the Charter.

As observed by Mercik [14], quite intuitively the right of veto will increase the power of a player in most cases. This is exactly the case in the UNSC, where the five permanent members have great power in the decision process for substantive matters, not only because of the absence of a term-period, but mainly because of their right to veto.

According to the definition of Tsebelis [21], in his theory of veto players, a veto player is an individual or collective actor whose agreement is necessary for policy changes. Policy stability, i.e. the impossibility of significant changes in the status quo, is strictly related to the role of veto players, as a significant policy change has to be approved by all of them. In his extensive literature on the topic, Tsebelis analyzes the connections between veto players and other important features: agenda control or production of significant laws, for example, are strictly related to this topic and the theory of veto players represents for Tsebelis a way to unify the understanding of politics.

Having stated that the power of veto represents a central topic in politics, it is natural to ask: how should we evaluate it? This question has led in recent years to an increasing number of papers and surveys on the topic but the attention paid to veto power indices in the literature is still less than that devoted to power indices. Two questions arise at first: are veto power and power analogous concepts? Can we evaluate them with the same instruments? The wide range of existing power indices take into account different features, like the ordering of the players in a coalition (Shapley-Shubik index [20]), different majorities (Banzhaf-Coleman index [1] and [5]), the importance of minimum winning coalitions (Public Good Index [9] and Deegan–Packel Index [6]), and of quasi-minimum winning coalitions (Johnston index [11]), as well as the weights of the players (weighted Shapley value [12]). In order to better study political situations, possible ideological affinities have been analyzed, accounting for possible existing relations among the players (Owen value [18]) or drawing graphs which represent the possible connections between the players (Myerson index [16] and FP indices [7] and [4]).

Our idea is that various features have to be considered in order to define an index suitable for analyzing the power of veto. A party, for example, could be able by itself

*For further details on the UNSC see http://www.un.org/Docs/sc/index.html to block a proposal voting against it, but it may not be able to pass any law without the support of other parties. This happens, for example, in the case of a permanent member of the UNSC, which has full veto power but not full power according to classical indices. Moreover, the concepts of a priori unions ([18, 19]) and/or of connected coalitions ([16, 7, 4]), which have been introduced to better represent the relations between parties, are no longer relevant when speaking about the power of a party which is against the approval of a proposal. In order to block a proposal, it is not necessary anymore to have a common ideological position: two parties very far from each other can, for opposing reasons, decide to vote against a law, even if this does not mean that they would agree in making a common alternative proposal.

In the work of Carreras [2, 3] there is a detailed analysis of blocking power in a simple game, both from a collective and individual point of view. The strict protectionism index and the Banzhaf strict blocking power index are proposed as suitable tools to deal with this problem. The blockability relation was formalized by Ishikawa and Inohara [10] and just a couple of years after, Kitamura and Inohara [13] proposed the blockability index, which is a power index for coalitions. Another important contribution to this problem is given by Mercik [15] who studies the problem of evaluating the power of veto starting from the Johnston power index and based on some suitable axioms that an index for veto power should satisfy.

In this paper, we present a quantitative approach to the problem of evaluating the power of veto. We start from the observation that it is not necessary anymore that the power of the agents sum to a given fixed number, which is normally assumed to be equal to 1, as one, two, or all the agents of a voting procedure may have full power to block a proposal. We will define an index which assigns full veto power (for simplicity equal to 1) to all those agents who are able to veto a proposal alone. All the other players will be given a non-negative veto power smaller than 1 according to their ability to block a motion of other players. After some preliminaries in Section 2, we recall the work of Carreras in Section 3, from which the idea of this paper is taken, and we present the proposed index to evaluate veto power in Section 4. In Section 5, we give Bayesian formulation of the model, which is a good way to deal with the problem from a non-cooperative point of view. Section 6 concludes.

2. Preliminaries

A cooperative TU game is a pair (N, v), where $N = \{1, 2, ..., n\}$ is a nonempty finite set of players and v: $2N \to R$ is the characteristic function which associates with every nonempty subset S of N, called coalition, a real number $v(S)$, which represents the worth of S. (N, v) is simple if $v(S) \in \{0, 1\}$ for each $S \subseteq N$, $v(N) = 1$ and given $S, T \subseteq N$, $S \subseteq T \Rightarrow v(S) \leq v(T)$. When $v(S) = 1$, we say that the coalition is winning, otherwise it is losing. For further details, we address the reader to the book by Osborne and Rubinstein [17].

We call W the set of all the winning coalitions. A veto player is a player i that belongs to all the winning coalitions, i.e. for each $S \in W$, $i \in S$. A dictator is a player that can win without any support, i.e. $\{i\} \in W$. Given a coalition $S \in W$, a player $j \in S$ is critical for S if $S\backslash\{j\} \notin W$. The quantity $v(S) - v(S\backslash\{j\})$ is called the marginal contribution of player j w.r.t. S. We say that a winning coalition is minimal if each proper subcoalition is losing and we denote the set of all the minimal winning coalitions by W^m.

Political situations are frequently described by simple games, which are often defined through weighted majority situations. Let N be the set of parties in a Parliament, in our model the players of the game. A vector of weights $w = (w_1, w_2, ..., w_n)$ is associated to N, where w_i, $i \in N$ is a non negative weight given to each party that may represent, for example, the number of seats it has. Fixing a majority quota q, i.e. the number of votes needed in order to pass a proposal, we obtain a weighted majority situation denoted by $[q; w_1, w_2, ..., w_n]$. Given a weighted majority situation, it is possible to define the corresponding weighted majority game (N, v) where the characteristic function v: $2^N \rightarrow \{0, 1\}$ is

$$v(S) = \begin{cases} 1 & \text{if } \sum_{j \in S} w_j \geq q \\ 0 & \text{otherwise} \end{cases} \qquad \forall S \subseteq N$$

A simple game can always be described by the set W (or simply the set W^m), and in this case it is denoted by (N, W), while it is not always possible to define it as a weighted majority game. Defining the game directly by giving the set of its winning coalitions allows us to describe a greater range of scenarios in which, for example, a player is endowed with veto power independently of the number of seats, as happens in the case of the UNSC.

3. Decisiveness and loose protectionism indices

In this section, we recall the work of Carreras [2] that is mainly based on the idea of providing a numerical measure of the ability to take action under some collective decision-making mechanism. Given a game (N, W), the set of coalitions 2^N splits into four classes, namely:

- D (decisive winning): $S \in W$ such that $N \backslash S \notin W$;
- C (conflictive winning): $S \in W$ such that $N \backslash S \in W$;
- Q (blocking): $S \notin W$ such that $N \backslash S \notin W$;
- P (strictly losing): $S \notin W$ such that $N \backslash S \in W$.

Thus, $W = D \cup C$. The family Q is called the blocking family. The game is strong if $Q = \varnothing$ and weak otherwise. The game is proper if $C = \varnothing$ and improper otherwise. When a game is proper and strong, it is called decisive.

Carreras defines the decisiveness index of the game (N, W) as

$$\delta(N,W) = \frac{|W|}{2^n}$$

where $n = |N|$. This gives the probability that an abstract proposal will pass in the game (N, W), where each agent $i \in N$ has only two options*: voting for the proposal (Y) or voting against it (N), each occurring with probability 1/2. We remark that the actions of the agents are assumed to be independent. The motion will pass if and only if the set of agents that vote for Y is a winning coalition, i.e. $S \in W$. Obviously $0 < \delta(N, W) < 1$ as $\varnothing \notin W$ and $N \in W$. Given two simple games with the same set of players, $|W| < |W'|$ implies that $\delta(N, W) < \delta(N, W')$. If a game is decisive, then $\delta(N, W) = 1/2$ independently of the number of players involved. Since no improper and weak weighted majority game exists, in this subclass the index 1/2 characterizes decisive games. Carreras also observes that when a game is weak and proper, the decisiveness index is smaller than 1/2 and when it is improper and strong, the index is greater than 1/2.

Let (N, W) be a simple game and (N, W^*) be the dual game where $W^* = \{S \subseteq N : N \backslash S \notin W\}$. Thus, $W^* = D \cup Q$ and

1. $\delta(N, W^*) + \delta(N, W) = 1$
2. $\delta(N, W^*) - \delta(N, W) = (|Q| - |C|)/2^n$.

The dual game clearly suggests the following definition of a protectionism index, which Carreras calls the loose protectionism index, based on the idea of providing a numerical measure of the inertia of any decision-making mechanism, using the formula

$$\delta^*(N, W) = \delta(N, W^*) = 1 - \delta(N, W)$$

This gives the probability that a proposal will not pass in (N, W), where again each agent has the options to vote for or against the proposal, each occurring with probability 1/2. This index is suggested as a possible choice to define a collective blocking index for simple games to measure the capability of blocking at collective level, analyzing the game from the viewpoint of protectionism. As $\delta^*(N, W) = 1 - \delta(N, W)$, in

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*Abstaining is allowed but it counts as a vote "against".

[3] Carreras observes that it does not provide any new information, preferring to adopt an index based on the strict notion of blocking. The strict protectionism index is thus defined as $\pi(N, W) = |Q|/2^n$.

Then he defines a blocking swing for player $i \in N$ as a pair $(S, S)\backslash\{i\})$ s.t. $S \in Q$ and $S\backslash\{i\} \notin Q$ and he calls the Banzhaf strict protectionism index an individual blocking power index for the players involved, defined as

$$\rho_i(N, W) = \xi_i(N, W)/2^{n-1}$$

where $\xi_i(N, W)$ is the number of blocking swings for player i.

4. Decisiveness and loose protectionism indices of a player

Now we introduce a new index to measure the decisiveness of a game from the perspective of each player, which gives the probability that a proposal will pass in (N, W) when we already know that player i votes in favour of the proposal (Y) and the other players vote in favour or against, each with probability 1/2. Let w_i be the set of winning coalitions including player i, i.e. $W_i = \{S \in W : i \in S\}$.

Definition 1. The decisiveness index of player i is defined as

$$\delta_i(N,W) = \frac{|W_i|}{2^{n-1}} \tag{1}$$

Obviously, $0 < \delta_i(N, W) \leq 1$, as $N \in W_i$. If the outcome A stands for "the proposal will pass" and B for "player i votes in favour", this index corresponds to the conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Proposition 1. When i is a veto player, $\delta_i(N, W) = 2\delta(N, W)$

Proof. Writing this conditional probability as $P(A|B) = P(B|A)P(A)/P(B)$ and noting that $P(B) = 1/2$, $P(A|B) = \delta_i(N, W)$ and $P(A) = \delta(N, W)$, we obtain $\delta_i(N, W) = 2P(B|A)\delta(N, W)$. When i is a veto player $P(B|A) = 1$, thus $\delta_i(N, W) = 2\delta(N, W)$.

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Similarly, we can define the probability that a proposal will not pass in (N, W) when we know that player i votes against the proposal (N) and the others vote in favor or against, each with probability 1/2.

Definition 2. The loose protectionism index of player i is defined as

$$\delta_i^*(N,W) = \frac{2^{n-1} - |W| + |W_i|}{2^{n-1}} = 1 - 2\delta(N,W) + \delta_i(N+W) \quad (2)$$

Obviously, $0 < \delta_i^*(N, W) \leq 1$ as $\varnothing \notin W$. The numerator counts the number of losing coalitions which do not include player i, i.e. the number of coalitions without player i voting in favor of the proposal and unable to approve it.

Remark 1. We may observe that player i is a dictator iff $\delta_i(N, W) = 1$ and is a veto player iff $\delta_i^*(N, W) = 1$. The indices $\delta_i(N, W)$ and $\delta_i^*(N, W)$ are strictly related and this relation depends on the decisiveness index of the game. In particular,

- if the game is weak and proper, as $\delta(N, W) < 1/2$, we obtain $\delta_i^*(N, W) > \delta_i(N, W)$ for each $i \in N$;
- if the game is strong and improper, by duality $\delta(N, W) > 1/2$ and $\delta_i^*(N, W) < \delta_i(N, W)$ for each $i \in N$;
- if the game is decisive, then a player is a veto player ($\delta_i^*(N, W) = 1$) iff it is a dictator ($\delta_i(N, W) = 1$). In general, when the game is decisive, $\delta_i(N, W) = \delta_i^*(N, W)$ for each $i \in N$. In a weighted majority game, as there are no weak and improper games, we are always able to say whether the players will have a higher power or a higher power of veto, or whether they are equivalent.

The two indices are also directly related as stated in the following proposition.

Proposition 2. $\delta_i^*(N, W) = \delta_i(N, W^*)$ for every $i \in N$.

Proof. We have $\delta_i(N, W^*) = \dfrac{|W_i|}{2^{n-1}}$ and $\delta^*(N, W) = \dfrac{2^{n-1} - |W| + |W_i|}{2^{n-1}}$.

Let $D_i = \{S \in D : i \in S\}$, $C_i = \{S \in C : i \in S\}$, $Q_i = \{S \in Q : i \in S\}$ and $P_i = \{S \in P : i \in S\}$. We want to show that

$$2^{n-1} - |W| + |W_i| = |W_i^*|$$

i.e.

$$|D_i| + |C_i| + |Q_i| + |P_i| - |D| - |C| + |D_i| + |C_i| = |D_i| + |Q_i|$$

$$|D_i| + 2|C_i| + |P_i| = |D| + |C|$$

and this is true as $|D_i| + |P_i| = |D|$ and $2|C_i| = |C|$. In fact,

$$|D_i| + |P_i| = |\{S \in W, i \in S: N \backslash S \notin W\}|$$

$$+ |\{S \notin W, i \in S: N \backslash S \in W\}| = |\{S \in W: N \backslash S \notin W\}| = |D|$$

and

$$|C_i| = |\{S \in W, i \in S: N \backslash S \in W\}| = |\{S \in W, i \notin S: N \backslash S \in W\}| = |C \backslash C_i| = |C| - |C_i|$$

■

As observed in [2], the Banzhaf index [1] also measures the decisiveness of a game from the perspective of each player. In particular, let (N, W) be a game and $i \in N$, then

$$\beta_i(N, W) = 2\delta(N, W) - 2\delta(N_{-\{i\}}, W_{-\{i\}})$$

where $(N_{-\{i\}}, W_{-\{i\}})$ denotes the residual game that arises when player i leaves, i.e. the subgame restricted to the players from the set $N \backslash \{i\}$. This basic relationship between the decisiveness index and the Banzhaf index suggests that we should look for a possible relation between the Banzhaf index and the indices defined in (1) and (2). When $i \in N$ is a veto player, as Carreras noticed $\beta_i(N, W) = 2\delta(N, W)$, then from Proposition 1 we simply obtain $\beta_i(N, W) = \delta_i(N, W)$.

The indices in (1) and (2) are quantitative indices and they evaluate the power of a player to pass (δ_i) or reject (δ_i^*) a proposal.

In Example 1, we compute these indices for a simple theoretical situation. For sake of completeness, we add the Banzhaf strict protectionism index, ρ, proposed by Carreras [3], and the Johnston index [11], J, as suggested by Mercik [15]. The comparison is carried out using the ratios of the indices for pairs of players, as not all the indices sum up to one.

Example 1. Consider a simple weighted majority situation [6; 2, 3, 5] representing a Parliament with only three parties. Thus $N = \{1, 2, 3\}$, with 2, 3 and 5 seats respectively and a majority quota of 6. The winning coalitions are $\{1, 3\}$, $\{2, 3\}$ and $\{1, 2, 3\}$.

The decisiveness index of the game is

$$\delta(N, W) = \frac{3}{8}$$

and the loose protectionism index is

$$\delta^*(N, W) = \frac{5}{8}$$

We now evaluate the decisiveness index and the loose protectionism index of the parties

$$\delta_1(N,W)=\frac{1}{2},\qquad \delta_2(N,W)=\frac{1}{2},\qquad \delta_3(N,W)=\frac{3}{4}$$

$$\delta_1^*(N,W)=\frac{3}{4},\qquad \delta_2^*(N,W)=\frac{3}{4},\qquad \delta_3^*(N,W)=1$$

We observe that player 1 has full veto power, i.e. is a veto player, while no player is a dictator. This is an example of a weak and proper game. Thus the loose protectionism indices of the players are greater than their decisiveness indices.

Evaluating the Banzhaf strict protectionism index and Johnston index, we obtain

$$\rho_1(N,W)=\frac{1}{4},\qquad \rho_2(N,W)=\frac{1}{4},\qquad \rho_3(N,W)=\frac{1}{4}$$

$$J_1(N,W)=\frac{1}{6},\qquad J_2(N.W)=\frac{1}{6},\qquad J_3(N,W)=\frac{2}{3}$$

The Banzhaf strict protectionism index assigns the same power of blocking to each party, in particular also to party 3, which is a veto player. The Johnston index assigns to party 3 four times the power given to the others, while according to our index of veto, it has only four thirds of the power of the other parties.

The sum of the veto power of the agents may be lower than 1, but this requires that there is a large number of winning coalitions, and consequently a small number of blocking coalitions. One simple situation is represented by a restricted committee that has to decide which proposal may be admitted to a large assembly vote (e.g. a parliamentary commission that has to decide which laws can be discussed by the Parliament). Usually, a very low majority is required, just to avoid wasting time on proposals that are of no interest to anybody. If we suppose that the committee is formed by 7 representatives and it is necessary that at least two of them vote in favor of discussing it, we see that the blocking coalitions are those of 6 or 7 players. Thus the veto power of each person is 7/64 and their sum is 49/64.

5. A Bayesian model

In this section, we present a new model, following the idea of Harsanyi [8] of a noncooperative game in which the players do not have complete information regarding the game itself. In this way, we are able to extend the model to the situation in which the agents do not vote in favor or against a proposal with probability 1/2 but with a different probability distribution. The lack of information is due to the fact that the agents can predict the preferences and, consequently, the behavior of the other players, but they cannot be sure about it. Thus they can only evaluate an expected payoff and try to maximize that.

A game with incomplete information played by Bayesian players, or simply a Bayesian game, is a 5-tuple (N, $\{C_i\}_{i\in N}$, $\{T_i\}_{i\in N}$, $\{p_{ik}\}_{i\in N,\, k\in T_i}$, $\{u_i\}_{i\in N}$), where:

- N is the set of players,
- C_i is the set of actions of player i,
- T_i is the set of types of player i,
- p_{ik} is the probability that player i is of type k, with $k \in T_i$, $\Sigma_{k\in T_i} p_{ik} = 1$,
- $u_i : \Pi_{j\in N} C_j \times \Pi_{j\in N} T_j \rightarrow \mathbb{R}$ is the utility function of player i.

A pure strategy for player i is a function $s_i : T_i \rightarrow C_i$ and Σ_i is the set of all the pure strategies of i. A mixed strategy for player i is a function $\sigma_i : C_i \times T_i \rightarrow [0, 1]$ with $\Sigma_{c\in C_i}\, \sigma_i(c, t) = 1$ for each $t \in T_i$.

In Example 1, $N = \{1, 2, 3\}$ is the set of players, i.e. the parties of the Parliament. Adopting a non-cooperative approach, we assume that the parties, instead of cooperating, vote independently. Each one has two choices: voting yes (Y) or voting no (N), then $C_i = \{Y, N\}$ for each $i \in N$. The types of the parties can be identified with their ideological position regarding the proposal: supporting (P) or in favor of the status quo (Q), then $T_i = \{P, Q\}$ for each $i \in N$. A probability is assigned to the types of the players, in our model equal to 1/2, then $p_{ik} = 1/2$ for each $i \in N$, $k \in T_i$. These probabilities may represent, more generally, the players' assessment of the probabilities with which each of the others are of a given type. However, we take a simplified situation in which these probabilities are all equal and given a priori. In our example, every voter knows that each other player can be in favor or against the proposal, both with probability 1/2. The outcome of the game is given by "the law is approved", if the parties which voted Y have a total number of seats greater than or equal to the majority quota, otherwise "the law is not approved". The payoff of each party is 1 if it is of type P and the law is approved or if it is of type Q and the law is not approved, otherwise the payoff is 0. Formally,

$$u_i(s_1, \ldots, s_n) = \begin{cases} 1 & \text{if } T_i = P \text{ and } \sum_{j\in N: s_j(T_j)=Y} w_j \geq q \\ 1 & \text{if } T_i = Q \text{ and } \sum_{j\in N: s_j(T_j)=Y} w_j < q \\ 0 & \text{otherwise} \end{cases}$$

In Table 1, we present the game in strategic form, where the payoffs of the parties are shown for 8 possible configurations, starting from the case where they are all of type P, in favor of the proposal, finishing with the case of where they are all of type Q, in favor of the status quo. In each situation, called a state of nature, we assume that party 1 chooses the row, 2 the column and 3 the matrix. The former choice for each of them corresponds to voting *Y*, the latter one to voting *N*. Each player has a utility of 1 when the outcome is consistent with the type of that player, 0 otherwise.

Table 1. Strategic form of the game

$(1_P, 2_P, 3_P)$	$\begin{pmatrix}(1,1,1) & (1,1,1)\\(1,1,1) & (0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0,0) & (0,0,0)\\(0,0,0) & (0,0,0)\end{pmatrix}$
$(1_P, 2_P, 3_Q)$	$\begin{pmatrix}(1,1,0) & (1,1,0)\\(1,1,0) & (0,0,1)\end{pmatrix}$	$\begin{pmatrix}(0,0,1) & (0,0,1)\\(0,0,1) & (0,0,1)\end{pmatrix}$
$(1_P, 2_Q, 3_P)$	$\begin{pmatrix}(1,0,1) & (1,0,1)\\(1,0,1) & (0,1,0)\end{pmatrix}$	$\begin{pmatrix}(0,1,0) & (0,1,0)\\(0,1,0) & (0,1,0)\end{pmatrix}$
$(1_P, 2_Q, 3_{Q)})$	$\begin{pmatrix}(1,0,0) & (1,0,0)\\(1,0,0) & (0,1,1)\end{pmatrix}$	$\begin{pmatrix}(0,1,1) & (0,1,1)\\(0,1,1) & (0,1,1)\end{pmatrix}$
$(1_Q, 2_P, 3_P)$	$\begin{pmatrix}(0,1,1) & (0,1,1)\\(0,1,1) & (1,0,0)\end{pmatrix}$	$\begin{pmatrix}(1,0,0) & (1,0,0)\\(1,0,0) & (1,0,0)\end{pmatrix}$
$(1_Q, 2_P, 3_Q)$	$\begin{pmatrix}(0,1,0) & (0,1,0)\\(0,1,0) & (1,0,1)\end{pmatrix}$	$\begin{pmatrix}(1,0,1) & (1,0,1)\\(1,0,1) & (1,0,1)\end{pmatrix}$
$(1_Q, 2_Q, 3_P)$	$\begin{pmatrix}(0,0,1) & (0,0,1)\\(0,0,1) & (1,1,0)\end{pmatrix}$	$\begin{pmatrix}(1,1,0) & (1,1,0)\\(1,1,0) & (1,1,0)\end{pmatrix}$
$(1_Q, 2_Q, 3_Q)$	$\begin{pmatrix}(0,0,0) & (0,0,0)\\(0,0,0) & (1,1,1)\end{pmatrix}$	$\begin{pmatrix}(1,1,1) & (1,1,1)\\(1,1,1) & (1,1,1)\end{pmatrix}$

In order to illustrate the complexity of the problem, in Fig. 1 we show the situation in extensive form, where black dots represent the choices of nature which selects the type of each party, both types occurring with the probability of 1/2. Then each player (white dots) has to take its own decision, selecting an action, either *Y* or *N*. Finally one of the 64 possible outcomes is selected.

Each party knows its own type, but not the types of the other two parties. As it assigns the probability of 1/2 to each possible type of other parties, it assigns the probability of 1/4 to each possible state of nature. If party 1, for example, is of type *P*, it will assign the probability of 1/4 to each one of the first four states of nature shown in Table 1. Assuming that party 2 will play $(p, 1 - p)$ and party 3 $(q, 1 - q)$, the expected payoff of player 1 when s/he plays *Y* is

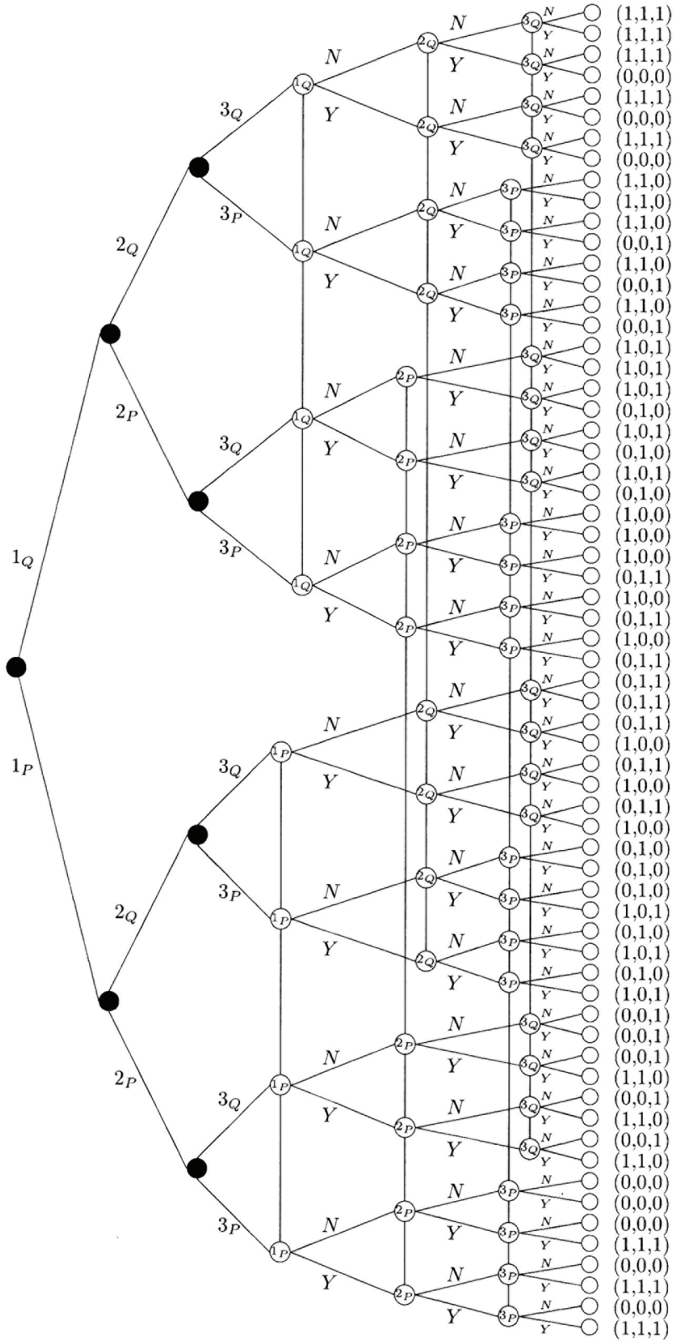

Fig. 1. Extensive form of the game

$$\frac{1}{4}\left[pq+(1-p)q\right]+\frac{1}{4}\left[pq+(1-p)q\right]+\frac{1}{4}\left[pq+(1-p)q\right]+\frac{1}{4}\left[pq+(1-p)q\right]=q$$

and when s/he plays N is

$$\frac{1}{4}[pq]+\frac{1}{4}[pq]+\frac{1}{4}[pq]+\frac{1}{4}[pq]=pq$$

Thus the best choice for party 1 when it is of type P is to play $(t, 1-t)$ with $t = 1$ if $p < 1$ and $t \in [0, 1]$ if $p = 1$. Deriving the best reply for both types of each player, we obtain the obvious result that the optimal strategy for the players is to choose Y if they are of type P and N if they are of type Q. This is the Bayesian pure equilibrium of the game. One interesting result is that when the probabilities of the types are both equal to 1/2, by playing the equilibrium strategy each party of type P can obtain an expected utility equal to its decisiveness index and each party of type Q an expected utility equal to its loose protectionism index.

6. Concluding remarks

In this paper, we have proposed a new quantitative index for measuring the veto power of each agent in a decisional situation. Our main aim was to evaluate veto power starting from the observation that a classical veto player, i.e. an agent whose approval is required to pass a proposal, has full veto power with the corresponding index being equal to one. Consequently, the power of the other agents represents the fraction of veto power they have in comparison to full veto power and the sum of the power of all the agents does not have to be equal to one.

Referring to the decisiveness and loose protectionism indices introduced by Carreras [2] for evaluating the characteristics of the game as a whole, we introduced the decisiveness and loose protectionism indices for a player, extending the results of Carreras, in order to have a measure of the role of each player, as well as analyzing some properties of these two indices.

Then we proposed a more specific index that takes into account from a quantitative point of view the influence of a player in rejecting a proposal when s/he votes against it. This index simply counts the frequency of the situations in which the final outcome is negative when her/his choice is negative. This approach provides a general evaluation of veto power, in any possible decision ballot. It is clear that in a more particular setting, i.e. a vote on a given proposal, the behavior and the preferences of each decision maker may be forecasted based on previous analogous situations or on specific information that the agents received in the past. We decided to represent this situation as a Bayesian game in which the agents may have two types, in favor of the proposal or in favor of the status quo. In Section 5 we assigned to each agent a fixed probability for each type, in order to have a unique representation of the game. Of course when they play the game, the agents may assign probabilities that better represent their beliefs about the types of the other players, accounting for their private information, their personal experience and expertise, and their knowledge of the situation at hand. We want to stress that the computationally simple loose protectionism index of the players coincides with the Bayesian equilibrium of the Bayesian game when the probabilities of both types are equal to 1/2 for each player.

This work is a step forward in the evaluation of the veto power of decision-makers, but other features of the problem may be introduced into the model, in order to have a more suitable measure of their power.

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