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EFFICIENCY OF THE RECEIVABLE ROTATION RATIO AS A MEASURE OF THE ACCOUNTS RECEIVABLE SETTLEMENT PERIOD

Accounts receivable forecasting, due to a wide range of applications in a number of financial decisions, is an important area of a company's financial forecasts. Estimation of accounts receivable involves finding solutions for two fundamental problems: forecasting the level of daily sales revenues and estimating the accounts receivable settlement period. This paper addresses the issue of constructing reliable models to be used in forecasting accounts receivable due on deliveries and services – the trend model and the simple regression model. The postulated models, together with a range of alternative methods suggested in the literature on the field, are tested to examine the quality of the resulting estimates of the A/R settlement period. The methods postulated in the paper prove to be functional – on the one hand, the results are reliable, on the other hand, the postulated models can be constructed using readily available data.

Keywords: *accounts receivable model, receivables collection period, accounts receivable forecast*

1. Introduction

The accounts receivable (A/R) variable is a subject of financial forecasting in a company ([1], p. 240). Forecasting of accounts receivable is used in the following areas of company finance:

- cash level management, where forecasted accounts receivable help predict company cash inflow,
- working capital management, with forecasted accounts receivable being one of the variables deciding on the level of demand for working capital in a company,
- company development policy, with forecasted values of accounts receivable and other variables serving as basis for predicting changes in current assets and, consequently, shaping the forecast of cash return on investment,

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- financing company development, with the forecast of accounts receivable being one of the forecast variables taken into consideration in predicted financial statements, i.e. the so-called *pro forma* balance sheet and cash flow account,
- short-term financing, where monitoring and forecasts of accounts receivable are used to determine parameters for a firm's trade crediting policy,
- management of a company's financial standing, with forecasts of accounts receivable being one of the aspects of determining predicted financial standing, including forecasted financial vulnerability and bankruptcy,
- company value management, with forecasts of accounts receivable being one of the factors used to predict changes in the value of current assets and, consequently, construction of the cash flow forecasts needed to evaluate the current value of a company,
- credit risk management, where forecasts of accounts receivable are one of the areas involved in the process of evaluating the financial standing of contractors.

The literature in this field presents various methods for forecasting accounts receivable. These include methods based on:

- receivable rotation ratio ([2], p. 66–72),
- DSO (days sales outstanding) index ([4], p. 137–142),
- regression models, describing accounts receivable in terms of a regression function ([3], p. 371–372).

The paper presents models based on trend and regression analysis of the accounts receivable due on company deliveries and services. These models, together with other methods postulated in the literature in this field, are tested with respect to their utility in predicting the outstanding period of receivables collection, i.e. one of the parameters used in a receivables forecast model.

All our analysis is based on trade receivables, as the conclusions relate solely to sales revenue.

2. Model of a company's accounts receivable

The accounts receivable due on deliveries and services result from postponed payment for shipped deliveries and services rendered to the recipient. Consequently, the level of accounts receivable, N , is influenced by the level of sales revenue and the established timeframe for payment.

Let us momentarily assume that a company offers their sales on payment terms of net 14 days, with all accounts receivable to be settled in due time. The payment term stated on the invoice is the theoretical period of account settlement. Let us denote this timeframe by the symbol T . At the end of the 14th working day, the level of company accounts receivable amounts to:

$$N_{14} = P_1 + P_2 + \dots + P_{14}. \quad (1)$$

On the following day (day 15), part of the accounts receivable will be settled, amounting to the sales figures from day 1. At the same time, the current level of accounts receivable will be augmented by new receivables, equal to the sales figures from day 15:

$$N_{15} = P_1 + P_2 + \dots + P_{14} - P_1 + P_{15}. \quad (2)$$

In general, therefore, the level of accounts receivable on day n ($n \geq T$) will be equal to the sum of the last T daily sales figures P_t :

$$N_n = \sum_{t=n-T+1}^n P_t. \quad (3)$$

In general, predicting the level of accounts receivable on deliveries and services requires solving the following two fundamental problems:

One problem is the estimation of the empirical A/R settlement period, denoted here by Te . The settlement period Te (known also as the days of sales outstanding) is usually different from the one stated on the invoice (T). At the same time, one may safely assume that Te is greater than T , since the decision to settle before the time due is irrational, because mercantile credit is a form of free external capital (settling before due time may, however, be considered useful, e.g. as an argument for price discounts).

The other problem is the forecasting of the daily sales revenues P_t^* . The forecast of accounts receivable on day n is the sum of the forecasts of daily sales revenues for a period of Te :

$$N_n^* = \sum_{t=n-Te+1}^n P_t^*. \quad (4)$$

There are five potential methods for estimating the period of accounts settlement Te .

1. According to the simplest method, the settlement period Te may be calculated from the date of issuing the first overdue invoice. The dates of invoices are numbered here in order 1, ..., n , with T^* being the index of the date of the first overdue invoice issued. Consequently, Te is given by:

$$Te = n - T^* + 1. \quad (5)$$

As an example, let us analyze the level of accounts receivable on the 20th sales day. The first overdue invoice was issued regarding the 2nd sales day, thus the A/R settlement period Te for the company under study will be $Te = 20 - 2 + 1 = 19$ days.

The above method for determining Te is applicable only for case studies involving just one contractor (customer) and requires access to much more detailed data. In practice, we are forced to estimate Te based on aggregated data, i.e. the level of accounts receivable shown in financial reports refers to the sum of trade transactions involving multiple recipients, with no reliable method of accessing detailed information on the dates of issue for particular overdue invoices. For the most part, we have to operate on the basis of cumulative daily sales revenue (commonly known as total revenue in financial statements). In such cases, the problem of estimating Te may be solved using analytical methods, based on an assumed form for the trend in daily sales revenues.

Let TP_n denote cumulative daily sales revenues for the whole of the period between 1 and n :

$$TP_n = \sum_{t=1}^n P_t. \tag{6}$$

2. If the cumulative daily sales revenue is decreased by the level of accounts receivable for the period of n , i.e. $TP_n - N_n$, the result will describe the level of cumulative daily sales revenue, for which the accounts receivable have already been settled. This fraction of the revenue will correspond to the period of $t = 1, \dots, T^*$ (see Figure 1).

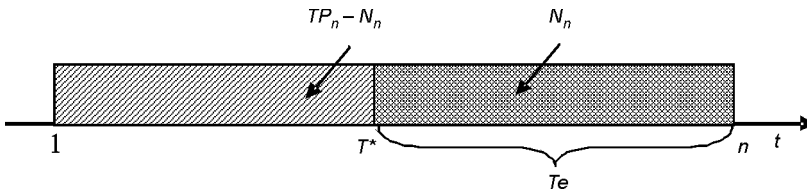


Fig. 1. Calculating the A/R settlement period Te based on the difference between cumulative daily sales revenue TP_n and accounts receivable N_n . Source – author’s study

In such a case, the A/R settlement period Te is calculated from the equation:

$$Te = n - T^*. \tag{7}$$

Both daily revenue and cumulative daily sales revenue may be expressed as functions of time:

$$P_t = f(t), \tag{8}$$

$$TP_t = f(t), \tag{9}$$

where t ($t = 1, \dots, n$) denotes the ordered numbers of consecutive sales days. By substituting TP_T by the difference $TP_n - N_n$, $t = T^*$ can be calculated, and one can calculate Te from T^* . Therefore, the problem of estimating Te is reduced to calculating the

function describing daily sales revenues, P_t which is then used to estimate the function describing cumulative daily sales revenue TP_T .

3. Another, the third, method for estimating the A/R settlement period Te involves estimating the parameters of the accounts receivable model, including the parameter Te . Models of accounts receivable may be computed on the basis of daily sales revenue models.

4. The most popular method of estimating the period of accounts settlement is the accounts receivable rotation ratio (known also as $DSO_{\bar{P}}$) and defined as:

$$W_n = \frac{N_n}{TP_n} n \quad (10)$$

The idea of this ratio is based on the assumption of constant daily sales revenues. It is thus possible to estimate the A/R settlement period by dividing the accounts receivable on day n by the average daily sales revenue \bar{P} . The relevant assumption is $\bar{P} = P_t$. In fact, this assumption is almost always unreal and using the average *cumulative daily* revenue gives very poor estimates of the A/R settlement period. This issue will be discussed in more detail later in this paper.

5. The fifth method is based on the DSO_{LIFO} ratio. Calculation of the DSO ratio using the LIFO method involves correction of the accounts receivable at time n by the amount of A/R from previous reporting periods, i.e. from $n, n - 1, \dots$, up to the moment of 'zeroing' the level of accounts receivable. The number of days required to 'zero' the accounts receivable level is recognized as the settlement period:

$$DSO_{LIFO} = Te + N_n - \sum_{t=n-Te+1}^n P_t \quad (11)$$

Below two different models of daily sales revenue have been considered, functions describing cumulative daily revenue and accounts receivable are constructed for each. Both models are then used to estimate the A/R settlement period.

2.1. Constant daily sales revenues

In general, the assumption that daily sales revenue is constant should be regarded as false, since it implies failure on the part of the company to develop with time. As such, the assumption may be considered valid only over a short timeframe and be regarded as transitory. However, this variant is addressed here to present the basic assumptions that underlie the most frequently used method for forecasting accounts receivable, namely the method based on the accounts receivable rotation ratio.

Assuming that the daily sales figures are constant, i.e.:

$$P_t = a \quad (12)$$

the cumulative daily sales revenue is a linear function, as expressed by the equation:

$$TP_t = at \quad (13)$$

Equation 11 is solved as follows: The form of the function describing cumulative daily sales revenue is derived from the equation describing the sum of the elements in an arithmetic progression:

$$S_n = n \frac{2a_1 + (n-1)r}{2} \quad (14)$$

Daily sales revenue figures P_t are elements of an arithmetic progression where the difference r between any two successive elements of the progression equals 0, while all elements equal a . The cumulative daily sales revenue for the period t equals the sum of the daily sales revenues for the period 1, ..., t :

$$TP_t = t \frac{2a + (t-1) \cdot 0}{2} = at. \quad (15)$$

To calculate the A/R settlement period, Te , i.e. to calculate the sales outstanding period for which payment has not yet been settled, the difference between the level of cumulative daily sales revenue on day n and the level of accounts receivable for the same period is applied to the function TP_T :

$$TP_n - N_n = at. \quad (16)$$

Hence, T^* is calculated as follows:

$$T^* = \frac{TP_n - N_n}{a}. \quad (17)$$

Example 1. The daily sales of a company on 20 consecutive days were 10 thousand PLN. On the 20th day, the amount of accounts receivable amounted to 100 thousand PLN, while the cumulative daily sales revenue equalled 200 thousand PLN. Let us calculate the A/R settlement period, Te , that determines the number of days for which the accounts have not yet been settled. For this purpose, let us determine T^* :

$$T^* = \frac{200 - 100}{10} = 10,$$

followed by Te : $Te = 20 - 10 = 10$.

To verify these findings, let us calculate the levels of: daily sales revenue, P_t , cumulative daily sales revenue, TP_t , and accounts receivable, N_t , for $Te = 10$ (see Table 1 and Figure 2).

Table 1. Sales revenue and accounts receivable, data for Example 1. Source – author’s study

T	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
P_t	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
TP_t	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
N_t	10	20	30	40	50	60	70	80	90	100	100	100	100	100	100	100	100	100	100	100

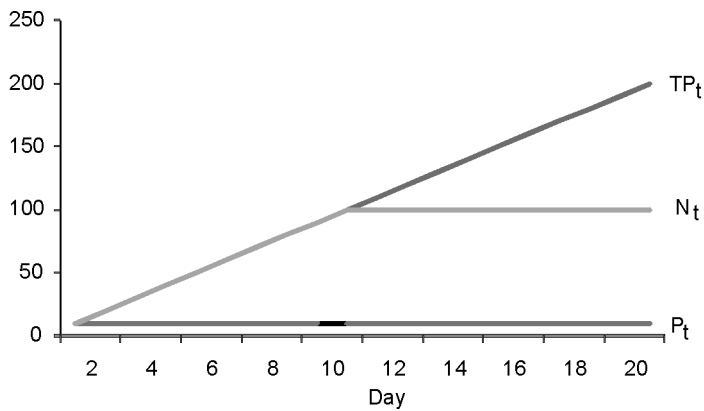


Fig. 2. Sales revenue and level of accounts receivable, data for Example 1. Source – author’s study

After defining the trend function describing daily sales revenue, we can also plot the trend function of accounts receivable for $t \geq Te$. In the case where $P_t = a$, the trend function describing accounts receivable takes the form:

$$N_t = Te \times a . \tag{18}$$

Equation 16 is solved as follows: The accounts receivable N_t are elements in an arithmetic progression where the progression difference r equals 0. The first element a_1 denoted by N_{Te} of the progression N_t is the sum of daily revenues P_t from the period $t = 1, \dots, Te$:

$$a_1 = N_{Te} = \sum_{t=1}^{Te} P_t = Te \times a . \tag{19}$$

The second element a_2 , denoted by N_{Te+1} , is given by

$$a_2 = N_{Te+1} = \sum_{t=Te+1-Te+1}^{Te+1} P_t = \sum_{t=2}^{Te+1} P_t = Te \times a . \quad (20)$$

The progression difference r is

$$a_2 - a_1 = N_{Te+1} - N_{Te} = Te \times a - Te \times a = 0 . \quad (21)$$

As the equation for the n th term of an arithmetic progression is expressed as follows:

$$a_n = a_1 + (n-1)r , \quad (22)$$

the function describing accounts receivable, N_t , is given by:

$$N_t = Te \times a + (t-1) \cdot 0 = Te \times a .$$

The trend functions describing accounts receivable and daily sales revenue are of the same analytical form. This fact bears some important consequences on another viable method of forecasting company accounts receivable – the regression function – which typically describes accounts receivable as a function of cumulative daily sales revenue:

$$N_t = f(TP_t) . \quad (23)$$

In the light of the above deliberations, such an approach proves false, since the functions describing N_t and TP_t do not share the same analytical form. In the case under study, i.e. with daily sales revenues P_t and accounts receivable N_t regarded as constant, and cumulative sales revenue TP_t being described by a linear trend function, accounts receivable and cumulative sales revenue are not correlated. However, there will be a dependence of accounts receivable on daily sales revenue:

$$N_t = Te \times a = TeP_t . \quad (24)$$

Assuming that daily sales revenues are constant, the period of settling accounts receivable Te will equal:

$$Te = \frac{N_t}{P_t} . \quad (25)$$

Using this relation, we can now calculate Te for the case described in Example 1:

$$Te = \frac{100}{10} = 10 .$$

The result is correct.

As stated before, the assumption that daily sales revenues are constant is a fundamental factor in the estimation of Te based on the accounts receivable rotation ratio, herein referred to as W_n . According to the notation used throughout this paper, the accounts receivable rotation ratio can be expressed as

$$W_n = \frac{N_n}{TP_n} \times n = \frac{Te \times a}{na} \times n = Te. \quad (26)$$

By calculating the accounts receivable rotation ratio and applying it to the calculation of Te in Example 1, we arrive at results consistent with those obtained using equation (25):

$$W_n = \frac{100}{200} \times 20 = 10 = Te.$$

It must be noted, however, that identical results from estimating Te can only be guaranteed under the assumption that daily sales revenues are constant.

The ratio DSO_{LIFO} for Example 1 equals 10, since

$$DSO_{LIFO} = 10 + 100 - \sum_{t=20-10+1}^{20} P_t = 10 + 100 - 100 = 10.$$

The estimated settlement period in this case corresponds to the one calculated using the accounts receivable model.

2.2. Linearly increasing daily sales revenues

Assuming that daily sales revenues can be expressed using a linear trend function,

$$P_t = at, \quad (27)$$

where a represents a constant rate of increase in daily sales revenues, the function describing cumulative sales revenue TP_t takes the following form:

$$TP_t = \frac{a}{2}t^2 + \frac{a}{2}t. \quad (28)$$

Equation (28) was solved as follows: As in the earlier example, the analytical form of the function describing TP_t was determined using the equation for the sum of an arithmetic progression. In this case, daily sales revenues are elements of an arithmetic progression where the progression difference is equal to a .

$$P_1 = a, P_2 = 2a, \dots, P_t = at. \quad (29)$$

By applying equation (14), we can now determine TP_t :

$$TP_t = t \frac{2a + (t-1)a}{2} = \frac{a}{2}t^2 + \frac{a}{2}t.$$

T^* is the positive root of the following quadratic equation:

$$\frac{a}{2}t^2 + \frac{a}{2}t - (TP_n - N_n) = 0. \quad (30)$$

The settlement period, Te , is calculated from Equation (7).

Example 2. Daily sales in a company increase on average by 10 thousand PLN each day. On the 20th day of sales, the level of cumulative sales revenue amounted to 2100, while the accounts receivable level grew to 1550. Let us calculate the A/R settlement period, Te , i.e. the number of days for which payments due have not yet been settled. The function TP_t takes the following form:

$$TPt = \frac{10}{2}t^2 + \frac{10}{2}t = 5t^2 + 5t.$$

The difference between the level of cumulative sales revenue and accounts receivable on the 20th day of sales equals

$$TP_{20} - N_{20} = 2100 - 1550 = 550.$$

T^* is the positive root of the following quadratic equation

$$5t^2 + 5t - 550 = 0.$$

Hence,

$$\Delta = 5^2 - 4 \cdot 5 \cdot (-550) = 11025$$

$$T^* = \frac{-5 + \sqrt{11025}}{2 \cdot 5} = 10$$

$$Te = 20 - 10 = 10.$$

To verify the above results, let us calculate the levels of: daily revenue P_t , cumulative daily revenue TP_t and accounts receivable N_t for $Te = 10$ (see Table 2 and Figure 3).

Table 2. Sales revenue and accounts receivable, data for Example 2. Source – author’s study

t	1	2	3	4	5	6	7	8	9	10
P_t	10	20	30	40	50	60	70	80	90	100
TP_t	10	30	60	100	150	210	280	360	450	550
N_t	10	30	60	100	150	210	280	360	450	550
t	11	12	13	14	15	16	17	18	19	20
P_t	110	120	130	140	150	160	170	180	190	200
TP_t	660	780	910	1050	1200	1360	1530	1710	1900	2100
N_t	650	750	850	950	1050	1150	1250	1350	1450	1550

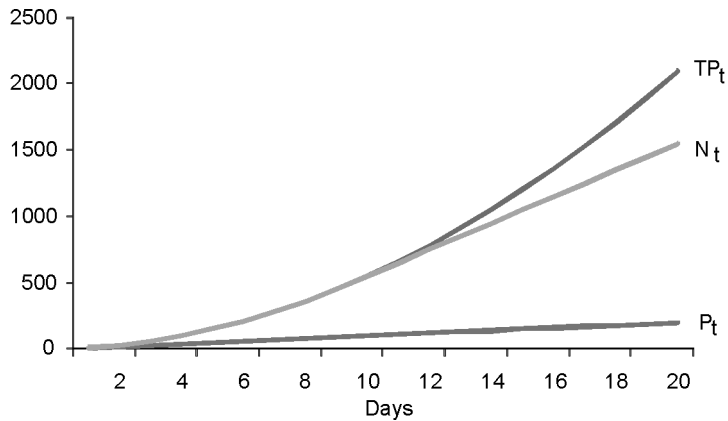


Fig. 3. Sales revenue and level of accounts receivable, data for Example 3. Source – author’s study

Based on the function describing daily sales revenue, we can determine the equation for the function describing accounts receivable for $t \geq Te$, which at $P_t = at$ takes the following form:

$$N_t = atTe - \frac{a}{2}Te^2 + \frac{a}{2}Te. \tag{31}$$

Equation (29) was solved as follows: To determine the function describing accounts receivable for $t \geq Te$ under the assumed linear form of the function describing daily sales revenue, let us use the fact that accounts receivable over a period of t days are the sum of Te daily sales revenues. Under the assumption of linearly increasing daily sales revenues, the accounts receivable, N_t for $t \geq Te$, are elements of an arithmetic progression with the difference r equal to $a \times Te$. The first element of this arithmetic progression is a sum of Te daily revenues P_t . However, because daily sales revenues

have a linear trend, sales revenue figures are elements of a different arithmetic progression with the difference r equal to a (see Equation (29)). By applying Equation (14), the first element of the progression describing accounts receivable is, therefore, a sum of Te elements of the progression describing daily sales revenues:

$$N_{Te} = \sum_{t=1}^{Te} P_t = Te \frac{2a + (Te-1)a}{2} = \frac{Te^2 a + Te \times a}{2}. \quad (32)$$

The second element is given by

$$N_{Te+1} = \sum_{t=2}^{Te} P_t = Te \frac{4a + (Te-1)a}{2} = \frac{Te^2 a + 3Te \times a}{2}. \quad (33)$$

The progression difference is given by

$$r = N_{Te+1} - N_{Te} = \frac{Te^2 a + 3Te \times a}{2} - \frac{Te^2 a + Te \times a}{2} = Te \times a. \quad (34)$$

The function describing accounts receivable, N_t , for $t \geq Te$, is calculated from the equation for the n th term of an arithmetic sequence (see equation (22)):

$$N_t = \frac{Te^2 a + Te \times a}{2} + (t - Te + 1 - 1)Te \times a = Te \times at - \frac{a}{2}Te^2 + \frac{a}{2}Te.$$

As in the previous calculations, accounts receivable may then be expressed as a function of daily sales revenues, by substituting at in equation (31) for P_t :

$$\begin{aligned} N_t &= TeP_t + b \\ b &= \frac{a}{2}Te(1 - Te). \end{aligned} \quad (35)$$

In this model, the coefficient of the variable P_t equals the A/R settlement period Te . Information gathered using this model is of great value to managers but practical application of this model brings up several problems. The first problem is related to limited access to daily sales revenue data. Another issue involves accounts receivable, as their level is measured in “continuous” fashion, i.e. part of the accounts receivable disclosed in a financial report for a given period refers to sales revenue effected in the previous reporting period. The contrary holds true in the case of sales revenues measured ‘afresh’ from zero in each consecutive reporting period. Due to these considerations, for the purpose of estimating Te , a better suited method is the one described above, i.e. the method based on the function describing the cumulative daily sales revenue. Information on the level of revenue is, in most cases, readily available.

For the purpose of illustration, let us append the calculations for Example 2 with the appropriate accounts receivable rotation ratio:

$$W_n = \frac{1550}{2100} \times 20 = 14.76 \approx 15.$$

In the case where daily sales revenues are described by a linear trend function, the accounts receivable rotation ratio provides a false estimate of the accounts receivable settlement period.

The value of DSO_{LIFO} for Example 2 equals

$$DSO_{LIFO} = 10 + 1550 - \sum_{t=20-10+1}^{20} P_t = 10 + 1550 - (200 + 190 + \dots + 120 + 110) = 10.$$

The estimated settlement period in this case corresponds to the one calculated using the accounts receivable model.

The literature in this field notes that estimation of the A/R settlement period using the DSO_{LIFO} index offers reliable results ([4], p. 137). Indeed, the results obtained in the examples presented above prove the validity of the DSO_{LIFO} index but this method requires access to reliable data on daily sales revenue. In practical applications, this type of data is often unavailable.

3. Conclusions

Estimation of the A/R settlement period based on the A/R rotation ratio leads, in general, to false results. The A/R rotation ratio calculated using cumulative daily revenue does not measure the A/R settlement period at all. The only exception to this rule is the case where daily sales revenues are constant, which is an unrealistic assumption.

Estimation of the settlement period based on the DSO_{LIFO} index offers correct results but in practical applications this method proves limited, since it requires access to reliable and detailed data on daily sales revenues. This problem mainly exists when access to financial data is limited.

The regression function $N_t = f(TP_t)$ is not a suitable tool for describing company accounts receivable – no dependence exists between accounts receivable and cumulative daily revenue. Any such relations found in practical applications are purely coincidental.

The paper postulates the following models of accounts receivable:

- assuming constant daily sales revenues, i.e. $P_t = a$;
- model for trend in accounts receivable: $N_t = Te \times a$,
- regression function for accounts receivable: $N_t = Te \times P_t$;

- assuming linearly increasing daily sales revenues, i.e. $P_t = at$:
 - model for trend in accounts receivable: $N_t = Te \times at - \frac{a}{2}Te^2 + \frac{a}{2}Te$
 - regression function for accounts receivable: $N_t = TeP_t - b$

In both cases, the level of accounts receivable in a company changes with the level of daily sales revenues and with the settlement period.

Estimation of the A/R settlement period based on the A/R models presented above offers reliable and valid results from readily accessible data, under the provision of the assumptions imposed on daily sales revenue.

The models of accounts receivable constructed herein may be utilized in forecasting accounts receivable.

Time series directly related to sales revenue, A/R time series included, often involve a periodic component. Consequently, the A/R models presented herein need to be complemented by a variable representing seasonal fluctuations in sales.

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