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DEPTH-DURATION-FREQUENCY RAINFALL MODEL FOR DIMENSIONING AND MODELLING OF WROCLAW DRAINAGE SYSTEMS

The paper presents the method for model formation to describe the maximum rainfall depth in Wrocław for practical applications. Two models have been formulated: a probabilistic model for rainfall frequency range from $C = 1$ year do $C = 100$ years, and a physical model for frequency range from $C = 1$ year do $C = 0.1$ year. The probabilistic model can be used for dimensioning and modeling effects of sewage or combined sewage systems, for the required EN 752 standard for drainage area. This model, due to the time range of rainfalls (up to 3 days) can be also suitable for determining the volume of the storage reservoirs. The physical model, in turn, allows determination of storm overflows number per year.

1. INTRODUCTION

Extreme natural phenomena such as sudden or long-lasting storms, often accompanied by floods or sewer outflows intensifying over the last decades bring about significant economical losses. This should force us to continuously improve dimensioning rules for sewage systems.

Safe design of sewage systems aims at ensuring proper standard of an area drainage, which is defined as a sewage system adaptation to take forecast precipitation water flows with a frequency equal to the permissible – socially acceptable frequencies of flooding occurrences [1]. According to the EU standard EN 752, recommended design rainfall frequencies are from $C = 1$ year for rural areas to $C = 10$ years for underground transportation facilities and underpasses (Table 1). The standard restricts the occurrence frequency of flooding from sewage systems to repeatability: once per

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10 years in case of rural areas and once per 50 years for underground transportation facilities and underpasses.

Table 1

The recommended design frequencies of rainfall and permissible flooding occurrence frequencies according to EN 752

Design rainfall frequency [1 per C years]	The area drainage standard category	Flooding occurrence frequency [1 per C years]
1 per 1	Out of town areas (rural)	1 per 10
1 per 2	Residential areas	1 per 20
1 per 5	City centers, service and industry area	1 per 30
1 per 10	Underground transportation facilities, underpasses, etc.	1 per 50

The sizing of sewage systems or combined sewage systems in Poland presents difficulties resulting from the lack of a reliable local IDF and DDF precipitation models. Model of Błaszczyk from 1954 lowers calculation results for rainfall intensities by 40%, which has been shown on the example of precipitation measured in Wrocław [2]. Precipitation model of IMGW (the Institute of Meteorology and Water Management) from 1998 [3] is inconsistent because of its underestimation of the rainfall intensity frequency to once a year as shown in [1, 4]. This has negative consequences when sizing drainage areas in Poland according to the recommendations of European standard EN 752 [1, 5–7]. This standard follows the Directive of the European Committee for Standardization (CEN) on harmonized requirements for the protection of areas against flooding from sewers in the member states of the European Union.

Previous precipitation models relate to the rain frequency no less than once a year ($C \geq 1$ year). However, some design tasks demand meteorological data of rainfall occurring several times a year ($C < 1$ year). Namely, when sizing the storm overflows, quantitative or qualitative criteria for the protection of the receiver waters against pollution should be included expressed as the maximum number of stormwater discharges during the year or allowable pollutant load. In Poland, the quantitative criterion is limited value of the average annual number of stormwater discharges, depending on the type of sewage system. For example, waste water from storm overflows of combined sewage system may be discharged to surface waters if the average annual number of discharges does not exceed 10 [1]. The argument that the designed storm overflow drains sewage to the receiver not more than 10 times a year can be tested only by hydrodynamical simulations under various scenarios of rainfall. These scenarios can be measured series of intense rainfalls, model rains (example of Euler type II) [8, 9], or randomly generated synthetic rainfalls [10–15].

The authors of the present paper assumed the criterion of precipitation amount $h \geq 0.75t^{0.5}$, based on the Chomicz criterion: $h_{U_0} \geq t^{0.5}$ related to the limit precipitation amount for strong rainfalls. The criterion was used to separate intensive rainfalls for statistical analyses of precipitation amount. The assumed criterion of interval precipitation amounts (16 intervals of durations from 5 min to 3 days) allowed one to isolate a number of the most intensive rainfalls in each year from the period of 1960–2009. The total of 514 synthetic rainfall instances were selected for detailed statistic analysis [2] from the period of 50 years of observations (Table 2).

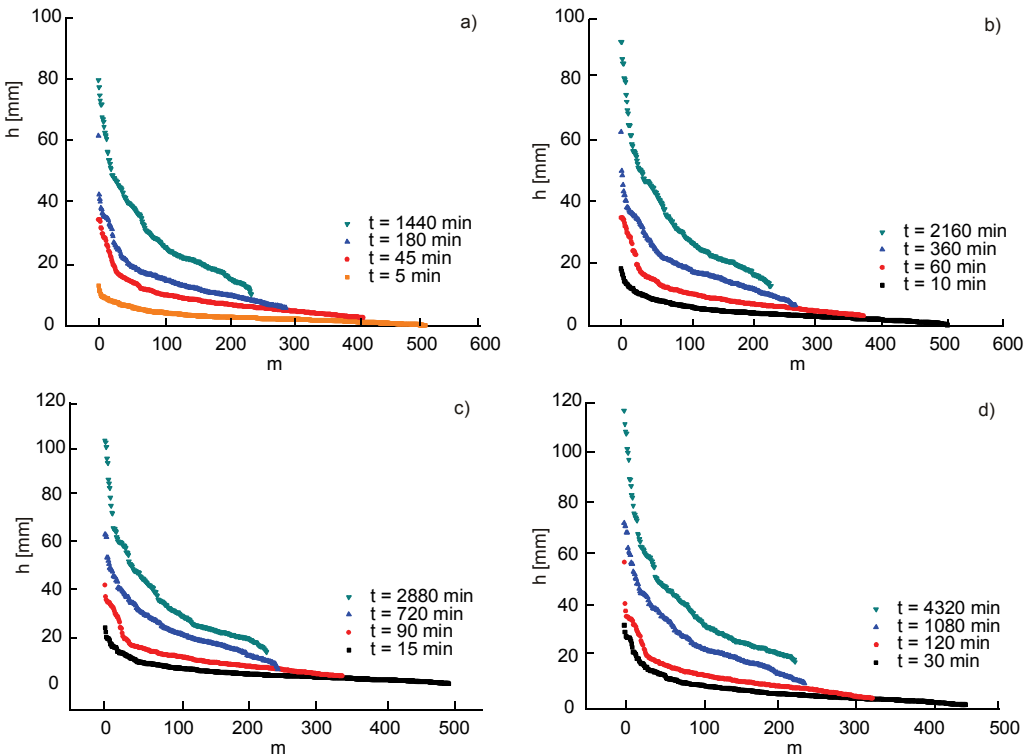


Fig. 1. Empirical cumulative distribution functions of the highest precipitation amounts in Wrocław

Empirical cumulative distribution functions of the highest precipitation amounts from the 50-year measurement period are presented in Fig. 1.

3. THE PROBABILISTIC PRECIPITATION MODEL FOR $C \geq 1$ YEAR

To determine the probability distribution of a random variable, the empirical non-exceeding probability should be assigned to particular sample elements. The empirical

probability distribution is constructed based on the observed time dependences of the precipitation amounts. From the period of 50 years of observations, the 50 highest precipitation amounts were selected. The concept of the empirical distribution results directly from a partial probability interpretation [2]:

$$p(m, N) = \frac{m}{N+1} = \frac{1}{C(m, N)} \quad (1)$$

where m is a line number (series) in an integral sequence: $m = 1, \dots, 50$, N – an observation sequence size, $N = 50$ years.

Based on the literature data for description of precipitation phenomena, the following distributions were used: Fisher–Tippett of type I_{\max} , Fisher–Tippett of type III_{\min} , lognormal, Pearson of type III and generalized exponential distribution [2, 23–25]. The estimates of the parameters of the density function were determined by maximum likelihood method. Then, using a λ -Kolmogorov test, the accuracy of the theoretical distributions with the empirical distribution was verified [26]. Test rejected lognormal distribution as unsuitable for the theoretical description of the measurement data on the significance level $\alpha = 0.05$ [2]. The other four probability distributions were evaluated by the Bayesian information criterion (BIC) [27], in order to determine the best one. BIC criterion allowed one to choose between two distributions: Fisher–Tippett of type III_{\min} and Pearson of type III, as suitable to describe the maximum precipitation amounts recorded at the meteorological station Wrocław-Strachowice. Then two probabilistic models allowing one to calculate the maximum precipitation amount for the duration of $t \in [5; 4320]$ min and frequency of $C \in [1; 100]$ years were formulated. Based on comparative analysis of two probabilistic models with the measured data, a model based on the Fisher–Tippett of type III_{\min} distribution was considered to be better [2]. This model has been discussed in details.

The density function of the distribution of the Fisher–Tippett type III_{\min} occurs in the following form [28]:

$$f(x) = \beta \alpha^\lambda (x - \varepsilon)^{\lambda-1} e^{-\alpha^\lambda (x - \varepsilon)^\lambda} \quad (2)$$

hence the credibility function in a logarithmic form is

$$\ln L = N \ln \lambda + N \lambda \ln \alpha + (\lambda - 1) \sum_{i=1}^N \ln (x_i - \varepsilon) - \alpha^\lambda \sum_{i=1}^N (x_i - \varepsilon)^\lambda \quad (3)$$

where: α is the scale parameter, λ – the shape parameter, ε – the lower limit, mm.

Values of the distribution lower limits were estimated as $\varepsilon_i = h_{\max i} - 0.1$ mm for $P(50, 50) = 0.98$. Applying the method of maximum likelihood, the parameters α and β were determined from the equations:

$$\begin{cases} \frac{1}{\alpha} - \alpha^{\lambda-1} \overline{(x-\varepsilon)^\lambda} = 0 \\ \frac{1}{\lambda} + \ln \alpha - \alpha^\lambda \ln \lambda \overline{(x-\varepsilon)^\lambda} + \overline{\ln(x-\varepsilon)} = 0 \end{cases} \quad (4)$$

Parameters of the distribution of Fisher's–Tippett's type III_{min} calculated for the highest precipitation amounts in Wrocław in the period of 1960–2009 and durations $t \in [5; 4320]$ min are given in Table 3.

Table 3

Values of parameters α , λ and ε
for the distribution of Fisher's–Tippett's type III_{min}

Time [min]	α	λ	ε [mm]
5	0.5034	1.1828	6.3
10	0.3208	1.1603	8.8
15	0.1964	1.3405	10.0
30	0.1603	1.1139	13.6
45	0.1357	1.0620	14.7
60	0.1241	1.0434	15.2
90	0.1088	1.0617	16.2
120	0.1052	1.0826	17.8
180	0.0934	1.3226	19.9
360	0.0923	1.6363	26.1
720	0.0829	1.5616	31.9
1080	0.0706	1.4920	36.4
1440	0.0710	1.2399	39.8
2160	0.0669	1.1289	45.1
2880	0.0613	1.0304	48.0
4320	0.0425	1.3294	48.9

Quantiles of a random variable for the distribution of Fisher's–Tippett's type III_{min} were calculated from the formula:

$$x_p = \varepsilon + \frac{1}{\alpha} (-\ln p)^{1/\lambda} \quad (5)$$

Empirical (obtained in measurements) and theoretical (calculated from the distribution of Fisher's–Tippett's type III_{min}) cumulative distribution functions are plotted in Fig. 2.

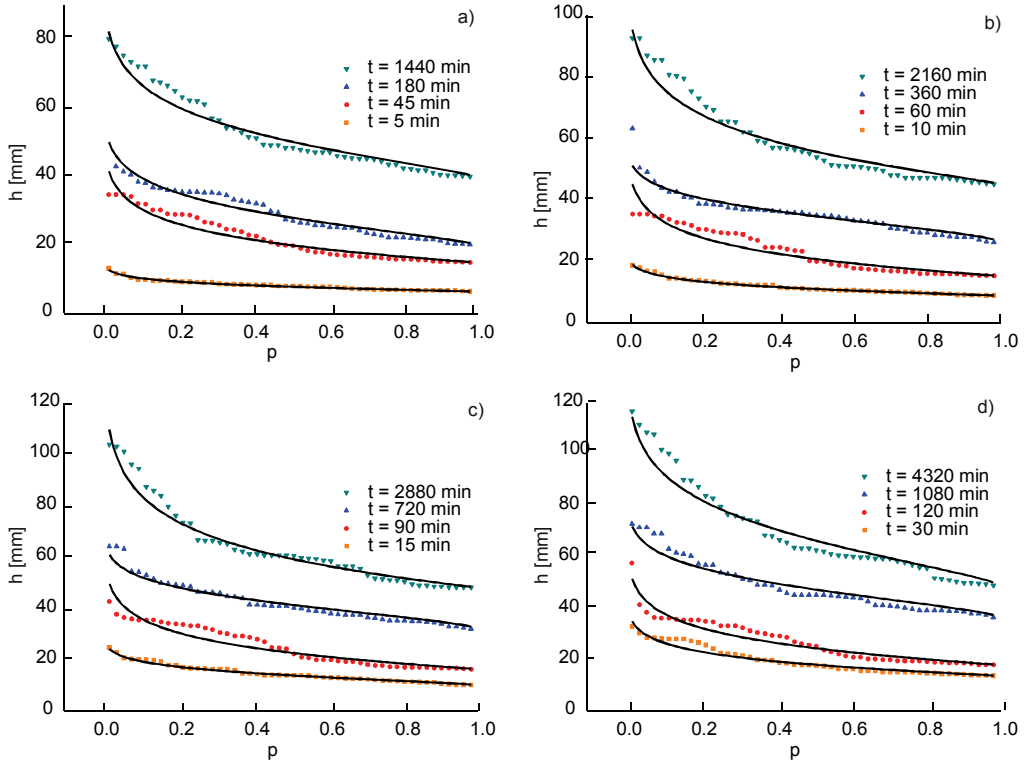


Fig. 2. Theoretical cumulative distribution functions for the distribution of Fisher's-Tippett's type III_{min}

Since there is no pattern of the dependence of λ on t , the mean value of $\bar{\lambda} = 1.2367$ was assumed for calculations (Table 3). The dependence of the coefficient α on the precipitation duration t was described (at $R^2 = 0.986$) with the function:

$$\alpha = (97.11t^{0.0222} - 98.68)^{-1} \quad (6)$$

while the dependence of the coefficient ε on t (at $R^2 = 0.992$) with the formula:

$$\varepsilon = -4.58 + 7.41t^{0.242} \quad (7)$$

Taking into account that $p = 1/C$, for maximum precipitation amounts in Wrocław for the duration of $t \in [5; 4320]$ min and frequency of $C \in [1; 100]$ years according to Eq. (5), the probabilistic model will assume the formula (h_{\max} in mm):

$$h_{\max}(t, C) = -4.58 + 7.41t^{0.242} + (97.11t^{0.0222} - 98.68) \left(-\ln \frac{1}{C} \right)^{0.809} \quad (8)$$

4. THE PHYSICAL PRECIPITATION MODEL FOR $C \leq 1$ YEAR

To determine the physical model for maximum precipitation amount in Wrocław with the frequency up to 10 times a year ($C = [0.1, 1]$), Eq. (7) was adopted as its upper limit. Assumption of the lower limit in the probabilistic model (for $C \geq 1$) as the upper limit in the physical model (for $C \leq 1$) will keep the continuity of the results of calculation for both models. Physical model was proposed as follows:

$$h_{\max} = \varepsilon - \beta \ln C = -4.58 + 7.41t^{0.242} - \beta \ln C \quad (9)$$

where β is an empirical coefficient.

Due to the incomplete measurement material (Table 2), the physical model was limited to rainfalls with duration up to 360 min. Considering practical applications, limiting the scope of applicability of model for 6 h appears to be justified. Rain of about 6 h duration, with an average sewage flow velocity of 1.0 m/s, allows verification of the system hydraulic capacity of the collector ca. 20 km long.

Estimation of the β parameter in Eq. (9) was carried with the method of least squares for 10 analyzed rainfall durations (Table 4).

Table 4

Results of calculations of parameters ε and β in Eq. (9)

Time t [min]	ε from Eq. (7), [mm]	β	R^2
5	6.36	-2.395	0.978
10	8.36	-2.961	0.978
15	9.69	-3.371	0.976
30	12.3	-4.361	0.985
45	14.04	-5.052	0.993
60	15.38	-5.753	0.993
90	17.44	-6.582	0.987
120	19.03	-7.207	0.981
180	21.46	-8.412	0.982
360	26.22	-10.254	0.980

Figure 3 shows the obtained relationships for the maximum precipitation amounts for $C \in [0, 1; 1]$ in the analyzed time intervals $t \in [5; 360]$ min.

The relationship of the β coefficient in Eq. (9) (shown in Fig. 4) was described by the function of time (at $R^2 = 0.996$) as follows:

$$\beta = -1.47t^{0.330} \quad (10)$$

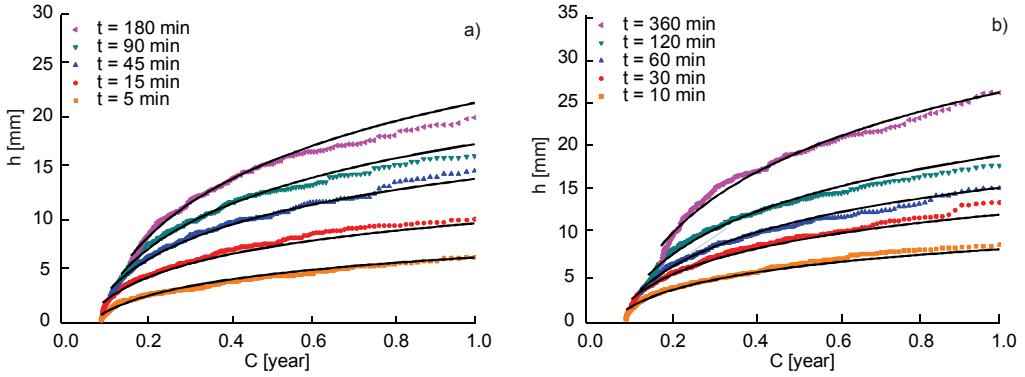


Fig. 3. Precipitation amounts at various time durations in function of their frequency (Eq. (9))

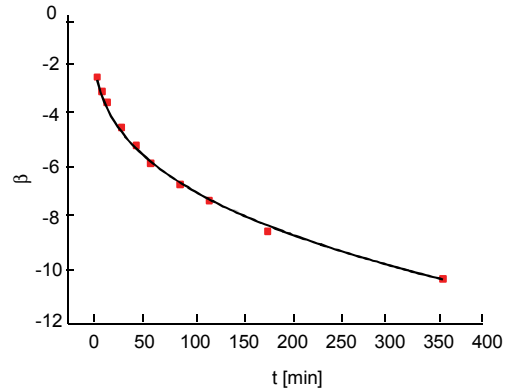


Fig. 4. Variability of empirical coefficient β as a function of time

Substituting Equation (10) to (9), the physical formula for maximum precipitation amount occurring from 1 to 10 times a year will be obtained:

$$h_{\max}(t, C) = -4.58 + 7.41t^{0.242} + 1.47t^{0.330} \ln C \tag{11}$$

Table 5 presents the measurement data from the meteorological station IMGW Wrocław-Strachowice from years 1960–2009 and results of calculations of maximum precipitation amounts from the probabilistic model in Eq. (8) and physical model in Eq. (11). The analysis was carried out for selected rainfall frequencies and their durations: $C \in [1; 50]$ years and $t \in [5; 4320]$ min – for model (8) and $C \in [0.1; 1]$ years and $t \in [5; 360]$ min – for model (11).

As shown in Table 5, the probabilistic model (8) accurately describes the results of measurements of the rainfall depths for durations up to 72 h and occurrences up to 50 years. Average discrepancy between measured data and calculation results is about $\pm 7\%$. The physical model (11) also describes a similar class of accuracy of results of measurement for the analyzed rainfall durations to 6 h and the incidence of up to

10 times a year. However, for the rain occurring 10 times a year ($C = 0.1$), differences between measured data and results of calculations are the highest. It can be explained at least partly by imprecision of measuring instruments for low rainfall intensities (caused by: evaporation, the accuracy of recording on paper pluviographs, etc. [22]).

Table 5

The comparison of measurement data for Wrocław (h_{max}) with results of calculation for maximum rainfall intensities from models (8) and (11)

C [year]	t [min]	h_{max} [mm]	h_{max} Eq. (8) [mm]	Δh_{max} [%]	C [year]	t [min]	h_{max} [mm]	h_{max} Eq. (8) [mm]	Δh_{max} [%]	C [year]	t [min]	h_{max} [mm]	h_{max} Eq. (11) [mm]	Δh_{max} [%]
50	5	13.1	12.28	-6.3	5	5	9.3	9.24	-0.6	0.5	5	4.5	4.63	2.9
	15	24.7	23.10	-6.5		15	17.7	16.23	-8.3		10	6.6	6.18	-6.4
	30	32.9	30.52	-7.2		30	26.7	21.18	-20.7		15	7.7	7.20	-6.5
	60	35.3	38.50	9.1		60	30.5	26.65	-12.6		30	9.6	9.17	-4.5
	120	57.7	47.12	-18.3		120	35.4	32.72	-7.6		45	10.3	10.46	1.6
	180	61.9	52.50	-15.2		180	35.7	36.59	2.5		60	10.9	11.44	5.0
	360	63.1	62.35	-1.2		360	38.7	43.83	13.3		90	12.8	12.94	1.1
	720	64.2	73.14	13.9		720	49.2	51.97	5.6		120	13.7	14.08	2.8
	1440	80.1	85.05	6.2		1440	65.0	61.18	-5.9		180	15.5	15.80	1.9
	2880	103.9	98.25	-5.4		2880	76.2	71.65	-6.0		360	19.2	19.10	-0.5
4320	116.9	106.66	-8.8	4320	87.5	78.44	-10.4	5	3.4	3.61	6.2			
25	5	11.6	11.41	-1.6	2	5	8.0	7.82	-2.3	0.33	10	5.2	4.90	-5.8
	15	22.8	21.14	-7.3		15	13.9	13.00	-6.5		15	6.2	5.74	-7.4
	30	30.3	27.86	-8.1		30	17.9	16.79	-6.2		30	7.8	7.33	-6.0
	60	35.3	35.13	-0.5		60	20.2	21.08	4.4		45	8.8	8.36	-5.0
	120	41.5	43.02	3.7		120	25.6	25.95	1.4		60	9.1	9.14	0.4
	180	42.8	47.97	12.1		180	27.3	29.11	6.6		90	10.1	10.31	2.1
	360	50.4	57.07	13.2		360	35.2	35.12	-0.2		120	11.2	11.18	-0.2
	720	64.2	67.12	4.5		720	40.8	42.02	3.0		180	12.4	12.50	0.8
	1440	77.9	78.25	0.4		1440	48.3	49.97	3.5		360	15.8	14.95	-5.4
	2880	103.2	90.67	-12.1		2880	60.6	59.15	-2.4		5	2.6	2.33	-10.4
4320	111.6	98.62	-11.6	4320	63.4	65.18	2.8	10	3.8	3.30	-13.2			
10	5	9.9	10.21	3.1	1	5	6.4	6.36	-0.6	0.2	15	4.5	3.91	-13.1
	15	20.1	18.42	-8.4		15	10.1	9.69	-4.1		30	5.4	5.03	-6.9
	30	28.2	24.17	-14.3		30	13.7	12.30	-10.2		45	6.0	5.73	-4.5
	60	34.7	30.44	-12.3		60	15.3	15.38	0.5		60	6.4	6.24	-2.5
	120	36.2	37.32	3.1		120	17.9	19.02	6.3		90	7.2	6.99	-2.9
	180	38.4	41.67	8.5		180	20.0	21.46	7.3		120	7.9	7.54	-4.6
	360	43.9	49.75	13.3		360	26.2	26.21	0.0		180	8.0	8.33	4.1
	720	54.2	58.74	8.4		720	32.0	31.84	-0.5		360	8.9	9.71	9.1
	1440	72.2	68.81	-4.7		1440	39.9	38.49	-3.5		5	0.6	0.60	0.0
	2880	94.5	80.15	-15.2		2880	48.1	46.35	-3.6		10	0.9	1.12	24.4
4320	101.9	87.46	-14.2	4320	49.0	51.60	5.3	15	0.9	1.42	57.8			

5. SUMMARY

The precipitation models presented in the paper, probabilistic (8) and physical (11) ones, for determination of maximum rainfall depth, can be a comprehensive solution to the problem of input data for designing and testing the hydraulic capacity of sewer systems in hydrodynamic modeling.

The probabilistic model of maximum precipitation (8) can apply to sewage systems or combined sewage systems dimensioning for the required EN 752 standard for drainage area (Table 1). Due to comprehensive measuring data from 50 years of observations and Eq. (8) and good fit of empirical distribution functions to theoretical ones, this formula can be most probably extrapolated to $C = 100$ years ($p = 0.01$). This model, due to the time range of rainfalls (up to 3 days) can also be suitable for determining the volume of the storage reservoirs, where the rainfall duration reliable for sizing of tanks proves to be several times longer than the sewage flow in the system.

The physical formula (11), allows one to construct the Euler rainfall model for frequencies up to 10 times a year. Based on the model, during hydrodynamic modeling, multiples of storm overflows activity or storm water separators can be determined.

The lower limit from probabilistic model (8) was assumed as the upper limit in physical model (11). It allowed one to maintain continuity of the results of calculations of the maximum rainfall for the frequency $C = 1$ year, with using both models.

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