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Grzegorz Koloch, Bogumił Kamiński*, Mateusz Żbikowski*, Marek Antosiewicz**

MODELLING HETEROGENEOUS ECONOMIES – ROBUSTNESS VS. FLEXIBILITY[1]

In this paper we present and compare two different modelling paradigms which compete within the field of heterogeneous agent macroeconomics: a dynamic stochastic general equilibrium approach, which is considered mainstream, and an agent-based approach. We concentrate on methodological aspects and provide an at-hand comparison of the two approaches. Our main conclusion is that, unlike in the case of DSGE, the development of ABMs is impeded by the lack of a baseline, reference specification. The development of such would result in synergy between the research agendas of individual researchers. However it may be difficult, if not impossible, to obtain one since in the ABM method decision or policy functions constitute degrees of freedom and a reference specification would have to abstract from pinning them down. Therefore, even with such a specification available, it can be impossible to produce a high-level theory – stylized predictions, which would be qualitatively robust over the discretionary implementations of particular mechanisms within the model. An issue of whether the assumptions of the DSGE paradigm reflect an optimal trade-off between the robustness of the predictions and the flexibility in the specification remains at this time undecided. We believe this question will be answered when commonly available computational technology will allow for full rationality within the ABM framework, which, in principle, is possible.

Keywords: heterogeneous agent macroeconomic modelling, dynamic stochastic general equilibrium, agent based modelling

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1. INTRODUCTION

Nowadays, two modelling paradigms are attracting most attention in the field of heterogeneous agent macroeconomic modelling: the Dynamic Stochastic General Equilibrium approach and the Agent Based Modelling approach. The first methodology (henceforth DSGE), as it is based on neoclassical foundations and follows the leads of rational expectations revolution, is considered to be mainstream, whereas the second one

* Department for Decision Analysis and Support, Institute of Econometrics, Warsaw School of Economics.

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(henceforth ABM), although it presents an attractive alternative, is still not popular to a comparable extent both among academic economists as well as among policy makers, most notably central bankers. The aim of this paper is to provide a review of both approaches in an attempt to answer the question: why? To do so, we outline both methodologies, compare their main assumptions and discuss the differences between them which we believe can provide a reasonable explanation of the situation in question.

The main conclusion which we draw is that the absence of a commonly agreed upon reference specification of a macroeconomic ABM constitutes a serious impediment for this strand of research to become mainstream. We believe the reason for this is twofold. Firstly, the lack of a reference ABM economy makes it difficult, if not impossible, to align the agendas of individual researchers so that synergy in the field could be achieved. This impedes the development of a paradigm in a non-local, non-case study based fashion, when each model specification would not constitute a research agenda on its own. Secondly and more importantly, the lack of such a reference model economy makes it difficult to produce high level, stylized predictions on economic dynamics or policy prescriptions which would be qualitatively robust over particular model implementations. A different situation can be observed in the field of DSGE models, where commonly agreed upon modelling principles (general equilibrium, rationality, first order optimality, neoclassical structure of the economy) imply a high level reference specification which allows for the non-local development of the paradigm and the development of stylized predictions. Each researcher in the domain contributes to the general framework in the sense that their results or predictions are most often robust – qualitatively valid, also within other model implementations, as long as they are in line with the commonly agreed upon principles, and therefore resemble the high level reference specification, which furthermore develops over time allowing for a better match with the stylized facts of empirical research.

This does not mean that each and every model is the same – this by all means is not the case, but in general terms, model dynamics responses to certain events (shocks), predictions, and the consequences of implementing particular mechanisms within the model economy (e.g. of Nash bargaining in the labour market), tend to be qualitatively similar among different models. Moreover, once a deficiency within the framework is recognized and considered to be a serious one, the paradigm can be fixed or developed by means of the non-local, implicit cooperation of researchers, i.e. by the entire community, since all researchers work, in principle, within the same reference economy architecture, within which predictions are fairly stable or robust. Such an effect could be observed for example in the aftermath of the financial crisis, when models involving such elements as financial intermediaries, financial frictions, zero lower bounds and defaults quickly emerged (and often were reinvented) and became standard when crisis-related issues are relevant.

In our opinion, since the lack of a commonly agreed upon reference specification of an ABM macroeconomy impedes non-local development in the field, a natural question arises: why does it lack one? We believe that the most fundamental reason for this is that decision functions (policy rules) in ABMs constitute a degree of freedom, i.e. they must be chosen fairly arbitrarily. This remark concerns not only parameter values but most importantly functional forms. As a consequence, even if the structure (markets, agents) of two ABM economies is similar, their predictions can be very different. The opposite situation is typical for DSGE models, where one does not assume the explicit functional forms of decision functions, but only agrees that agents behave optimally – with respect to commonly agreed objectives (utility or profit maximization), within a structure which closely resembles the reference specification. Utility functions can have diverse forms and arguments, but still, they are based on common principles like utility derived from consumption, discounting and decreasing marginal effects. As a consequence, economic dynamics and qualitative predictions tend to be qualitatively robust to changes in the models' structure and to particular versions of the implemented mechanisms.

2. HETEROGENEOUS AGENT DSGE MODELS

Starting from the 1970s, when the so-called rational expectations revolution emerged, the agenda of macroeconomic modelling shifted from *ad hoc* specification paradigm towards the then novel real business cycle models, which thereafter became what is now called DSGE models. In this class of models microfoundations – individual decision making, agents' expectations formation, their rational (optimal) choices, constituted altogether a new philosophy of thinking about the way macroeconomic dynamics can be described at the aggregate level. Aggregate dynamics became resultant to exogenous, unpredictable, yet interpretable (structural) shocks. Shocks, having hit the economy, propagate through it, driving economic aggregates. The resulting dynamics are in line with the imposed structure of the economy and agents' policy functions which are endogenous and optimal within this structure, subject to their expectation formation, see e.g. Wickens (2012). With forward looking agents, optimal policy functions, deep parameters – and therefore robustness to the "Lucas critique", the new approach started to be considered the only valid method for policy evaluation and for the assessment of the welfare related ramifications of macro policy interventions, see Lucas (1976) and Lucas (1987).

Along the lines put across in the seminal work of Kydland and Prescott (1982), the new models were based within the classical general equilibrium framework, the Walrasian equilibrium, founded in the works of, e.g. Arrow and Debreu (1954). Agents make their decisions regarding consumption, savings, labour, money holdings, production, etc. in an optimal way, with objective functions given by the discounted stream of expected utility or the discounted stream of profits. Optimization is subject to the structure of markets, exogenous shocks and internal consistency conditions such as market clearing or transversality. With agents of a given type being alike, a complete market structure being imposed, the perfect insurance of idiosyncratic or individual risk was possible, and a representative agent framework could therefore be utilized and, in fact, was exploited extensively. As Heathcote et al. (2009) point out however, "*the most important reason for this choice was that economists lacked the tools to solve dynamic models with heterogeneous agents and incomplete markets*".

Since the 1980s, and especially in the 1990s, the focus of structural macroeconomic modelling within the general equilibrium framework has visibly shifted from the analysis of equilibrium allocation within the representative agent framework, towards heterogeneous agent DSGE models where the concept of equilibrium allocation (both in the stationary as well as in the stochastic economy) must have been extended, so that it allowed for the consequences of market incompleteness. Such models do not rely anymore on the representative agent assumption, but instead account for the entire distribution of economic variables such as employment status, income, savings or wealth, as equilibrium objects and state variables. DSGE models started to account for one of the most important feature of the real world economies: agents' heterogeneity and therefore, inequality. As Heckman (2001) comments on the observations and conclusions drawn from the then recent investigation of economic data on the micro level: "*the most important discovery was the evidence on the pervasiveness of heterogeneity and diversity in economic life*".

This transition would not have been possible if not for the availability of (relatively) cheap and fast computing machines, but it is not clear that if such machines had readily been available in the 1970s, then the representative agent framework would not have been exploited anyway. As Heathcote et al. (2009) suggest, "*it was not obvious that incorporating household or firm heterogeneity was of first-order importance for understanding the business cycle dynamics of aggregate quantities and prices*".

In the 1990s most DSGE models were based on a representative agent assumption and were considered by the ABM community to be straight-jacket specifications, lacking the ability of representing complex interactions among agents in real economies, see e.g. Smets and Wouters (2003) and Adolfson et al. (2007) for influential models based on a representative agent assumption, and Fagiolo and Roventini (2008) and Colander et al. (2008) for a discussion. Nevertheless, starting from seminal papers of Hugget (1993), Ayiagari (1994), and Krusell and Smith (1998), the extent to which different dimensions of heterogeneity are exploited in otherwise standard DSGE models is becoming more and more substantial.

The introduction of heterogeneity into the mainstream macro engine turned out to produce diverse consequences. First and foremost a new class of issues and questions could now be addressed, i.e. issues which relate to welfare distribution and redistribution, especially in the context of the cost of the business cycle and policy evaluation. These questions could not have been answered using the representative agent framework. Once DSGE embraced heterogeneity, welfare analysis would have been re-evaluated, taking now into account income inequality and redistribution, where the same aggregate shock can have different effects for different agents, depending on their current positioning in the state space. In such a setting, the cross-sectional welfare cost of aggregate fluctuations can be much bigger than the cost which is incurred by agents, when the perfect risk sharing mechanism is allowed for and enforced. Furthermore, the volatility of economic quantities on the micro level turns out to be much larger than the volatility of the corresponding aggregates. Krusell et al. (2009) investigate the welfare effects of eliminating business cycles in a model with substantial consumer heterogeneity, which arises from uninsurable and idiosyncratic uncertainty in preferences and employment status, see also Krusell and Smith (1999). They find fairly large effects. For the benchmark model, they find that the welfare effects of business cycle elimination, on average across all consumers, are of more than one order of magnitude larger than those computed by Lucas (1987). In addition, there are large differences across the groups.

The seminal papers in the field of heterogeneous agent DSGE models were presented by Hugget (1993) and Aiyagari (1993), as far as economies driven by individual risk are concerned, and by Krusell and Smith (1998), who proposed an approximate numerical technique for solving stochastic economies, which, along with market incompleteness, also allow for the old-fashioned aggregate risk (which used to drive the original representative agent economies). A broad review of this strand of research can be found in Heathcote et al. (2009). Here we focus on the most fundamental mechanisms, which serve as a basis and constitute an anatomy which is typical of many particular model implementations.

Hugget (1993) considers an economy with agents' two idiosyncratic states: employment and unemployment. He concludes that within an incomplete market framework with (exogenous) credit constraints, when agents cannot get completely insured against individual risk and, simultaneously, cannot roll their debts infinitely, they build up precautionary savings – a form of a buffer against a stream of bad luck (i.e. idiosyncratic unemployment) in the future. The precautionary savings motive increases aggregate savings and drives down the equilibrium real interest rate. This is a way of solving the so called real interest puzzle, which states that real interest rates observed in the real world are lower than those implied by calibrated or estimated RBC or DSGE models.

Within a similar framework, Aiyagari (1993) discusses capital transactions as an (imperfect) means of substituting the mechanism of agents' risk sharing. At the end of this paper, the author suggests that the framework be extended so that aggregate risk is also included, being aware of the technical difficulties this entails. The next landmark contribution, as in Krusell and Smith (1998), exploits this avenue. They consider an economy in which both individual and aggregate (business cycle) risk is at play. It was a well-known fact that aggregate risk makes real prices (wages, interest rate) within the incomplete market set-up variable over time and dependent on the entire distribution of the state variables (in the considered case on the distribution of wealth or assets). Market prices are variable because they depend on (the expected next period) aggregate state variables (in Krusell and Smith (1998) framework on the next period aggregate capital), and these, due to aggregate productivity shocks, fluctuate over time. Distributions of state variables constitute therefore an equilibrium object since agents are unable to predict the aggregates on the basis of their own choices alone. Instead, they must know all other agents' choices and, as agents with different characteristics may also make different decisions (e.g. may have different savings rates, different incomes at disposal), they must know the entire distribution of the individual states (e.g. wealth, income), and thus they must know the entire distribution of individual choices in the economy so that these can be integrated over all agents to get the next-period aggregates, and therefore the prices which depend upon them.

The standard DSGE model specification based on Krusell and Smith (1998) can be summarized as follows:

1. There is a continuum of agents represented as points on a unit interval.
2. Each agent maximizes their expected stream of discounted utility derived from consumption (and possibly leisure).
3. There is a representative firm, which maximizes expected profits, the production function is neoclassical.
4. Markets are competitive, i.e. factor prices (real wage, real rate of interest) are set at marginal factor productivities.
5. Agents are rational. Bounded rationality (a limited amount of it) is introduced only for technical reasons through the first moment based approximation of the law of motion approximation of the aggregate quantities (the Krusell and Smith algorithm).
6. Agents face idiosyncratic productivity or labour status shocks.
7. Asset markets are incomplete, therefore perfect risk sharing is impossible.
8. The economy is stochastic, i.e. aggregate shocks are at work, therefore stationary distribution as a concept of the steady state is not attainable.

A DSGE model is specified in terms of the decision problems of agents and equilibrium conditions. To make it operational it must be solved, i.e. the policy functions of agents must be derived. A handful of numerical techniques exist in this respect, e.g. value function iteration, perturbation or function approximation methods, see e.g. Heer and Maussner (2009). Independent of which solution method is used, economic dynamics is provided by a transformation which implements the policy functions of agents and governs the transition rules of all the state variables. Given the realization of shocks, they transform the current joint distribution of state variables into the next-period one. Without aggregate shocks, an equilibrium object which represents a state of the economy in a given moment of time is given by the stationary distribution of state variables. When aggregate shocks are at play and the economy does not obtain a stationary distribution, the notion of stationary equilibrium is replaced by the so-called recursive dynamic equilibrium. The model must be calibrated and the initial conditions in the form of the distribution of endowments must be assumed.

3. AGENT BASED MODELS

As noted by Tesfatsion (2006), agent-based computational economics within which ABMs are specified, is a set of techniques for studying complex adaptive systems involving many interacting agents with exogenously given behavioural rules. The specifications of ABMs resemble methods typical of dynamic system modelling utilized for example in the field of computational cosmology and biology. The ABM approach can be positioned within the paradigm of non-orthodox, post-Walrasian economics. Models are built according to a bottom-up approach, which means that the interactions and behaviours of individual agents are specified on a micro-level and macro-level dynamics is observed as the emergent outcome, see e.g. Oeffner (2009), Colander (2006), Colander et al. (2008). As Farmer and Folley (2009) point out, "*An agent-based model is a computerized simulation of a number of decision-makers (agents) and institutions, which interact through prescribed rules. (...) Such models do not rely on the assumption that the economy will move towards a predetermined equilibrium state, as other models do. Instead, at any given time, each agent acts according to its current situation, the state of the world around it and the rules governing its behavior*".

The first attempt to take an agent-based approach in economic modelling was presented in the early 1990s by Holland and Miller (1991). They tried to address the issue of how economic agents adapt their behaviour to changes in a constantly evolving environment. They treat the economy as a complex adaptive system usually operating far from equilibrium. In particular, they point out that the classical approach – based on agents' optimization – should be considered as an infeasible strategy, since real agents are not able to derive solutions of their optimization problems (or even state the problems explicitly). Even if one assumes that agents only behave as if they were optimizing, the classical framework does not allow for validating such a conjecture, see e.g. Friedman (1953). It should be noted that this idea was not new back then in social sciences – since the 1970s there have been important contributions in the field, see e.g. Schelling (1971), Sakoda (1971), Axelrod and Hamilton (1981). However, the field of sociological science was not strongly focused on formal mathematical modelling and agent-based simulations constituted a means of making the discussion more formal, whilst allowing for the desired flexibility in assumptions. On the other hand, in the area of economics mathematical modelling was the mainstream approach. Therefore the key purpose of ABMs was to explain the unexplained by targeting the identified shortcomings of existing mainstream models, e.g. Marimon et al. (1990), Albin and Foley (1992), Marks (1992), Gode and Sunder (1993), Palmer et al. (1993), Arthur (1994), Vriend (1995).

The three key phenomena that are often a challenge to models using the traditional approach, and which agent-based approach tries to solve are: 1) the dynamics of an economy out-of-equilibrium, 2) taking into account the heterogeneity of agents in terms of their attributes, policies and interactions with each other, 3) the direct modelling of learning (without the need of assuming that agents directly optimize or behave as if they optimized). All three elements are considered crucial when structural changes take place, when one cannot naturally assume that the economy is in equilibrium and all types of agents are able to find their optimal policies. In particular, the agenda was to build models that are more realistic, i.e. strongly rooted in empirical and experimental microeconomic evidence, see e.g. Fagiolo and Roventini (2012), and which would be capable of escaping the theoretical requirements or limitations of the DSGE approach, see e.g. Fagiolo and Roventini (2017).

On a high level of abstraction, summarizing the observations of Fagiolo et al. (2007) and Oeffner (2009), the following features of ABMs can be emphasized:

1. Bottom–up perspective. Macro-level dynamics emerge as a result of the behaviour and explicit interactions of individuals on the micro level, Tesfatsion (2002), Pyka and Fagiolo (2005).

2. Heterogeneity. Agents are heterogeneous in their behaviour, competencies, (bounded) rationality, computational skills etc.

3. Evolving a complex system approach modelled by a network of direct interactions. All agents live in a network which is a complex dynamically evolving system (Kirman, 1997), aggregated properties emerge after repeated interaction between agents take place, agents' decisions are based on present and past experience, trading of goods and services are modelled explicitly and as a result, the general equilibrium does not hold.

4. Non-linearity. Interactions between agents are highly non-linear, ABMs can contain feedback loops between micro and macro levels (small scale interactions create a macro level dynamics, which in turn influences activity on the micro level).

5. Direct interactions. Agents interact with each other directly, their decisions depend on the past and present choices made by other agents (Fagiolo, 1998): subgroups of agents (local networks) can emerge and their

structure can change endogenously over time, agents can decide with whom to interact according to the expected payoffs (in a bounded-rational way).

6. Bounded rationality. Agents live in a world which is too complex for exact (hyper) rationality, only local or partial rationality can be imposed, agents behave as rational individuals with adaptive expectations.

7. Learning. In numerous ABMs learning algorithms are introduced, such as Windrum and Moneta (2007), where agents engage in an open-ended search within a dynamically changing environment, observed patterns constitute a relevant ingredient for learning and adaptation, initial conditions often put agents as units without any knowledge about the environment in which they live.

8. Dynamics. ABMs, due to adaptive expectations, are characterized by dynamics, which is irreversible.

9. Endogenous and persistent novelty. Economic systems are non-stationary with constantly introduced novelty, which leads to the emergence of new behaviour patterns, which in turn drive adaptation and learning, on top of which agents find it difficult to adapt and learn in such a turbulent and changing environment, e.g. firms introducing new products into the market in order to increase payoffs while the results of research and development cannot be known *ex ante* (Dosi et al., 2006).

10. Selection mechanisms in the market. Goods and services produced by companies are filtered and selected by consumers, the selection criteria are complex and involve numerous dimensions (e.g. product features), additional turbulence can be created by firms entering or dropping out of the market (Windrum, 2005).

In a more explicit, implementation-oriented manner, a minimalist ABM consists of the following ingredients:

1. Agents. They are specified as objects of predefined types (e.g. households, firms, banks, the government) and implemented within the simulated economic environment as autonomous and interactive entities. Agents are characterized by micro parameters according to which they can differ (e.g. education type, age or productivity). Micro parameters can be fixed or variable over simulation iterations. Each agent has a set of decision micro variables attached, which are updated according to *ex ante* assumed decision rules (e.g. consumption, labour demand, wage offered).

2. Interaction structure. Agents interact with each other exchanging the resources they have at their disposal (e.g. trading consumption goods, hiring labour supply, borrowing money holdings) and information contained in

their information sets (e.g. wages, prices, labour market status). Interaction structure defines who interacts with whom and how.

3. Time. Models are simulated in discrete time steps, e.g. days in Legnick (2013), weeks in Ashraf et al. (2011), months in Giovanni (2010), quarters in Gaffeo (2008). Different kinds of decisions can be made in different timeframes.

4. Macro variables. Result as an explicit aggregation of micro variables. Some can be defined exogenously on the macro level (e.g. a rate of interest).

The above outlined requirements are very high-level. Should they be assumed as a definition of the modelling paradigm, one should not be surprised that ABMs are diverse in nature and, unlike DSGE models, no single one can be thought of as a union (an approximate one) of all the other ones, as the origin. However, these are examples of baseline specifications which involve only basic macroeconomic relations in a business cycle environment and generate only generic properties of markets, e.g. the model of Legnick (2013).

The structure of the Legnick (2013) model is motivated by other well-known ABMs – see Dosi et al. (2008) and Gaffeo et al. (2008). There are two types of agents: households and firms. The main characteristics of the environment in which agents interact with each other are as follows: time is indexed by days and months to allow for different actions to take place in different time intervals (there are 21 days in a month), prices and wages are chosen according to simple adaptive rules, agents knowledge depends only on local information (not on aggregate statistics), households and firms are fixed in number and infinitely lived, production technology is fixed, there is no government and no central bank, different types of goods are typically traded in different time intervals, consumption takes place on a daily basis, labour is bought monthly, agents are connected through a network of trading relationships and in the short run this network is fixed, whereas over time agents cut unsatisfying trading connections to create new ones, therefore the network is allowed to change in the medium term.

In the case of an ABM, its specification in the form of the explicit decision functions of agents and of a system (network) of the relations (connections) between them is sufficient for model implementation and, after calibration, sufficient for simulation. Simulation dynamics, and stationary dynamics in particular, can be perceived as an equilibrium of an ABM system. Initial conditions in the form of the distribution of endowments must be assumed for simulation. Often, a burn-in period is also assumed.

DISCUSSION

Table 1 summarizes the main features of both methodologies that were discussed in previous sections, which are often contrasted with each other. ABMs can be perceived as flexible specifications, whereas DSGE as the straight-jacket but robust ones. In particular, in the context of the recent financial crisis, it has been emphasized that mainstream macroeconomic models could not have predicted a severe recession since they are not capable of representing complex dynamic interactions between economic agents. Moreover, the mainstream attributes to itself what Caballero (2010) calls a pretense of knowledge, which is dangerous for research activity in any field of science. Blind reliance on DSGE predictions can and should be criticized, but it should be emphasized that such reliance on any kind of modelling method should be deemed improper. The DSGE models did not predict the financial crisis because most of them, and especially most of

Table 1

Summary of main concepts which underline DSGE and ABM methods

Feature	DSGE	ABM
Model structure	Agents, Markets, Decision Making	Agents, Markets, Decision Making, Interaction structure
Notion of dynamic equilibrium	Stationary distribution, Recursive dynamic equilibrium	No explicit notion of dynamic equilibrium, Out-of-equilibrium dynamics, Simulated dynamics, Stationary dynamics possible
Market clearing	General equilibrium	Out-of-equilibrium dynamics, but markets clear on average
Expectation formation	Rational expectations, Bounded rationality possible	No explicit expectation formation, Naïve expectations, Adaptive expectations
Decision rules	Emergent, resultant to intertemporal optimization	Assumed *a priori*, e.g. adaptive, resultant to learning
Micro-level interaction structure	No explicit micro-level interaction, implicit coordination of micro-level decisions *via* expectations on aggregate variables	Explicit interaction between agents on the micro-level (includes randomness)
Macro-level allocation	Resultant to micro-level decisions	Resultant to micro-level decisions
Drivers of the dynamics	Exogenous stochastic shocks (interpretable)	Endogenous mechanisms (include randomness)
Analogies to other fields	Lagrangian formalism of classical physics (dynamics)	System dynamics approach
Reduced form	A simulation model	A simulation model

Source: own elaboration.

those which were used by the financial authorities, did not have mechanisms which would allow for such phenomena to happen. However after the crisis, a great deal of models with financial frictions, intermediaries, spreads, defaults, zero lower bound and large asset purchases were constructed, and they are capable of representing concepts similar to the ones which were observed in and after 2007, see e.g. Coibion et al. (2012). It is a common feature of both DSGEs and ABMs that they cannot predict phenomena which are not allowed for within their specifications, regardless of their degree of complexity.

Although based on different principles, from a technical perspective a reduced form DSGE model with aggregate risk and an ABM belong to the same class – they are both simulation models. The main difference is that in the case of ABMs, exogenous shocks are not assumed to be the main drivers of the stochastic economy, despite the obvious fact that simulations are not deterministic – randomness is extensively involved. Also, a significant qualitative difference is that a simulated ABM economy does not obtain a general equilibrium, since it lacks a coordination mechanism which would imply markets clearing exactly each period. On average, however, ABM markets tend to clear. ABMs are closer to reality than DSGE models in the sense that they better resemble the topology, links and connections between agents, which can, in principle, be taken to be as close as it gets to the true structure of economic systems. Within an ABM approach, even a one-to-one low-level correspondence between the model and the structure of real economy is, at least in principle, possible. Whether this makes them closer to reality regarding their predictions is an open question. One can argue whether such a low-level correspondence makes much practical sense, since a large number of arbitrary choices is needed, especially regarding the decision making of all groups of agents. Free parameters make ABMs unidentified. These could be set on the basis of micro or experimental studies, but even so these would to a large extent be arbitrary assumptions since a variety of possible options exist. In such a situation it is difficult, if not impossible, to preserve the stability or robustness of the model's predictions, even if the structure of the economy is fixed. This lack of robustness is a crucial feature of ABMs, and, arguably, a feature which impedes development in this field. It is not that the diversity of ABMs is substantial, but there is a lack of understanding as to how different models are interconnected and how they relate one to another. This is not the case, or at least to a far lesser extent, in DSGE modelling where the model's predictions (e.g. economic dynamics) are far more robust to changes in their specification. On the other hand, the existence of a reference specification and standardized predictions can be considered an obstacle for structural change and the improvement of the paradigm, which, especially after the recent crisis, is called for in macroeconomics.

From a more practical standpoint, the most far reaching difference between DSGE and ABMs is that the latter do not have a commonly agreed upon technology for agents' decision making process. In the case of ABMs, the decision making rules of agents constitute a degree of freedom which must be pinned down each time a model is specified. Adaptive learning, naïve decision making, or even artificial intelligence can be utilized as decision making vehicles. The resulting flexibility is on the one hand appealing, but on the other it precludes the systematization of discourse within the field. This feature of ABMs can be contrasted with the extremely different approach adopted within the DSGE field where the decision making processes, i.e. agents' policy functions, do not constitute a degree of freedom but are emergent and resultant to the model's structure and the assumption that, given the model's structure, agents optimize in line with their expectations. These principles are commonly agreed upon in the field and different models can be thought of as different instances specified within the same technology. Model instances can be very different one to another, but their main workings remain the same. Moreover, DSGE also allows for such features as bounded rationality, limited or asymmetric information, heterogeneous expectations or cognitive constraints like rational inattention.

As was pointed out in Richiardi (2003), creating standardized prescriptions on how ABMs should be built, standardized protocols would benefit the ABM method. Also Richiardi (2015) states that the ABM field requires a development of appropriate application programming interfaces, routines, protocols, and tools which would define the functionalities used internally by agents (e.g. learning algorithms) or used by agents to interact with other agents (e.g. exchange of information, goods and services) that are independent of their respective implementations. In our opinion the development of such standards, although very helpful for model construction and development, would not address the core difficulty which is inherent when decision rules are degrees of freedom. At the moment an objective, commonly agreed upon decision making technology within the field of learning and bounded rationality does not exist and it is not clear if it is possible to develop one.

Both the ABMs and the DSGE models relate to the same economic reality and their specifications are inspired by the same real economy. As

Grabner (2014) points out, the selection of a particular tool is not enough to constitute a scientific paradigm – the discussion should be about the underlying theory and not about the modelling approach itself. On the other hand, technical limitations bind the extent to which underlying theories can be made operational. In this context, the relaxation of neoclassical models to ABMs is a process of giving up axioms. As this is done, convenient properties get lost along the way, the robustness of the predictions is lost, and as a result the mainstream tries to avoid this step. DSGE models are less complex but more robust than ABMs. Although ABMs are formally not inferior to DSGE models, from a practical perspective the theories are desired to be operational, hence robust, which explains much of the resistance against ABMs.

Scientific theories are based on abstractions and since ABMs aim the explicit replication or description of reality, they can be thought of as an overload with respect to scientific theory. According to Leijonhufvud (2006), models should not be as complex as reality. But this argument seems to be valid if simplicity is needed, i.e. when the purpose of modelling is generalization. Generalization, however, does not have to be the only purpose of economic modelling. Apart from abstraction, one can be interested in predicting how a given system behaves or can in principle behave under given circumstances, and in such cases a close correspondence between the model and the reality seems desirable. Since empirically we observe only one trajectory of reality, we do not know if some other system, even a very similar one, would have exhibited similar behaviour under the same circumstances. The question of whether real life economic dynamics is a non-robust, fragile, knife-edge phenomenon remains of course undecided. For example, in a recent paper, Fagiolo and Roventini (2017) suggest that in order to avoid the pitfalls of the DSGE framework such as overconfidence or missed policy prescriptions, economies should rather be considered and, as a consequence, modelled as complex, evolving, far-from-equilibrium systems whose structures undergo continuous changes. They point out that the last financial crisis did in fact prompt a strand of research in which DSGE models were somehow adjusted to the circumstances, e.g. by the introduction (in fact sometimes re-introduction) of specific frictions in the financial sector or fat-tailed shocks, but they argue that such practices do not and will not solve the problem which is rather at the level of the paradigm's fundamentals. In this context however, one should also consider the question, which also remains open, of whether it is not the case that a DSGE method (in its present or future form) does not perhaps constitute an optimal trade-off between flexibility and robustness, especially when the purpose of the inquiry is generalization.

In our opinion this is a question which can be answered along with the developments in computing technology. This is because rationality is, in principle, possible within the ABM framework. Methods such as iterated best responses and n^{th} order rationality can be used to arrive at a rational expectations equilibrium (or close to one) within ABMs, see e.g. Reeves and Wellman (2004) and Łatek et al. (2009). If full rationality was implementable within ABMs, then, up to general equilibrium, they would settle for a DSGE approach. The amount of rationality could be controlled for, amalgamated with bounded rationality, and a trade-off between flexibility and robustness could be investigated. It may turn out that the DSGE approach is close to an optimal trade-off. At the moment however, the computational resources needed to implement full rationality within (at least the medium scale) ABMs are not readily available. While DSGEs seem to be descriptively incompatible with real life complexity, the flexibility of ABMs comes at a substantial cost. Rationality is an arbitrary choice, but a very convenient one.

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