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STOPPING RULES FOR THE ESTIMATION OF THE PARAMETERS OF BIVARIATE AND TRIVARIATE BINOMIAL DISTRIBUTIONS

In this work one step look-ahead rules for the estimation of the parameters of the bivariate and trivariate distributions are given. They turn out to be asymptotically optimal and may be useful in many statistical contexts, for example, in statistical quality control and customer satisfaction analyses.

Keywords: *binominal distribution, estimation, success categorization*

1. Introduction

Several categorical data analyses consider dichotomous characters imposing either statistical independence or multinomial dependence, which natural statistical models of dependence ignore.

The binomial bivariate distribution, defined by Zenga [5], is the natural structure of the number of successes of two characters in n independent extractions. Zini [7] generalises it to three characters, which exhibit univariate margins, binomial and bivariate ones, as *binomial bivariate*. For the general properties of these distributions, see the cited works. In this article, a sequential problem of estimating the parameters of these models in a Bayesian framework is considered. Assuming conjugate prior (Dirichlet) and conjugate loss function (including quadratic loss), a *one step look-ahead* stopping *rule* is deduced for both bivariate and trivariate binomial statistical models. By the principle of independence between the stopping rule and the estimation method (Berger [1]), this rule, which exhibits asymptotical optimality properties,

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turns out to be appropriate to the problem, allowing one to take into account information costs, both of inference and sampling.

2. The statistical models

This section presents the bivariate and trivariate binomial distributions.

The *binomial bivariate* model $(X, Y \mid P_{11}, P_{10}, P_{01})$, Zenga [5]:

$$P(X = x, Y = y \mid P_{11}, P_{10}, P_{01}) = \sum_{n_{11}=\max(0,x+y-n)}^{\min(x,y)} \frac{n!}{n_{11}!(x-n_{11})!(y-n_{11})!(n-x-y+n_{11})!} \times P_{11}^{n_{11}} \cdot P_{10}^{x-n_{11}} \cdot P_{01}^{y-n_{11}} P_{00}^{n-x-y+n_{11}} . \quad (1)$$

Let $P = (P_{111}, P_{110}, ..., P_{001})'$. The binomial trivariate model, Zini [7]:

$$\begin{aligned} \mathrm{P}(X = x,\ Y = y,\ Z = z \mid P) = & \sum_{n_{111}} \sum_{n_{11\cdot}} \sum_{n_{1\cdot 1}} \sum_{n_{\cdot 11}} \frac{n\ !}{n_{111}!(n_{11\cdot} - n_{111})!(n_{1\cdot 1} - n_{111})!(n_{\cdot 11} - n_{\cdot 11})!} \\ & \times \frac{1}{(x - n_{11\cdot} - n_{1\cdot 1} + n_{111})!} \\ & \times \frac{1}{(y - n_{11\cdot} - n_{\cdot 11} + n_{111})!(z - n_{\cdot 11} - n_{1\cdot 1} + n_{111})!} \\ & \times \frac{1}{(n - x - y - z - n_{111} + n_{11\cdot} + n_{1\cdot 1} + n_{\cdot 11})!} \\ & \times P_{111}^{\ n_{111}} P_{110}^{\ n_{11\cdot} - n_{111}} P_{101}^{\ n_{1\cdot 1} - n_{111}} P_{011}^{\ n_{\cdot 11} - n_{111}} P_{100}^{\ x - n_{11\cdot} - n_{1\cdot 1} + n_{111}} \\ & \times P_{010}^{\ y - n_{\cdot 11} - n_{11\cdot} + n_{111}} P_{001}^{\ z - n_{1\cdot 1} - n_{\cdot 11} + n_{111}} \\ & \times P_{000}^{\ n - x - y - z - n_{111} + n_{11\cdot} + n_{\cdot 11} + n_{1\cdot 1}}, \end{aligned} \quad (2)$$

where:

$$\begin{aligned} & n_{111} \le \min(n_{\cdot 11};\quad n_{1\cdot 1};\quad n_{11\cdot};\quad n - x - y - z + n_{11\cdot} + n_{1\cdot 1} + n_{\cdot 11}), \\ & n_{11\cdot} \le \min(x - n_{1\cdot 1} + n_{111};\quad y - n_{\cdot 11} + n_{111}), \\ & n_{1\cdot 1} \le \min(z - n_{\cdot 11} + n_{111};\quad x - n_{11\cdot} + n_{111}), \\ & n_{\cdot 11} \le \min(z - n_{1\cdot 1} + n_{111};\quad y - n_{11\cdot} + n_{111}), \end{aligned} \quad (3)$$

and

$$\begin{aligned} &n_{111} \geq \max\left(0;\quad n_{1\cdot1}+n_{\cdot11}-z;\quad n_{11\cdot}+n_{1\cdot1}-x;\quad n_{\cdot11}+n_{11\cdot}-y\right)\\ &n_{11\cdot} \geq \max(n_{111};\quad x+y+z-n-n_{1\cdot1}-n_{\cdot11}+n_{111}),\\ &n_{1\cdot1} \geq \max(n_{111};\quad x+y+z-n-n_{11\cdot}-n_{\cdot11}+n_{111}),\\ &n_{\cdot11} \geq \max(n_{111};\quad x+y+z-n-n_{1\cdot1}-n_{11\cdot}+n_{111}). \end{aligned} \tag{4}$$

3. The one step look-ahead rules and their properties

In this section, one step look-ahead rules for both models are given. De Groot's [3] notation will be used in the sequel.

Let write model (1) in the following way:

$$P(X=x, Y=y|P) = \sum_{n_{11}=\max(0,x+y-n)}^{\min(x,y)} \frac{n!}{n_{11}!(x-n_{11})!(y-n_{11})}, \tag{5}$$
$$\times P_1^{n_{11}} \cdot P_2^{x-n_{11}} \cdot P_3^{y-n_{11}} P_4^{n-x-y+n_{11}},$$

where $P=(P_1,P_2,P_3)'$, and $P_4 = 1-\sum_{i=1}^{3} P_i$.

The prior is assumed to be conjugate (Dirichlet):

$$h(P_1,P_2,P_3) = \frac{\Gamma\left(\sum_{i=1}^{4}\alpha_i\right)}{\prod_{i=1}^{4}\Gamma(\alpha_i)} \prod_{i=1}^{4} P_i^{\alpha_i-1} \cdot I_{S_3}(P), \tag{6}$$

where $S_k \equiv \left\{(P_1,\cdots,P_k): P_i>0,\ \sum_{i=1}^{k} P_i<1\right\}$. In the sequel the covariance matrix will be called $V(\alpha)$, $\alpha \equiv (\alpha_1,\alpha_2,\cdots,\alpha_{k+1})'$.

So, the posterior has the form:

$$h(P|x,y) = \frac{\prod_{i=1}^{4} P_i^{\alpha_i-1} \sum_{n_{11}} \frac{n!}{n_{11}!(x-n_{11})!(y-n_{11})!(n-x-y+n_{11})!} P_1^{n_{11}} \cdot P_2^{x-n_{11}} \cdot P_3^{y-n_{11}} P_4^{n-x-y+n_{11}}}{\int_{S_3} \prod_{i=1}^{4} P_i^{\alpha_i-1} \sum_{n_{11}} \frac{n!}{n_{11}!(x-n_{11})!(y-n_{11})!(n-x-y+n_{11})!} P_1^{n_{11}} \cdot P_2^{x-n_{11}} \cdot P_3^{y-n_{11}} P_4^{n-x-y+n_{11}} dP}$$

$$= \frac{\sum_{n_{11}} \frac{n!}{n_{11}!(x-n_{11})!(y-n_{11})!(n-x-y+n_{11})!} \cdot P_1^{\alpha_1+n_{11}-1} \cdot P_2^{\alpha_2+x-n_{11}-1} \cdot P_3^{\alpha_3+y-n_{11}-1} P_4^{\alpha_4+n-x-y+n_{11}-1}}{\int_{S_3} \sum_{n_{11}} \frac{n!}{n_{11}!(x-n_{11})!(y-n_{11})!(n-x-y+n_{11})!} P_1^{\alpha_1+n_{11}-1} \cdot P_2^{\alpha_2+x-n_{11}-1} \cdot P_3^{\alpha_3+y-n_{11}-1} P_4^{\alpha_4+n-x-y+n_{11}-1} dP}$$

$$= \frac{\sum_{n_{11}} \frac{1}{n_{11}!(x-n_{11})!(y-n_{11})!(n-x-y+n_{11})!} \cdot \left\{ P_1^{\alpha_1+n_{11}-1} \cdot P_2^{\alpha_2+x-n_{11}-1} \cdot P_3^{\alpha_3+y-n_{11}-1} P_4^{\alpha_4+n-x-y+n_{11}-1} \right\}}{\sum_{n_{11}} \frac{\Gamma(\alpha_1+n_{11})\Gamma(\alpha_2+x-n_{11})\Gamma(\alpha_3+y-n_{11})\Gamma(\alpha_4+n-x-y+n_{11})}{n_{11}!(x-n_{11})!(y-n_{11})!(n-x-y+n_{11})!\,\Gamma\left(n+\sum_{i=1}^{4}\alpha_i\right)}}. \tag{7}$$

The loss function (quadratic, as a particular case) is assumed to be conjugate:

$$L(P,d) = (P-d)' A (P-d) \prod_{i=1}^{4} P_i^{b_i} , \tag{8}$$

with $\underset{k\times k}{A}$ semidefinite positive (symmetric) matrix.

The decision risk at stage n, imposing $\delta_i = \alpha_i + b_i$ in order to exist, is as follows:

$$\rho_0[h(x,y)] = \inf_{d\in S_3} \int_{S_3} L(P,d) h(P|x,y) dP$$

$$= \inf_{d\in S_3} \int_{S_3} \frac{\sum_{n_{11}} \frac{(P-d)' A (P-d)}{n_{11}!(x-n_{11})!(y-n_{11})!(n-x-y+n_{11})!} \cdot \{ P_1^{\delta_1+n_{11}-1} \cdot P_2^{\delta_2+x-n_{11}-1} \cdot P_3^{\delta_3+y-n_{11}-1} P_4^{\delta_4+n-x-y+n_{11}-1} \}}{\sum_{n_{11}} \frac{\Gamma(\alpha_1+n_{11})\Gamma(\alpha_2+x-n_{11})\Gamma(\alpha_3+y-n_{11})\Gamma(\alpha_4+n-x-y+n_{11})}{n_{11}!(x-n_{11})!(y-n_{11})!(n-x-y+n_{11})!\,\Gamma\left(n+\sum_{i=1}^{4}\alpha_i\right)}} dP$$

$$= \frac{\sum_{n_{11}} \frac{\Gamma(\delta_1+n_{11})\Gamma(\delta_2+x-n_{11})\Gamma(\delta_3+y-n_{11})\Gamma(\delta_4+n-x-y+n_{11})}{n_{11}!(x-n_{11})!(y-n_{11})!(n-x-y+n_{11})!\,\Gamma\left(n+\sum_{i=1}^{4}\delta_i\right)} Tr\{A\cdot V[\delta_n(x,y;n_{11})]\}}{\sum_{n_{11}} \frac{\Gamma(\alpha_1+n_{11})\Gamma(\alpha_2+x-n_{11})\Gamma(\alpha_3+y-n_{11})\Gamma(\alpha_4+n-x-y+n_{11})}{n_{11}!(x-n_{11})!(y-n_{11})!(n-x-y+n_{11})!\,\Gamma\left(n+\sum_{i=1}^{4}\alpha_i\right)}}, \tag{9}$$

where $\delta_n(x,y;n_{11}) = [(\delta_1+n_{11});(\delta_2+x-n_{11});(\delta_3+y-n_{11});(\delta_4+n-x-y+n_{11})]'$, "*Tr*" is the trace operator. The expected risk with respect to the predictive at stage (n+1) has the following form:

$$E\{\rho_0[h((X,Y);(R,S)|(X,Y)=(x,y))]\}$$

$$=\sum_{r=0}^{1}\sum_{s=0}^{1}\left\{\frac{\sum_{n_{11}}\frac{\Gamma(\delta_1+n_{11})\Gamma(\delta_2+x+r-n_{11})\Gamma(\delta_3+y+s-n_{11})\Gamma(\delta_4+n+1-x-r-y-s+n_{11})}{n_{11}!(x+r-n_{11})!(y+s-n_{11})!(n+1-x-r-y-s+n_{11})!\Gamma\left(n+1+\sum_{i=1}^{4}\delta_i\right)}Tr\{A\cdot V[\delta_{n+1}(x+r,y+s;n_{11})]\}}{\sum_{n_{11}}\frac{\Gamma(\alpha_1+n_{11})\Gamma(\alpha_2+x+r-n_{11})\Gamma(\alpha_3+y+s-n_{11})\Gamma(\alpha_4+n+1-x-r-y-s+n_{11})}{n_{11}!(x+r-n_{11})!(y+s-n_{11})!(n-x-r-y-s+n_{11})!\Gamma\left(n+1+\sum_{i=1}^{4}\alpha_i\right)}}\right.$$

$$\left.\times\frac{\sum_{n_{11}}\frac{\Gamma(\alpha_1+n_{11})\Gamma(\alpha_2+x+r-n_{11})\Gamma(\alpha_3+y+s-n_{11})\Gamma(\alpha_4+n+1-x-r-y-s+n_{11})}{n_{11}!(x+r-n_{11})!(y+s-n_{11})!(n+1-x-r-y-s+n_{11})!\Gamma\left(n+1+\sum_{i=1}^{4}\alpha_i\right)}}{\sum_{n_{11}}\frac{\Gamma(\alpha_1+n_{11})\Gamma(\alpha_2+x-n_{11})\Gamma(\alpha_3+y-n_{11})\Gamma(\alpha_4+n-x-y+n_{11})}{n_{11}!(x-n_{11})!(y-n_{11})!(n-x-y+n_{11})!\Gamma\left(n+\sum_{i=1}^{4}\alpha_i\right)}}\right\}$$

$$=\sum_{r=0}^{1}\sum_{s=0}^{1}\left\{\frac{\sum_{n_{11}}\frac{\Gamma(\delta_1+n_{11})\Gamma(\delta_2+x+r-n_{11})\Gamma(\delta_3+y+s-n_{11})\Gamma(\delta_4+n+1-x-r-y-s+n_{11})}{n_{11}!(x+r-n_{11})!(y+s-n_{11})!(n+1-x-r-y-s+n_{11})!\Gamma\left(n+1+\sum_{i=1}^{4}\delta_i\right)}Tr\{A\cdot V[\delta_{n+1}(x+r,y+s;n_{11})]\}}{\sum_{n_{11}}\frac{\Gamma(\alpha_1+n_{11})\Gamma(\alpha_2+x-n_{11})\Gamma(\alpha_3+y-n_{11})\Gamma(\alpha_4+n-x-y+n_{11})}{n_{11}!(x-n_{11})!(y-n_{11})!(n-x-y+n_{11})!\Gamma\left(n+\sum_{i=1}^{4}\alpha_i\right)}}\right\}$$

$$=\frac{\left(n+\sum_{i=1}^{4}\delta_i\right)}{\left(n+1+\sum_{i=1}^{4}\delta_i\right)}\frac{\sum_{n_{11}}\frac{\Gamma(\delta_1+n_{11})\Gamma(\delta_2+x-n_{11})\Gamma(\delta_3+y-n_{11})\Gamma(\delta_4+n-x-y+n_{11})}{n_{11}!(x-n_{11})!(y-n_{11})!(n-x-y+n_{11})!\Gamma\left(n+\sum_{i=1}^{4}\delta_i\right)}Tr\{A\cdot V[\delta_n(x,y;n_{11})]\}}{\sum_{n_{11}}\frac{\Gamma(\alpha_1+n_{11})\Gamma(\alpha_2+x-n_{11})\Gamma(\alpha_3+y-n_{11})\Gamma(\alpha_4+n-x-y+n_{11})}{n_{11}!(x-n_{11})!(y-n_{11})!(n-x-y+n_{11})!\Gamma\left(n+\sum_{i=1}^{4}\alpha_i\right)}}.$$

(10)

So, by (9) and (10), the one step look-ahead rule for the problem (5), (6) and (8) has the form:

$$T_L^1=\inf\{n\geq 0:\rho_0[h(x,y)]-E\{\rho_0[h((X,Y);(R,S)|(X,Y)=(x,y))]\}\leq c\}$$

$$=\inf\left\{n\geq 0:\frac{\sum_{n_{11}}\frac{\Gamma(\delta_1+n_{11})\Gamma(\delta_2+x-n_{11})\Gamma(\delta_3+y-n_{11})\Gamma(\delta_4+n-x-y+n_{11})}{n_{11}!(x-n_{11})!(y-n_{11})!(n-x-y+n_{11})!\Gamma\left(n+\sum_{i=1}^{4}\delta_i\right)}Tr\{A\cdot V[\delta_n(x,y;n_{11})]\}}{\left(n+1+\sum_{i=1}^{4}\delta_i\right)\sum_{n_{11}}\frac{\Gamma(\alpha_1+n_{11})\Gamma(\alpha_2+x-n_{11})\Gamma(\alpha_3+y-n_{11})\Gamma(\alpha_4+n-x-y+n_{11})}{n_{11}!(x-n_{11})!(y-n_{11})!(n-x-y+n_{11})!\Gamma\left(n+\sum_{i=1}^{4}\alpha_i\right)}}\leq c\right\}.$$

(11)

Let us write the binomial trivariate model (2) (Zini [7]) the following way:

$$P(X=x,Y=y,Z=z|P)=\sum_{n_{111}}\sum_{n_{11.}}\sum_{n_{1.1}}\sum_{n_{.11}}\frac{n!}{n_{111}!(n_{11.}-n_{111})!(n_{1.1}-n_{111})!(n_{.11}-n_{.11})!}$$
$$\times\frac{1}{(x-n_{11.}-n_{1.1}+n_{111})!}$$
$$\times\frac{1}{(y-n_{11.}-n_{.11}+n_{111})!(z-n_{.11}-n_{1.1}+n_{111})!}$$
$$\times\frac{1}{(n-x-y-z-n_{111}+n_{11.}+n_{1.1}+n_{.11})!} \tag{12}$$
$$\times P_1^{n_{111}}P_2^{n_{11.}-n_{111}}P_3^{n_{1.1}-n_{111}}P_4^{n_{.11}-n_{111}}P_5^{x-n_{11.}-n_{1.1}+n_{111}}$$
$$\times P_6^{y-n_{.11}-n_{11.}+n_{111}}P_7^{z-n_{1.1}-n_{.11}+n_{111}}$$
$$\times P_8^{n-x-y-z-n_{111}+n_{11.}+n_{.11}+n_{1.1}},$$

where $P=(P_1,P_2,\cdots,P_7)'\in S_7$, $P_8=1-\sum_{i01}^{7}P_i$, and limits given by (3) and (4).

It is possibile to show that, assuming conjugate prior and loss (like in (6) and (8)) for the model (12), the one step look-ahead rule has the following form:

$$T_L^1=\inf\Bigg\{n\geq 0:\Bigg[\sum_{n_{111}}\sum_{n_{11.}}\sum_{n_{1.1}}\sum_{n_{.11}}\frac{Tr\{A\cdot V[\delta_n(x,y;z)]\}\cdot\Gamma(\delta_1+n_{111})\Gamma(\delta_2+n_{11.}-n_{111})\Gamma(\delta_3+n_{1.1}-n_{111})\Gamma(\delta_4+n_{.11}-n_{111})}{\Gamma\left(n+\sum_{i=1}^{8}\delta_i\right)\cdot n_{111}!(n_{11.}-n_{111})!(n_{1.1}-n_{111})!(n_{.11}-n_{111})!}\times$$
$$\times\frac{\Gamma(\delta_5+x-n_{11.}+n_{111})\Gamma(\delta_6+y-n_{.11}-n_{11.})\Gamma(\delta_7+z-n_{1.1}-n_{.11}+n_{111})\Gamma(\delta_8+n-x-y-z-n_{111}+n_{11.}+n_{1.1}+n_{.11})}{(x-n_{11.}+n_{111})!(y-n_{.11}-n_{11.})!(z-n_{1.1}-n_{.11}+n_{111})!(n-x-y-z-n_{111}+n_{11.}+n_{1.1}+n_{.11})!}\Bigg]$$
$$\times\Bigg[\sum_{n_{111}}\sum_{n_{11.}}\sum_{n_{1.1}}\sum_{n_{.11}}\frac{\cdot\,\Gamma(\alpha_1+n_{111})\Gamma(\alpha_2+n_{11.}-n_{111})\Gamma(\alpha_3+n_{1.1}-n_{111})\Gamma(\alpha_4+n_{.11}-n_{111})}{\Gamma\left(n+\sum_{i=1}^{8}\alpha_i\right)\cdot n_{111}!(n_{11.}-n_{111})!(n_{1.1}-n_{111})!(n_{.11}-n_{111})!}$$
$$\times\frac{\Gamma(\alpha_5+x-n_{11.}+n_{111})\Gamma(\alpha_6+y-n_{.11}-n_{11.})\Gamma(\alpha_7+z-n_{1.1}-n_{.11}+n_{111})\Gamma(\alpha_8+n-x-y-z-n_{111}+n_{11.}+n_{1.1}+n_{.11})}{(x-n_{11.}+n_{111})!(y-n_{.11}-n_{11.})!(z-n_{1.1}-n_{.11}+n_{111})!(n-x-y-z-n_{111}+n_{11.}+n_{1.1}+n_{.11})!}\Bigg]^{-1}$$
$$\leq\left(n+1+\sum_{i=1}^{8}\delta_i\right)\cdot c\Bigg\}, \tag{13}$$

where

$$\delta_n(x,y;z)' \equiv [(\delta_1 + n_{111});(\delta_2 + n_{11.} - n_{111});(\delta_3 + n_{1.1} - n_{111});(\delta_4 + n_{.11} - n_{111});(\delta_5 + x - n_{11.} + n_{111});$$

$$(\delta_6 + y - n_{.11} - n_{11.});(\delta_7 + z - n_{1.1} - n_{.11} + n_{111});(\delta_8 + n - x - y - z - n_{111} + n_{11.} + n_{1.1} + n_{.11})]'.$$

For $M_n \equiv (n + \sum \delta_i) \cdot \rho_0(h)$ is a limited martingale, see Zini [6] and Shapiro et al. [4], the stopping rules (11) and (13) are asymptotically optimal in the sense of Bickel et al. [2].

4. Conclusions

In this work, a one step look-ahead rule has been deduced for binomial bivariate and trivariate statistical models under very general assumptions. Next step will be the generalisation of that rule when, at any sampling stage, *m* observations are drawn from a binomial bivariate or trivariate distribution.

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Reguła stopu w estymacji parametrów uogólnionych rozkładów dwumianowych

W pracy przedstawiono rozszerzenie technik estymacji parametrów w uogólnionych rozkładach dwumianowych. Rozważane uogólnienia dotyczą kategoryzacji sukcesów. W niektórych przypadkach w analizie danych jakościowych rozważa się dychotomiczny charakter narzucającej się statystycznej zależności lub wielomianowej niezależności, ignorując istnienie naturalnych modeli zależności.

Uogólniony w kontekście dwóch kategorii sukcesów rozkład dwumianowy, wprowadzony przez Zenga (1968), określa naturalną strukturę liczby sukcesów dwóch kategorii w *n* niezależnych próbach. Zini rozszerzył kategoryzację sukcesów do trzech klas oraz podał właściwości rozszerzonego w ten sposób rozkładu dwumianowego. W artykule zaprezentowano problem sekwencyjnej estymacji parametrów uogólnionego rozkładu dwumianowego z wykorzystaniem podejścia bayesowskiego. Założenia dotyczące rozkładów *a priori* parametrów w rozkładzie dwumianowym uogólnionym oraz przyjęta *a priori* funkcja straty stanowiły bazę do konstrukcji reguły stopu dla estymacji sekwencyjnej parametrów uogólnionego do dwóch oraz trzech kategorii sukcesów rozkładu dwumianowego. Mając na uwadze fundamentalną zasadę niezależności między regułą stopu a techniką estymacji (Berger) prezentowana reguła stopu, posiadająca asymptotycznie optymalne właściwości jest adekwatna do określonego problemu nawet w przypadku uwzględnienia kosztów wnioskownia i próbkowania. Omawiana reguła może być użyteczna w wielu obszarach zastosowań statystyki, np. w kontroli jakości lub analizie satysfakcji konsumentów.

Słowa kluczowe: *rozkład dwumianowy, estymacja, kategoryzacja sukcesu*