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## **NOVEL 2D REPRESENTATION OF VIBRATION FOR LOCAL DAMAGE DETECTION**

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**Abstract:** In this paper a new 2D representation for local damage detection is presented. It is based on a vibration time series analysis. A raw vibration signal is decomposed via short-time Fourier transform and new time series for each frequency bin are differentiated to decorrelate them. For each time series, auto-correlation function is calculated. In the next step ACF maps are constructed. For healthy bearing ACF map should not have visible horizontal lines indicating damage. The method is illustrated by analysis of real data containing signals from damaged bearing and healthy for comparison.

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**Keywords:** local damage detection, bi-frequency plane, partial autocorrelation function, fault pattern surface

### 1. INTRODUCTION

Local damage detection is one of the most often investigated problems in the literature related to diagnostics of rotating machinery. Such interest is caused by high importance of the local damage detection in maintenance of the considered machinery. Vibro-acoustic diagnostic methods are very popular since they do not require a long-time and expensive visual inspection. A local damage of rotating machinery results in a pulse train in the time domain occurred while two surfaces (e.g. two teeth from a gear-pair, rolling element and a raceway) are in contact during operation and at

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least one of them is damaged. Many vibro-acoustic methods rely on the time-frequency relation which associates an impulse in the time domain with a wideband excitation in the frequency domain. This relation is the principle of time-frequency methods that visualize energy flow in particular frequency bands through the whole signal. One of the most known time-frequency representations of the signal is the spectrogram, which is just a spectral density estimator obtained by the Fourier transform in relatively short-time windows (short-time Fourier transform, STFT). There are also other time-frequency representations and their enhancements, e.g. Vigner-Ville distribution, scalogram, ARgram, spectrogram enhanced by the local maxima method, etc. (Staszewski et al. 1997, Lin & McFadden 1997, Peng et al. 2002, Martin 1986, Makowski & Zimroz 2014, De Sena & Rocchesso 2007, Obuchowski et al. 2014). A detailed description of these methods might be found in extensive review works (Feng et al. 2013, Klein et al. 2014, Cohen 1995, Cohen 1989, Flandrin 1999). In the case of rotating machinery, due to its rotation, there is always a pulse train while a local damage occurs. If the rotating speed is constant in time, the time lapse between each pulse is expected to be constant (excl. some phenomena related to rolling elements, e.g. high jitter). Thus, it is reasonable to consider the vibration signal in a two-dimensional space, where dimension is the (modulated) frequency and the second is the time lapse between impacts (or, equivalently, its inverse in the unit of frequency). Such representation is the answer to the question about the frequency band excited by a damage revealed by a particular fault frequency. In the literature one can find a few existing bi-frequency representations and their enhancements, e.g. Spectral Correlation Density (SCD), Modulation Intensity Distribution (MID), bispectrum, higher-order polyspectra, etc. (Antoni 2009, Urbanek et al. 2012, Guoji et al. 2014, Hickey et al. 2009, Collis et al. 1998). The bi-frequency plane might be also applied for signals that represent vibration of a machine which operates in a time-varying rotational speed regime. In this case an additional pre-processing step (e.g. angular resampling) must be applied to make distances between impulses constant in time. In this paper we propose a novel frequency-frequency map which is based on second-order properties of the investigated signal, namely the autocorrelation function (ACF) and the partial autocorrelation function (PACF, as known as reflection coefficients). These functions are widely-used in the field of signal processing for machine diagnostics. The main applications of these functions are related to autoregressive modeling with constant, periodic or time-varying autoregression coefficients (Wang & Wong 2002, Wang 2008, Wang & Makis 2009, Endo & Randall 2007, Wyłomańska et al. 2014, Poulimenos & Fassois 2006, Spiridonakos & Fassois 2014, Makowski & Zimroz 2011). In this paper we investigate their benefits without referring to the autoregressive modeling, thus showing that these functions might be informative by themselves. Moreover, since the proposed frequency-frequency map is an enhancement of the spectrogram obtained by the short-time Fourier transform we analyze properties

and benefits of the novel representation over the spectrogram. We also compare the properties to the existing bi-frequency plane.

The paper is organized as follows. In Section 2 we present methodology which is consisted of STFT transform, decomposition, ACF and PACF calculation. In Section 3 real data analysis results are presented which illustrate properties of the proposed methodology. Last section contains conclusion.

## 2. METHODOLOGY

In this section we present the algorithm that is used for the local damage detection in rotating machines. After obtaining the raw signal, we first transform it into time-frequency map through the short-time Fourier transform (STFT). The STFT is defined as (Allen 1977):

$$STFT(t, f) = \int_{-\infty}^{\infty} w(t - \tau) X(\tau) \exp(2j\pi\tau) d\tau \quad (1)$$

where  $w(t - \tau)$  is the shifted window and  $X(\tau)$  is the input signal. The discrete version of equation (1) for observations  $X_1, X_2, \dots, X_N$ , time point  $t \in T$  and frequency  $f \in F$  is defined as follows:

$$STFT(t, f) = \sum_{k=0}^{N-1} w(t - k) X(k) \exp\left(\frac{2j\pi k}{N}\right) \quad (2)$$

In our analysis we use the Kaiser window. In the proposed procedure each sub-signal is a time series corresponding to a narrow frequency band that arises after the mentioned time-frequency decomposition. However, since STFT matrix is complex, absolute value needs to be taken to obtain the spectrogram. Moreover, in our analysis we examine the increments of the sub-signals, because differentiation of signal is frequently used in time series processing to decorrelate signal. In the next step the sample autocorrelation function (ACF) and partial autocorrelation function (PACF) are calculated for each sub-signal corresponding to a given frequency bin. Let us remind that the ACF for vector of observations  $X_1, X_2, \dots, X_N$  is defined as follows (Box et al. 1994):

$$R(t) = \frac{\frac{1}{n} \sum_{i=1}^{n-t} (X_{i+t} - \bar{X})(X_i - \bar{X})}{\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2}, \quad (3)$$

and, moreover, the PACF at lag  $k$  is defined as (Box et al. 1994):

$$\alpha(k) = \varphi_{kk}, \quad k \geq 1, \quad (4)$$

where  $\varphi_{kk}$  is last component of vector, where:

$$\Gamma_k = [\gamma(i-j)]_{i,j=1}^k, \quad \gamma(j) = \frac{1}{n} \sum_{i=1}^{n-j} (X_{i+j} X_i) - \bar{X}^2, \quad (5)$$

where  $\bar{X}$  is mean. In Matlab PACF is calculated as reflection coefficient scaled by  $-1$ . It is known that the autocorrelation function and partial autocorrelation function show dependency in time between time series and shifted by certain number of samples version of it. Higher/lower than zero value shows positive/negative dependency. Moreover, on the basis of ACF and PACF we can observe also the cyclic behavior of corresponding time series.

After calculating the ACF and PACF of sub-signals we construct the corresponding time-frequency maps based on the mentioned measures of dependence. The main advantages of the proposed two-dimensional representations over the spectrogram are clearer visibility of the fault pattern due to higher resolution (STFT shows energy flow in frequency bin, while ACF shows cyclicity in frequency bin) of time-frequency map. Their benefits over the STFT-based time-frequency map might be also noticed in the case of healthy bearing – lack of noise in informative band, possible proof of dependence/correlation for time series in frequency bin (due to properties of ACF/PACF map, it shows cyclicity/dependency in frequency bin). Furthermore, these benefits are not reflected by significantly higher computational complexity. Even a well-known Levinson-Durbin recursion calculates the PACF in a reasonable time. The ACF is calculated using FFT-based algorithm which incorporates the Wiener-Khinchin theorem. It is also worth mentioning that the proposed methodology does not require any additional knowledge about the expected fault frequency. Below, there is presented equation allowing easy calculation of frequency in input signal, given lag from correlation map.

$$lag = \frac{N}{N_1}[s] = \frac{1}{f_1[\text{Hz}]}, \quad (6)$$

where  $N$  is equal to length of input signal in seconds,  $N_1$  is length of vector of sub-signal and  $f_1$  is frequency for raw signal. Therefore this two-dimensional representation can be treated as both a time-frequency and bi-frequency map.

### 3. REAL DATA ANALYSIS

In this section we present real data analysis that validates the methodology presented in the previous section. The investigated machine is a belt conveyor driving unit which operates in an open-pit mine. It is consisted of a motor, two-stage gearbox and drive pulley. We analyze signals that were acquired on the drive pulley's housing in horizontal direction. Each signal represents vibration of a bearing in different condition, i.e. one bearing was healthy and one revealed a local damage (outer ring local damage). The length of both signals is 2.5 s, sampling frequency - 19.2 kHz. The signals were acquired under approximately constant rotational speed and the machine was loaded, i.e. bulk material was transported by the conveyor belt. The case analyzed in this paper was also investigated in a few previous works, i.e. (Makowski & Zimroz 2011, Zimroz & Bartelmus 2012, Obuchowski et al. 2014), where the Reader might find the scheme, photos, characteristic frequencies and full description of the machine. Recall, that the theoretical characteristic frequencies are: FTF (fundamental train frequency) – 0.51 Hz, BSF (ball spin frequency) – 4.45 Hz, BFF (ball fault frequency) – 8.9 Hz, BPFO (ball passing frequency outer race) – 12.34 Hz and BPFI (ball passing frequency inner race) – 16.06 Hz.

Fig. 1 and 2 present raw vibration signals and spectrograms of both signals, respectively. The spectrograms are obtained using the STFT with following parameters: Kaiser window of length 125, overlapping ratio – 88%, number of points at which the FFT was calculated - 512. In both cases high energy, low frequency contamination from the gearbox is dominating in the signal by means of amplitude. It covers frequency bands lower than 1 kHz. One can also observe high frequency low energy noise above 7kHz. Middle frequency band from 1 kHz to 7 kHz contains several cyclic wideband excitations which might be related to a local damage. Analysis performed in previous works proved that the frequency of occurrence of these excitations meet the characteristic frequency of the outer race damage, although the estimated frequency slightly differ from the theoretical one due to the jitter effect. The aim of this paper is to provide an enhanced two-dimensional plane where one could observe the modulated frequency band (informative frequency band) and corresponding modulation frequencies, which might be related to theoretical characteristic frequencies.

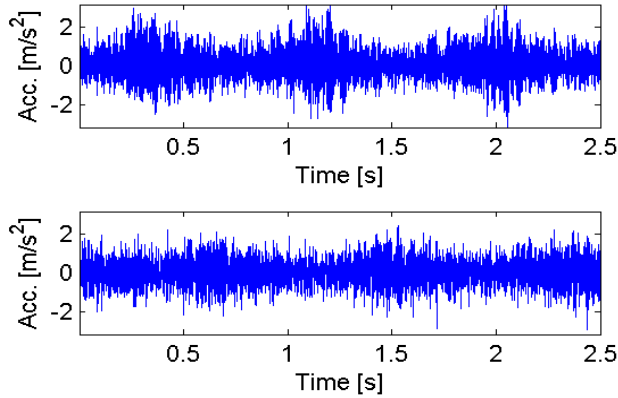


Fig. 1 Raw vibration signals from bearings (top panel – healthy bearing, bottom panel – damaged bearing)

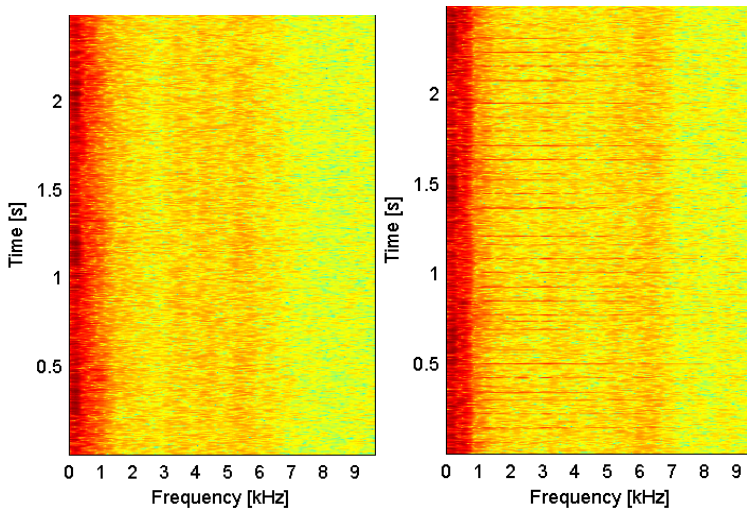


Fig. 2. Spectrograms of signals from bearings (left panel – healthy bearing, right panel – damaged bearing)

Fig. 3 presents ACF map for both signals. Both maps (healthy and outer race damage case) are calculated for lags up to 400. Low frequency bands (c.a. 200 Hz for both cases and c.a. 900 Hz for healthy case) reveal autocorrelation through all lags on the map (Fig. 3). The ACF-based map related to the signal from the damaged bearing clearly presents that for a certain number of lags and for multiplies of this number the ACF is higher in frequency bands between 1 and 6 kHz. Since the time lapse between

wideband lines in Fig 3 (right panel) varies from 0.078 to 0.079 s, the corresponding frequency varies from 12.66 to 12.82 Hz.

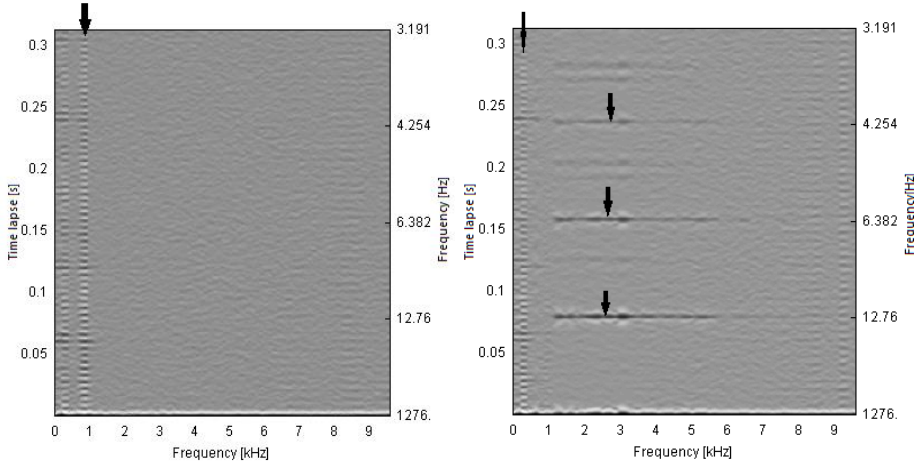


Fig.3 ACF Maps of signal from bearings (left panel – healthy bearing, right panel – damaged bearing, arrows indicate major differences between maps)

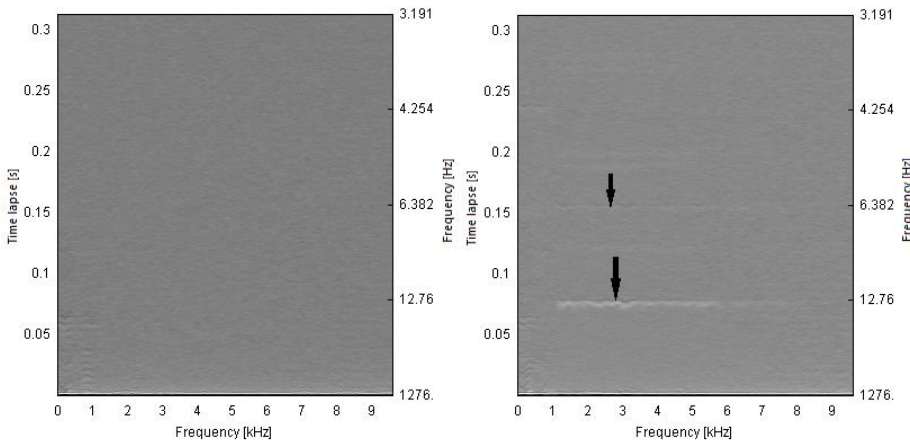


Fig.4 PACF Maps of signal from bearings (left panel – healthy bearing, right panel – damaged bearing, arrows indicate major differences between maps)

In Fig. 4 we observe the PACF-based maps of the signals. As it can be seen, the only significant difference between both plots is a wideband line of range 1–6 kHz at about 0.078–0.079 s. Comparing to the ACF-based maps the peculiar effect in frequency bands lower than 1 kHz cannot be observed. The PACF-based maps do not

distinguish high-energy low-frequency bands from high-frequency low-energy ones. Thus, the only significant pattern can be seen in the informative frequency band and it follows the characteristic frequency related to the local damage of the outer race.

#### 4. CONCLUSION

In this paper two novel two-dimensional representations of a vibrations signal from a heavy-duty mining machinery is proposed. Since they are based on indicators of dependency and cyclicity of the data (the autocorrelation function and partial autocorrelation function) they might be treated as both time-frequency and bi-frequency planes. Also formula for fast switching between these two domains is provided. We discussed properties of the two novel representations which might be interesting from the point of view of heavy-duty mining machinery diagnostics. The methodology is illustrated by deep analysis of the signals representing vibrations of bearing from a belt conveyor driving unit. We proved that such two-dimensional representations might provide better results than the spectrogram in both analyzed cases, i.e. healthy and damaged bearing.

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