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Physical and mathematical model of the digital coherent optical spectrum analyzer

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A physical and mathematical model of digital coherent optical spectrum analyzers is discussed. In digital coherent optical spectrum analyzers the input signal is forming as a two-dimensional transparency by means of a spatial light modulator. After Fourier transformation with a lens, multiplication by a spatial filter and second Fourier transformation, the signal is captured by a matrix detector for further computer processing. A lot of digital coherent optical spectrum analyzers and their components (laser, lighting system, spatial light modulator, Fourier lens and matrix detector) models were developed to calculate the signal at the matrix detector output. They use the impulse response and transfer function to evaluate the effectiveness of digital coherent optical spectrum analyzers. The analysis of mathematical relationships shows that the use of a discrete spatial light modulator for the signal input and a matrix detector for light field registration in the spectral domain when combined with computer technology greatly extends the functionality of digital coherent optical spectrum analyzer. The formulas for impulse response and transfer function calculations were obtained, which allows to analyze and optimize the digital coherent optical spectrum analyzer basic characteristics.

Keywords: optical spectrum analyzer, optical correlator, spatial light modulator, matrix detector.

1. Introduction

Laser optoelectronic devices, namely coherent optical spectrum analyzers (COSA) of spatial signals, became widespread in modern information systems [1, 2]. COSA input signal forms a transparency which is determined by the amplitude of the monitored signal. This signal is usually written onto a spatial light modulator (SLM) with pixel structure (matrix). Light transmission of the pixels is determined by the investigated spatial signal [3]. The output signal of COSA is recorded by a matrix detector (MD) with further computer processing that extends the functionality of optical information processing system [4]. Therefore, these devices are called digital coherent optical spectrum analyzers (DCOSAs). There is a large number of scientific works, where an analog COSA model with continuous transmittance transparency input and output detection with scanning microphotometers was investigated in details [5–7]. The research of DCOSA, which uses a modern digital matrix SLM and MD, is almost absent [3].

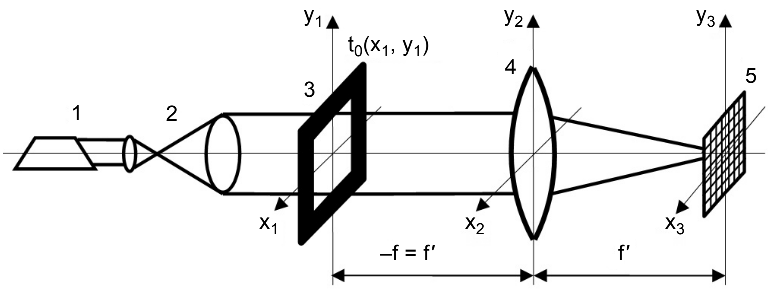

Fig. 1. DCOSA scheme; 1 – laser, 2 – optical system which forms a parallel beam, 3 – spatial light modulator, 4 – Fourier lens, 5 – matrix detector.

The purpose of this paper is to develop the physical and mathematical model of the modern digital coherent optical spectrum analyzer, which improves the characteristics of the spectrum analyzer.

2. Results and discussion

2.1. Functional transformation in DCOSA

A generalized scheme of DCOSA is shown in Fig. 1. It consists of a coherent light source (laser), optical lighting system, SLM, Fourier lens and MD.

A flat coherent light wave from the laser illuminates SLM where the diffraction of light takes place. If SLM has a transmittance coefficient $t_0(x_1, y_1)$ and is located in the front focal plane $x_1 y_1$ of Fourier lens, then the amplitude distribution in the rear focal plane $x_3 y_3$, where MD is situated, is defined by [2, 8]

$$V(x_3, y_3) = C_0 F\Big[V(x_1, y_1) t_0(x_1, y_1) t_{\text{oa}}(x_1, y_1) P_{0,\,\text{eff}}(x_1, y_1)\Big] \tag{1}$$

where C_0 is the complex coefficient, F is the operator of two-dimensional Fourier transform for spatial frequencies $v_x = x_3/(\lambda f)$ and $v_y = y_3/(\lambda f)$, λ is the light wavelength, f is the Fourier lens focal length, $V(x_1, y_1)$ is the input amplitude on the modulator, $t_{\text{oa}}(x_1, y_1)$ is the aperture function that limits the size of SLM, $P_{0,\,\text{eff}}(x_1, y_1)$ is the function of an effective lens aperture, which is modified to the modulator plane $x_1 y_1$. Illumination on MD is equal to the square of amplitude $E(x_3, y_3) = |V(x_3, y_3)|^2$.

MD converts the distribution of illumination into an electrical signal which in the sequel is processed by a microprocessor to determine the parameters of the input signal spectrum.

2.2. Mathematical models of DCOSA elements

To calculate the MD output signal, it is necessary to develop models and identify features of particular components of a generalized DCOSA model, that is: laser, lighting system, SLM, the Fourier lens and MD.

2.2.1. Coherent radiation source

A helium-neon or semiconductor laser is typically used as a source of coherent radiation in DCOSA. High coherence, small angle of divergence and radiation stability are the main advantages of the He-Ne laser, and large size and small coefficient of efficiency are its shortcomings. On the contrary, small size and high coefficient of efficiency (up to 70%) are advantages of a semiconductor laser and low coherence, big angle of divergence and radiation instability are disadvantages.

For mathematical modeling of DCOSA lasers we use such features as: radiation power Φ_L, laser wavelength λ, character of radiation (single-mode and multimode), laser beam angle of divergence θ_L.

The wavelength of a He-Ne laser is 632.8 nm, and it has a narrow spectrum width of 1.4 GHz. It can operate in a single mode as well.

The angle of divergence θ_L of a laser beam is determined by its confocal parameter R_L [9]

$$\theta_L = \sqrt{\frac{2\lambda}{\pi R_L}} = \frac{\lambda}{\pi \omega_L} \tag{2}$$

where ω_L is the beam radius of the fundamental mode in the beam waist,

$$\omega_L = \sqrt{\frac{\lambda R_L}{2\pi}} \tag{3}$$

The radius of the beam increases with the distance z to the beam waist by law

$$\omega(z) = \omega_L \sqrt{1 + \frac{2z}{R_L}} \tag{4}$$

The value θ_L is the divergence of the beam on its axis to reduce the level of intensity to e^2 times the maximum value. To find the radius of the beam r_0, which corresponds to a reduction of intensity to the specified level I_0, one can use the formula for the intensity distribution in the laser beam [9]

$$I_0(r_0) = I_{max} \exp\left(-\frac{2r_0^2}{\omega_L^2}\right) \tag{5}$$

hence

$$r_0 = \omega_L \sqrt{0.5 \ln(I_{max}/I_0)} \tag{6}$$

2.2.2. Laser beam expander optical system

Two-lens optical systems, which expand the laser beam diameter from small ω_L to bigger ω_o diameter (Fig. 2), are used for uniform illumination of input transparency. The requirement to convert the small diameter laser beam to the large diameter beam

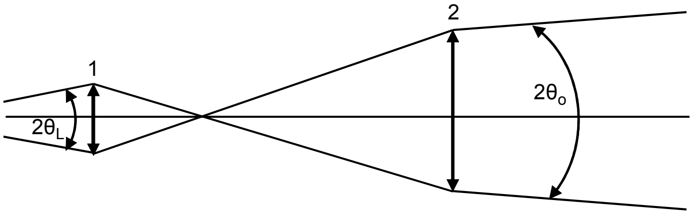

Fig. 2. Collimation laser beam of the telescopic system; 1 – first lens, and 2 – second lens.

coincides with the requirement to convert the beam of large angular divergence θ_L to the beam of low divergence θ_o.

The relationship between the output beam divergence and the input beam divergence for a telescopic system is

$$\frac{\theta_o}{\theta_L} = \frac{\omega_L}{\omega_o} = \left|\frac{f_1'}{f_2'}\right| \tag{7}$$

where f_1' and f_2' are the focal lengths of optical system components.

2.2.3. Spatial light modulator

A spatial light modulator is an optical device (such as mask, diffraction grating, aperture, phase plate, *etc*.), which is located towards the beam of light and modulates the incident plane wave in magnitude and phase. The modulator impact is described by the complex transmittance (reflectance) function $t_0(x_1, y_1)$

$$V'(x_1, y_1) = t_0(x_1, y_1)V(x_1, y_1) = t_s(x_1, y_1)t_m(x_1, y_1)V(x_1, y_1) \tag{8}$$

where $V(x_1, y_1)$ and $V'(x_1, y_1)$ are the amplitude distributions of the light field before and after the modulator, respectively, $t_s(x_1, y_1)$ is the input signal and $t_m(x_1, y_1)$ is the SLM transmission mask. A SLM inputs the signal to DCOSA for further research. The input signal typically is recorded as amplitude transmittance that varies in space and in time [10].

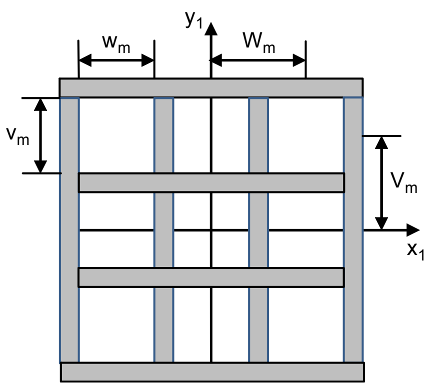

Fig. 3. Matrix structure of LCSLM.

Light modulation in most cases is based on the change of the complex refractive index, which leads to amplitude or phase modulation. Modulators are electrically (ECT) or optically (OCT) controlled transparency and are characterized by the following parameters: linear dimensions of the aperture $X_m \times Y_m$ (mm × mm), pixel size $v_m \times w_m$, pixel quantity $p_m \times q_m$, resolution (line/mm), operable and driving (for the OCT) wavelength (nm), control voltage V (for the ECT), optical contrast, operation speed, transfer function nonlinearity, and noise.

Liquid-crystal spatial light modulators (LCSLMs) with pixel transmissive structure (Fig. 3) have been widely used as SLMs in recent years. Transmittance of LCSLM is given by [3]

$$t_m(x_1, y_1) = \left\{\left[\mathrm{rect}\left(\frac{x_1}{v_m}\right)\mathrm{rect}\left(\frac{y_1}{w_m}\right)\right] ** \left[\frac{1}{V_m W_m}\mathrm{comb}\left(\frac{x_1}{V_m}\right)\mathrm{comb}\left(\frac{y_1}{W_m}\right)\right]\right\} \times \left\{\mathrm{rect}\left(\frac{x_1}{p_m V_m}\right)\mathrm{rect}\left(\frac{y_1}{q_m W_m}\right)\right\} \tag{9}$$

where $**$ is the two-dimensional convolution operator, and $V_m \times W_m$ – period of the modulator matrix structure, and

$$\mathrm{comb}\left(\frac{x_1}{V_m}\right)\mathrm{comb}\left(\frac{y_1}{W_m}\right) = \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\delta\left(\frac{x_1}{V_m} - n\right)\delta\left(\frac{y_1}{W_m} - m\right) \tag{10}$$

$$\mathrm{rect}\left(\frac{x_1}{p_m V_m}\right) = \begin{cases} 1 & \text{if } \left|\dfrac{x_1}{p_m V_m}\right| \le \dfrac{1}{2} \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

The first braces in Eq. (9) define the impulse response of the infinite matrix and the expression in second braces limits the matrix size to $p_m V_m \times q_m W_m$.

2.2.4. Fourier lens

The development of the Fourier lens is an important step in DCOSA optical system design. To ensure high accuracy of the two-dimensional Fourier transform, the lens must satisfy high requirements. There are aberrations in real lens with finite aperture, which significantly affect the accuracy of the two-dimensional Fourier transform and resolution of the device. The especially important condition for the usage of Fourier lens in DCOSA is to provide the linear dependence between the coordinates in the back focal plane of the lens and spatial frequencies in the spectrum, which is determined by Fourier lens distortion.

For Fourier lens modeling we will use the following features: focal length f (mm), diameter of the entrance pupil D_o (mm), linear field of view R_o (mm), circle of confusion radius r_o (μm), and modulation transfer function $M_0(v_x, v_y)$.

Full-length lenses for SLR cameras can be used as Fourier lenses. For example, a camera Nikon D610 has a suitable panoramic lens whose focal length varies from 24 to 85 mm, and the aperture value – from 3.5 to 4.5.

2.2.5. Matrix detector

CCD or CMOS detectors capture the signal spectrum in modern DCOSA [4]. A CCD is one of the main elements of digital cameras and camcorders. For modeling MD we will use the following features: matrix format $p_D \times q_D$, pixel size $v_D \times w_D$ (μm^2), detector matrix structure period $V_D \times W_D$ (μm^2), spectral sensitivity R_D (V/(lx · s)), frame rate f_f (Hz), clock speed readout f_d (Hz), threshold exposition H_n (lx · s), and signal acquisition (integration) time t_i (s).

The process of spatial sampling carried out by the MD can be represented by convolution (9):

$$R_D(x_3, y_3) = \left\{ R_{D,00}(x_3, y_3) ** \left[\frac{1}{V_D W_D} \text{comb}\left(\frac{x_3}{V_D}\right) \text{comb}\left(\frac{y_3}{W_D}\right) \right] \right\} \times \left\{ \text{rect}\left(\frac{x_3}{p_D V_D}\right) \text{rect}\left(\frac{y_3}{q_D W_D}\right) \right\} \quad (12)$$

where $R_{D,00}(x_3, y_3)$ is the sensitivity of the on-axis pixel defined as

$$R_{D,00}(x_3, y_3) = \begin{cases} R_D & \text{if } -\dfrac{v_D}{2} \le x_3 \le \dfrac{v_D}{2}, \quad -\dfrac{w_D}{2} \le y_3 \le \dfrac{w_D}{2} \\ 0 & \text{otherwise} \end{cases} = R_D \, \text{rect}\left(\frac{x_3}{v_D}\right) \text{rect}\left(\frac{y_3}{w_D}\right) \quad (13)$$

The real alternative to CCD is CMOS matrix that combines photosensitive matrix and analog processing circuit (including an analog-to-digital converter) on a single chip, which results in ease of usage and low cost.

2.3. DCOSA main characteristics

The efficiency of DCOSA can be measured by its main characteristics. Main DCOSA quality characteristics that determine their properties and functional capabilities are [11]: operating range of spatial frequencies, space bandwidth product, spatial spectral resolution, energy resolution, and spectrum amplitude and spatial frequency measurement errors.

Operating range of spatial frequencies is the range of spatial frequencies within which all frequencies of the research spectrum pass through DCOSA optical system. At a certain spatial frequency $\nu_{x,\,\max}$ the researched spectrum will disappear. This maximum spatial frequency determines the operating spatial spectral range.

Space bandwidth product (SBP) is determined by the number of distinguishable dots which are formed by the spectrum analyzer [12]. This band is similar to a working spectral range. The largest spatial frequency $\nu_{x,\,\max} = \nu_{\mathrm{res}}$ which is resolvable by DCOSA, is determined by the largest diffraction angle $\varphi_{\mathrm{d}} = \theta_{\max}$ of the grate which is located in SLM plane. At orthogonal incidence of the plane wave to the diffraction grating which is located in the front focal plane of the Fourier lens, for the first-order diffraction grating with the basic grating equation, we have: $d\sin(\varphi_{\mathrm{d}1}) = \lambda$, where $d = 1/\nu_{\mathrm{res}}$ is the period of the diffraction grating. Hence

$$\nu_{\mathrm{res}} = \frac{1}{\lambda}\sin(\varphi_{\mathrm{d}1}) \tag{14}$$

Then, the spatial bandwidth product of the spectrum analyzer is

$$\mathrm{SBP} = \frac{D_{\mathrm{m}}}{d} = D_{\mathrm{m}}\nu_{\mathrm{res}} = \frac{D_{\mathrm{m}}}{\lambda}\sin(\varphi_{\mathrm{d}1}) \tag{15}$$

where D_{m} is SLM diameter.

Spatial spectral resolution $\delta\nu_x$ is characterized by the frequency range in which the system can resolve (differentiate) two spectral components of equal amplitude with frequencies ν_x and $\nu_x + \delta\nu_x$. Since the output signal is recorded by a MD, then the spectral resolution of the system is determined, above all, by the size of its sensitive element (pixel).

Energy resolution determines whether MD is able to detect signals in background noise. The main feature that determines the energy resolution is MD sensitivity threshold. The sensitivity threshold is the least flow of radiation or least illuminance that can be registered with the receiving system. It corresponds to a minimum signal level at which MD output ratio signal/noise is equal to unity.

Spectrum amplitude measurement error is the difference between the amplitude of light field in the plane of analysis and the real signal spectrum amplitude for a specific spatial frequency.

Spatial frequency measurement error is the difference between the estimated and actual spatial frequencies in a certain analysis plane point.

2.4. DCOSA output signal

Let the signal at each MD pixel be formed by integration of light exposure within all pixel area. Then, the output signal is defined by

$$u_{\mathrm{s}} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E(x_3, y_3) R_{\mathrm{D}}(x_3, y_3)\,\mathrm{d}x_3\mathrm{d}y_3$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |V(x_3, y_3)|^2 R_{\mathrm{D}}(x_3, y_3)\,\mathrm{d}x_3\mathrm{d}y_3 \qquad (16)$$

where $R_{\mathrm{D}}(x_3, y_3)$ is MD sensitivity.

After the substitution of MD sensitivity (12) into (16), we obtain

$$u_{\mathrm{s}} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |V(x_3, y_3)|^2 \left\{ R_{\mathrm{D,\,00}}(x_3, y_3)** \left[\mathrm{comb}\left(\frac{x_3}{V_{\mathrm{D}}}\right)\mathrm{comb}\left(\frac{y_3}{W_{\mathrm{D}}}\right)\right]\right\}$$

$$\times \left[\mathrm{rect}\left(\frac{x_3}{p_{\mathrm{D}}V_{\mathrm{D}}}\right)\mathrm{rect}\left(\frac{y_3}{q_{\mathrm{D}}W_{\mathrm{D}}}\right)\right]\mathrm{d}x_3\mathrm{d}y_3 \qquad (17)$$

In the monograph [13] it was proved that the integral (17) can be written as

$$u_{\mathrm{s}} = \sum_{n=-n_x}^{n_x}\sum_{m=-m_y}^{m_y} u_{nm} \qquad (18)$$

where $p_{\mathrm{D}} = 2n_x + 1$, $q_{\mathrm{D}} = 2m_x + 1$ and u_{nm} is the amplitude of the output signal of *nm*-th pixel,

$$u_{nm} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |V(x_3, y_3)|^2 R_{\mathrm{D,\,00}}(x_3 - nV_{\mathrm{D}}, y_3 - mW_{\mathrm{D}})\,\mathrm{d}x_3\mathrm{d}y_3$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left| C_0 F\left[V(x_1, y_1)t_{\mathrm{s}}(x_1, y_1)t_{\mathrm{m}}(x_1, y_1)t_{\mathrm{oa}}(x_1, y_1)P_{0,\,\mathrm{eff}}(x_1, y_1)\right]\right|^2$$

$$\times R_{\mathrm{D,\,00}}(x_3 - nV_{\mathrm{D}}, y_3 - mW_{\mathrm{D}})\,\mathrm{d}x_3\mathrm{d}y_3$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T_{\mathrm{s}}^2(v_x, v_y)**h_{\mathrm{sa},\,nm}(v_x, v_y)\,\mathrm{d}v_x\mathrm{d}v_y \qquad (19)$$

where $h_{\mathrm{sa},\,nm}(v_x, v_y)$ is the impulse response of *nm*-th pixel,

$$h_{\mathrm{sa},\,nm}(v_x, v_y) = (C_0\lambda f)^2 \left| F\left[V(x_1, y_1)t_{\mathrm{m}}(x_1, y_1)t_{\mathrm{oa}}(x_1, y_1)P_{0,\,\mathrm{eff}}(x_1, y_1)\right]\right|^2$$

$$\times R_{\mathrm{D,\,00}}(\lambda f v_x - nV_{\mathrm{D}}, \lambda f v_y - mW_{\mathrm{D}}) \qquad (20)$$

The analysis of the obtained function (19) shows that the amplitude of the output signal of *nm*-th pixel depends linearly on the square of the spatial spectrum $|T_{\mathrm{s}}(v_x, v_y)|^2$.

The spectrum is impacted by DCOSA impulse response $h_{\mathrm{sa},\,nm}(v_x, v_y)$. The discrete value of spatial frequency which corresponds to the center position of nm-th pixel is determined by

$$v_{xn} = \frac{x_{3n}}{\lambda f} = \frac{nV_{\mathrm{D}}}{\lambda f} \tag{21a}$$

$$v_{ym} = \frac{y_{3m}}{\lambda f} = \frac{mW_{\mathrm{D}}}{\lambda f} \tag{21b}$$

It can be shown [8] that DCOSA optical system is a coherent linear invariant system with the impulse response $h_{\mathrm{sa,\,c}}(v_x, v_y)$, which for the proposed mathematical model (1) is given by

$$h_{\mathrm{sa,\,c}}(v_x, v_y) = C_0 F\Big[V(x_1, y_1)t_{\mathrm{m}}(x_1, y_1)t_{\mathrm{oa}}(x_1, y_1)P_{0,\,\mathrm{eff}}(x_1, y_1)\Big] \tag{22}$$

The amplitude of the light field $V(x_3, y_3)$ is defined by the convolution of the ideal spectrum $T_{\mathrm{s}}(v_x, v_y)$ with the coherent impulse response $h_{\mathrm{sa,\,c}}(v_x, v_y)$ of the spectrum analyzer. Therefore, the efficiency of DCOSA can be characterized by the coherent transfer function (CTF) of the spectrum analyzer $K_{\mathrm{sa}}(x_1, y_1)$ [2]. CTF $K_{\mathrm{sa}}(x_1, y_1)$ is defined by the normalized Fourier transform of an impulse response of the system $h_{\mathrm{sa,\,c}}(v_x, v_y)$, *i.e.*,

$$K_{\mathrm{sa}}(x_1, y_1) = \frac{F\Big[h_{\mathrm{sa,\,c}}(v_x, v_y)\Big]}{K_{\mathrm{sa}}(0, 0)} \tag{23}$$

After substituting the impulse response (22) into the normalized Fourier transform (23), we obtain

$$K_{\mathrm{sa}}(x_1, y_1) = V(x_1, y_1)t_{\mathrm{m}}(x_1, y_1)t_{\mathrm{oa}}(x_1, y_1)P_{0,\,\mathrm{eff}}(x_1, y_1) \tag{24}$$

The analysis of functions (24) and (9) shows that DCOSA optical system CTF is equal to unity within SLM pixels and is equal to zero otherwise. Pixel structure of SLM transmission leads to the distortion of the researched spectrum due to the spatial sampling of the input signal.

3. Conclusions

Physical and mathematical model of the digital coherent optical spectrum analyzer that allows to explore the process of converting an input signal from a discrete spatial light modulator to an output signal of the matrix detector was developed. The analysis of mathematical relationships showed that:

1. The usage of a discrete SLM for recording the input signal and MD for registration of the light field in the spectral domain makes it possible to create a new class of digital

coherent optical spectrum analyzers, which being combined with computer technology greatly expands the functionality of the system.

2. The main feature of DCOSA is the transition from analog measurements of the input signal and its spectrum to discrete measurements, which results in distortions of the measured spectrum. These distortions are determined by the impulse response and the coherent transfer function of the optical system and the impulse response of DCOSA.

3. The equations which were obtained to calculate the impulse response and the transfer function enable to analyze and optimize main DCOSA characteristics.

4. In the future it is advisable to investigate the impact of SLM and MD pixels size and Fourier lens aberrations on the modulation transfer function of DCOSA.

References

[1] STARK H. [Ed.], *Applications of Optical Fourier Transforms*, Academic Press, 1982.

[2] ZENKOVA C.YU., GORSKY M.P., SOLTYS I.V., ANGELSKY P.O., *Use of motion peculiarities of test particles for estimating degree of coherence of optical fields*, Ukrainian Journal of Physical Optics **13**(4), 2012, pp. 183–195.

[3] CURATU G., HARVEY J.E., *Analysis and design of wide-angle foveated optical systems based on transmissive liquid crystal spatial light modulators*, Optical Engineering **48**(4), 2009, article ID 043001.

[4] NEUYMIN O., DYACHENKO S., "CCD", Visnyk NTUU "KPI", Ser. Radiotekhnika, Radioaparatobuduvannya, No. 41, 2010, pp. 182–189.

[5] KOLOBRODOV V.G., TYMCHYK G.S., *Analysis of optical systems of coherent spectrum analyzers*, Rus. J. Opto-Mechanical Industry **10**, 1982, pp. 4–7.

[6] SALEH B.E.A., TEICH M.C., *Fundamentals of Photonics*, Wiley, New York, 1991.

[7] KOLOBRODOV V.G., TYMCHYK G.S., *Applied Diffractive Optics*, Politekhnika, Kyiv, 2014.

[8] KOLOBRODOV V.G., TYMCHYK G.S., KOLOBRODOV M.S., *Geometrical noise bandwidth of the optical spectrum analyzer*, Ukr. J. Visnik Kremenchuk Mykhailo Ostohradskyi National University **5**, 2015, pp. 50–55.

[9] KLIMKOV YU.M., *Bases of Calculation of Optoelectronic Devices with Lasers*, Sovetskoe Radio, Moscow, 1978.

[10] BOGATYIREVA V.V., DMITRIEV A.L., *Optical Methods of Information Processing*, ITMO University, St. Petersburg, 2009.

[11] KOLOBRODOV V.G., TYMCHYK G.S., QUYNH ANH, *Defining generalized characteristics of coherent spectrum ANALYZER*, Ukr. J. Naukovi visti NTUU "KPI", No. 5, 2012, pp. 102–108.

[12] BURALLI D.A., MORRIS G.M., *Design of a wide field diffractive landscape lens*, Applied Optics **28**(18), 1989, pp. 3950–3959.

[13] KOLOBRODOV V.G., TYMCHYK G.S., KOLOBRODOV M.S., *Coherent Optical Spectrum Analyzer: Monograph*, Politekhnika, Kyiv, 2015.

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