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Highly nonlinear solitary wave velocity measurement with a modified Michelson interferometer

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We have measured the velocity of a highly nonlinear solitary wave, generated by the collision of a ball with a linear set of pre-compressed balls of the same material, using a double Michelson interferometer. One of the mirrors of each interferometer was mounted on one of two balls of the array. The measured soliton velocity was $u = (346 \pm 40)$ m/s for brass balls with pre-compression force $F = (0.8 \pm 0.1)$ N, and $u = (102 \pm 3)$ m/s for Teflon balls with pre-compression force $F = (0.4 \pm 0.1)$ N.

Keywords: nonlinear waves, granular media, modified Michelson interferometer.

1. Introduction

Granular matter is ubiquitous in Nature. It is found in agriculture as grains, in pharmaceutical industry as pills, in soils as sand, and even in outer space as in Saturn rings. This matter could be dry or wetted and it should not come as a surprise that there is so much interest in understanding its dynamics [1–4]. The simplest example of this kind of matter is a line of spherical beads, which are modelled as a line of point masses connected by nonlinear springs. Nesterenko has shown that if these systems are struck at one end, highly nonlinear waves are generated [5]. The main interaction between non-conforming solids of elliptical shapes was derived by Hertz in 1882 [6], assuming a pure elastic contact. This force law is proportional to the compression of the surfaces in contact to the power 3/2. Under very restricted circumstances, this force law gives reasonable results when compared with experiments (see, *e.g.*, [7–11]). However, when the impact between grains is such that energy dissipating phenomena become relevant, Hertz theory fails. Hertz theory gives a clear functional form for the velocity of highly nonlinear solitary waves as a function of the compression force F_{m}; it is proportional to F_m to the power 1/6. Then any deviation of this force law could be established by measuring the propagation velocity of these waves.

In this work we present an optical method for measuring the propagation velocity of highly nonlinear solitary waves (sometimes called solitons) in a one-dimensional granular media, modelled by a chain of spheres. It is based on the use of two Michelson interferometers with tiny mirrors mounted on two beads. With this method we are able to measure the soliton velocity without modifying the balls as is usually done by inserting a piezoelectric sensor in some balls (see, *e.g.*, [5, 12]). We performed two experiments: in the first one, we used brass balls and Teflon balls in the second one.

2. Experiment results

The experimental setup is shown in Fig. 1. The incident ball on the left side collides with impact velocity v with the linear array of 13 balls on the right-hand side. The impact velocity was measured with a laser and a photodiode placed 2 cm in front of the first ball of the linear ball array. Two (4 × 4 × 2) mm first surface mirrors (M1 and M2) obtained from an old CD-player were glued with Loctite on the top of two balls (B1 and B2), separated by the ball center-to-center distance d. The mass of each mirror was 400 mg which is 5% of the mass of the brass ball-mirror set and 17% of the mass of the Teflon ball-mirror set. The ball B1 was the fourth ball in the array in all the experiments. The ball B2 was located in position 6 to 11 for different tests.

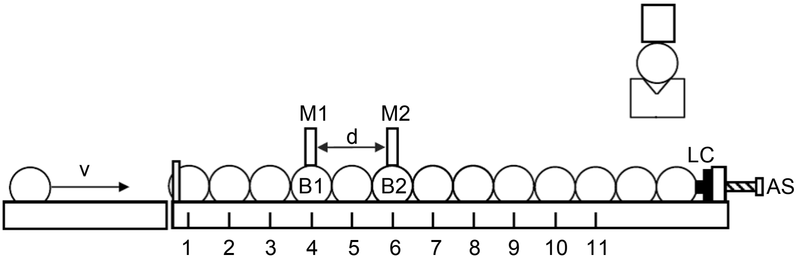

Fig. 1. Experimental setup for the array of beads, v – impact velocity, M1 and M2 – mirrors (not to scale), B1 and B2 – balls with mirrors, d – distance between centers of mirrors, AS – adjusting screw, and LC – load cell.

In order to avoid perturbations produced by the rolling of the striker, the striker system and the linear array were installed in two different but aligned Teflon rails, separated by a 2 mm gap, both mounted on different supports. In addition, the supports of the incident ball rail were placed into two boxes filled with raw rice to reduce vibrations. To avoid producing a gap between the beads after being struck, balls in the chain were compressed with a load cell (Omegadyne, Mod. LCMKD-50N). The charge cell was calibrated, fixed with an adjust screw, and used to measure the compression force.

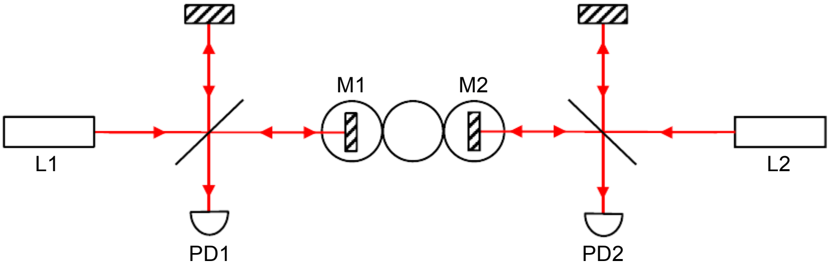

Fig. 2. Diagram of the two interferometers. M – mirrors, L – lasers, and PD – photodiodes.

The setup of the two independent Michelson interferometers is depicted in Fig. 2. The interferometers were precisely aligned to obtain high-contrast, independent interference patterns on two separated photodiodes (Thorlabs, Mod. DET10A). The photodiode voltages were recorded simultaneously in two channels of a digital storage oscilloscope (Tektronix, Mod. 5040). The alignment of the mirrors M1 and M2 glued on the top of the balls was done with a translation stage and optomechanical elements, not shown here, that were removed after alignment by moving the translation stage. After each shot a small correction of the ball-mirror alignment was necessary.

Figure 3 shows a typical oscilloscope trace of the photodiode voltage, starting a short time before the impact of the brass colliding ball. In this example, the number of balls between B1 and B2 was four. The ball diameter was 0.5 inches. In Fig. 3 the time of the setup of the perturbation on each ball is indicated by a vertical line. The phases are inverted as the voltage signal corresponding to one interferometer was at its maximum while the other was at its minimum. The time interval between the arrivals of the soliton at each ball-mirror set was 212 μs. As the distance between mirrors was 63.5 mm, then

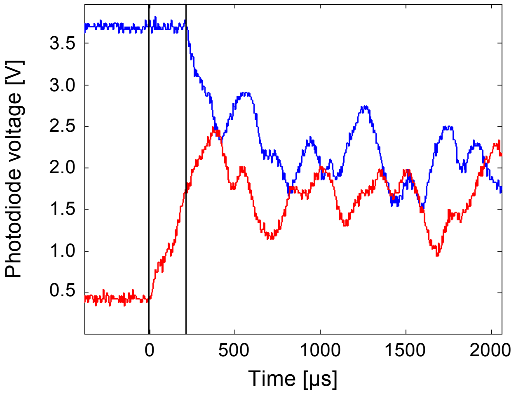

Fig. 3. Typical oscilloscope traces for brass balls. The red curve is the voltage on photodiode PD1 and the blue curve is the PD2 voltage. The vertical lines mark the time interval $\Delta t = 212$ μs between perturbations on the two mirrors.

the soliton velocity is approximately $u = 334$ m/s. The large voltage oscillations, of order 0.6 V, are due to the vibrations of the system after the collision. For brass balls, the time interval measurement was repeated 35 times for each position of the B2 ball, and averaged. The experimental uncertainty was taken as the standard deviation of the 35 measurements. In the case of Teflon balls only 6 and 12 measurements were necessary as the rail-balls system has lower vibration.

3. Results

Table 1 shows the average impact velocity $\overline{v}$ for each B2 mirror position, the distance between mirrors, and the time interval between the arrivals of the soliton on each ball with mirror.

Table 1. Average striker ball velocity $\overline{v}$ and time interval Δt between perturbations of brass balls separated by a distance d. The number N represents the number of balls between balls B1 and B2. The compression force was $F = (0.8 \pm 0.1)$ N.

N	$\overline{v}$ [cm/s]	d [mm]	Δt [μs]
1	6.0 ± 0.7	25.4 ± 0.1	67 ± 14
2	5.6 ± 0.8	38.1 ± 0.1	136 ± 14
3	5.5 ± 0.7	50.8 ± 0.1	155 ± 11
4	9.3 ± 1.0	63.5 ± 0.1	189 ± 10
5	11.9 ± 0.5	76.2 ± 0.1	200 ± 9
6	11.9 ± 0.7	88.9 ± 0.1	265 ± 8

We repeated the experiment with Teflon balls. Because Teflon has a lower friction coefficient than brass, it was necessary to apply a lower force; at larger forces one ball glided out of the array. Table 2 shows the distance between the mirrors and the time interval between the arrivals of the solution on each ball with the mirror. The average impact velocity of the Teflon ball in each case was $\overline{v} = (23 \pm 1)$ cm/s.

Figure 4 shows the distance-time plot for the solitons in brass and Teflon arrays of balls. Blue and red dots represent data from the experiments, for brass and Teflon balls respectively.

Table 2. Time interval Δt between perturbations of Teflon balls separated by a distance d. The number N represents the number of balls between balls B1 and B2. The compression force was $F = (0.4 \pm 0.1)$ N. The average impact velocity of the Teflon ball in each case was $\overline{v} = (23 \pm 1)$ cm/s.

N	d [mm]	Δt [μs]
1	25.4 ± 0.1	277 ± 16
2	38.1 ± 0.1	383 ± 10
3	50.8 ± 0.1	536 ± 4
4	63.5 ± 0.1	633 ± 7
5	76.2 ± 0.1	787 ± 23
6	88.9 ± 0.1	884 ± 23

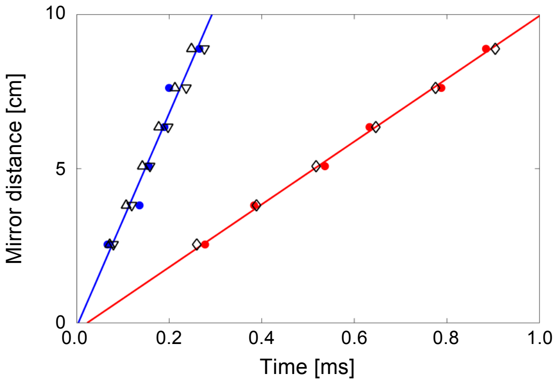

Fig. 4. Soliton experimental *x*-*t* plot for linear array of brass and Teflon balls (blue and red dots, respectively). Blue and red solid lines are obtained from least squares fit for brass and Teflon experimental data, respectively. Theoretical data for brass for impact velocities $v = 11.9$ cm/s (upward pointing triangles), $v = 5.5$ cm/s (downward pointing triangles), and Teflon $v = 23$ cm/s (diamonds). The soliton velocity obtained from least squares fit for brass was $u = (346 \pm 40)$ m/s and for Teflon $u = (102 \pm 3)$ m/s.

The overall mean standard deviation of the time measurement with Teflon balls was 14.0 μs. For short distances ($N = 1, 2, 3, 4$) we repeated each time measurement 6 times. The mean standard deviation of these time measurements was 9.7 μs. For longer distances ($N = 5, 6$) we repeated the time measurement 12 times and the typical standard deviation was 22.7 μs.

In our experiments with brass and Teflon balls, the mirrors attached to the balls were misaligned after each shot, due to rotation of the balls. After each shot, we realigned carefully the mirrors. The dispersion in the travel times can be attributed to a small change of the distance between them after each re-alignment. This produced a slight change in the fringe position on each photodiode. Another cause of the travel time dispersion was the fact that the exact position and compression of the beads slightly differs from one experiment to the other, which increased the variability of the travel times.

To appreciate the value of our experimental findings we could have an estimate of the soliton velocity using a model proposed in [12]. The balls (provided by Hoover Precision, Inc.) have Young's modulus $Y = 110$ GPa, density $\rho = 8484.6$ kg/m^3 and Poisson ratio $\nu_P = 0.31$ for brass (Alloy 260), and $Y = 685$ MPa, $\rho = 2172.4$ kg/m^3 and Poisson ratio $\nu_P = 0.46$ for Teflon. With $R = 6.35 \times 10^{-3}$ m and compression forces $F = 0.8$ N on brass and $F = 0.4$ N on Teflon, one gets a soliton velocity of $u = (355 \pm 3)$ m/s and $u = (100 \pm 1)$ m/s for brass and Teflon, respectively. Theoretical errors (not shown in the figure) originate by adjustment of phenomenological couplings in the model. We observe that the experimental and theoretical results agree well within the experimental uncertainties.

These results show that our novel optical method is appropriate for measuring the velocity of highly nonlinear solitary waves in one dimensional granular media. Being an optical method, it is not necessary to insert a piezoelectric transducer in two beads in the array as done elsewhere. The method can be improved by measuring with more precision the impact velocity. If the linear array is put into a tube, with apertures for positioning the mirrors, one could also use different compression forces.

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