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One shot profilometry using iterative two-step temporal phase-unwrapping

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This paper reviews two techniques that have been recently published for three-dimensional profilometry and proposes one shot profilometry using iterative two-step temporal phase-unwrapping by combining the composite fringe projection and the iterative two-step temporal phase unwrapping algorithm. In temporal phase unwrapping, many images with different frequency fringe pattern are needed to project, which would take much time. In order to solve this problem, Ochoa proposed a phase unwrapping algorithm based on phase partitions using a composite fringe. However, we found that the fringe order determined through the construction of phase partitions tended to be imprecise. Recently, we proposed an iterative two-step temporal phase unwrapping algorithm, which can achieve high sensitivity and high precision shape measurement. But it needs multiple frames of fringe images which would take much time. In order to take into account both the speed and accuracy of three-dimensional shape measurement, we get a new, and more accurate unwrapping method based on a composite fringe pattern by combining these two techniques. This method not only retains the speed advantage of Ochoa's algorithm, but also greatly improves its measurement accuracy. Finally, the experimental evaluation is conducted to prove the validity of the proposed method.

Keywords: phase unwrapping, composite fringe pattern, Fourier transform, two-step temporal phase-unwrapping.

1. Introduction

Fringe-projection profilometry is a well-developed technique and powerful tool in current three-dimensional (3D) shape measurement for its non-contact nature, full-field measurement capability, high profile sampling density and low environment vulnerability, which has been an extensively studied research area due to the diversity of potential applications [1–4]. The basic operation of the process is shown below, first of all, sinusoidal fringes are projected onto an object surface, images of the fringe patterns deformed by the object surface are captured by a camera, then a phase map is calculated from the pixel in the images. Because the retrieved phase distribution corresponding to the object height is wrapped in the range, phase unwrapping has become one of the key steps in grating projection 3D shape measurement [5–7]. In recent decades, a variety of phase unwrapping algorithms have been proposed, they can be divided into two categories: spatial phase unwrapping method and temporal phase unwrapping method. Spatial phase unwrapping is a process of integral accumulation; once the error appears, it will spread to the point around, so noise, shadow and discontinuous points in the actual measurement will affect the phase unwrapping quality [8, 9]. A temporal phase unwrapping method can solve this problem. The temporal phase unwrapping method is proposed by SALDNER and HUNTLEY [10], which is realized by projecting a series of different frequency gratings to the surface of the measured object, capturing the fringe sequence modulated by the object surface, then unwrapping the phase along the time series for each pixel independently, so it can avoid the phase unwrapping error propagation. However, it is unable to meet the requirements of real-time dynamic 3D shape measurement. How to realize the phase unwrapping algorithm in time domain with fewer projection fringes is now a challenge to everyone. KAI LIU *et al.* [11] proposed the dual-frequency method, which combines a unit-frequency fringe pattern with a high -frequency. SHOUQI LIU *et al.* [12] also proposed the tri-frequency heterodyne method. SHAOYAN GAI and FEIPENG DA proposed the amplitude modulation method, where the unit frequency is obtained from amplitude modulation of the fringes [13]. SERVIN *et al.* proposed a two-step temporal phase unwrapping algorithm [14], which only needs the two extreme phase-maps to achieve exactly the same results as the standard temporal unwrapping method. Recently, we proposed an iterative two-step temporal phase unwrapping algorithm [15] to achieve high sensitivity and high precision shape measurement. In order to further improve the measuring speed, WEI-HUNG SU and HONGYU LIU [16] proposed a single-shot measurement method, which requires an unwrapping process for both the high and low frequency phases. GARCÍA-ISÁIS and OCHOA [17] proposed a phase unwrapping algorithm based on phase partitions using a composite fringe, which only needs the projection of one composite fringe pattern with four kinds of frequency information to complete the process of unwrapping. However, according to our own experiments with Ochoa's method, we found that the fringe order determined through the construction of phase partitions tended to be imprecise. In order to take into account both the speed and accuracy of 3D shape measurement, we get a new, and more accurate unwrapping method based on a composite fringe pattern by combining the composite fringe projection in [17] and the iterative two-step temporal phase unwrapping algorithm [15]. This method not only retains the speed advantage of Ochoa's algorithm, but also greatly improves its measurement accuracy. The capability of the presented method is demonstrated by both theoretical analysis and experiments.

The paper is organized as follows. Section 2 reviews these two methods and combines these two techniques into a new and more accurate 3D profilometry. Section 3 presents the experimental results. Section 4 summarizes this paper.

2. Theory

2.1. Phase partitions unwrapping algorithm using a composite fringe

Ochoa *et al.* [17] proposed a phase partitions unwrapping algorithm using a composite fringe pattern, the composite pattern to be projected is described by the following equation:

$$I(x, y) = \frac{G}{8}\Big\{4 + \cos(2\pi f_1 x) + \cos(2\pi f_2 x) + \cos(2\pi f_2 y) + \cos\big[2\pi(f_2 + 1)x + 2\pi f_2 y\big]\Big\} \tag{1}$$

where f_1 and f_2 are medium and high carrier frequencies ($2f_1 < f_2$), G is a constant that represents the maximum gray level range, (x, y) are the normalized pixel coordinates, and $I(x, y)$ is the image with its gray levels in the range $[0, G]$. The four carrier terms are given by:

$$C_{x_1}(x, y) = 2\pi f_1 x \tag{2a}$$

$$C_x(x, y) = 2\pi f_2 x \tag{2b}$$

$$C_y(x, y) = 2\pi f_2 y \tag{2c}$$

$$C_{xy}(x, y) = 2\pi(f_2 + 1)x + 2\pi f_2 y \tag{2d}$$

Thus the following relation should hold [18],

$$C_{xy}(x, y) - C_x(x, y) - C_y(x, y) = 2\pi x \tag{3}$$

which is a unit frequency vertical fringe.

The intensity profile that we will obtain after projecting Eq. (1) onto the object's surface will be given by

$$I = a + b\Big[\cos(C_{x_1} + \varphi^{x_1}) + \cos(C_x + \varphi^{x}) + \cos(C_y + \varphi^{y}) + \cos(C_{xy} + \varphi^{xy})\Big] \tag{4}$$

where a and b are background and amplitude terms that depend on the object's reflectivity, respectively, and φ^{x_1}, φ^{x}, φ^{y} and φ^{xy} are the phase functions related to the surface height. Ochoa used Fourier analysis to demodulate the composite fringe patterns and obtain the wrapped phase $W[C_{x_1} + \varphi^{x_1}]$, $W[C_x + \varphi^{x}]$, $W[C_y + \varphi^{y}]$ and $W[C_{xy} + \varphi^{xy}]$, where $W[\cdot]$ is the wrapping phase operator. According to Eqs. (8)

and (9) in [17], we can obtain the vertical phase $2\pi x + \varphi^{\mathrm{eq}}(x, y)$ only with one period, where $\varphi^{\mathrm{eq}} = \varphi^{xy} - \varphi^{x} - \varphi^{y}$ represents the equivalent phase (unwrapped) of the phase differences. In order to unwrap the high-frequency phase $W[C_x + \varphi^x]$, Ochoa proposed a phase partitions unwrapping algorithm. First, determine n and m by the following relationship:

$$\frac{n(x, y) - 1}{f_1} 2\pi \leq 2\pi x + \varphi^{\mathrm{eq}}(x, y) < \frac{n(x, y)}{f_1} 2\pi \tag{5a}$$

$$\frac{m(x, y) - 1}{f_2} f_1 2\pi \leq W[C_{x_1} + \varphi^{x_1}(x, y)] < \frac{m(x, y)}{f_2} f_1 2\pi \tag{5b}$$

where $n(x, y)$, $m(x, y)$ are integer values and $n \in [0, f_1 - 1]$, $m \in [0, (f_2/f_1) - 1]$, then the fringe order of $W[C_x + \varphi^x]$ can be determined as

$$N(x, y) = n(x, y) f_1 + m(x, y) \tag{6}$$

which lets us calculate the absolute unwrapped high-frequency phase through the following equation:

$$\varphi^x(x, y) = W[C_x + \varphi^x(x, y)] + 2\pi N(x, y) - 2\pi f x \tag{7}$$

As noted in [17], some erroneous fringe order can be generated because of the Fourier transform and filtering operation. So the unwrapped phase should be corrected by the algorithm in [19]. Ochoa's method can calculate the unwrapped phase and the profile information of discontinuous, isolated and complex objects, when only one shot is used through Fourier techniques, which has the advantage of speed and suits for fast 3D surface measurement. However, according to our own experiments with Ochoa's method, we found that the fringe order determined through the construction of phase partitions tended to be imprecise. Even with the correction method in [19, 20], the imprecise fringe order cannot be completely eliminated, leading to many errors in the final unwrapped phase.

2.2. Iterative two-step temporal phase unwrapping algorithm

Recently, we proposed an iterative two-step temporal phase unwrapping algorithm [15], to achieve high sensitivity and high precision shape measurement. Assuming that the intensity mathematical formula for three fringe patterns with different phase modulation sensitivities is as follows:

$$I_1(x, y) = a(x, y) + b(x, y)\cos[\varphi(x, y)], \qquad \varphi(x, y) \in (-\pi, \pi) \tag{8a}$$

$$I_2(x, y) = a(x, y) + b(x, y)\cos[g_1 \varphi(x, y)], \qquad g_1 > 1, \qquad g_1 \in \mathrm{R} \tag{8b}$$

$$I_3(x, y) = a(x, y) + b(x, y)\cos[g_1 g_2 \varphi(x, y)], \quad g_2 > 1, \quad g_2 \in \mathrm{R} \tag{8c}$$

where $\varphi(x, y)$ is a 1λ sensitive phase (λ is wavelength) and $G\varphi(x, y)$ is G-times more sensitive, $G = g_1 g_2$. We can use the phase demodulation algorithm to obtain the three demodulated wrapped phase-maps as,

$$\varphi_1(x, y) = W[\varphi(x, y)], \quad \varphi(x, y) \in (-\pi, \pi) \tag{9a}$$

$$\varphi_{2\mathrm{w}}(x, y) = W[g_1 \varphi(x, y)], \quad g_1 > 1, \quad g_1 \in \mathrm{R} \tag{9b}$$

$$\varphi_{3\mathrm{w}}(x, y) = W[g_1 g_2 \varphi(x, y)], \quad g_2 > 1, \quad g_2 \in \mathrm{R} \tag{9c}$$

The first demodulation $\varphi_1(x, y)$ is not wrapped because it is less than 1λ. So, we have $\varphi_1(x, y) = \varphi(x, y)$. The $\varphi_{2\mathrm{w}}(x, y)$ and $\varphi_{3\mathrm{w}}(x, y)$ are the wrapped phase, and we can unwrap them by,

$$\varphi_2(x, y) = g_1 \varphi_1(x, y) + W[\varphi_{2\mathrm{w}}(x, y) - g_1 \varphi_1(x, y)] \tag{10a}$$

$$\varphi_3(x, y) = g_2 \varphi_2(x, y) + W[\varphi_{3\mathrm{w}}(x, y) - g_2 \varphi_2(x, y)] \tag{10b}$$

As the Eqs. (3) and (4) given in [14], due to the use of high sensitivity fringe, the iterative two-step temporal phase unwrapping algorithm can increase the signal-to-noise power-ratio, achieve higher sensitivity and more accurate measurement, correct the erroneous fringe orders of Ochoa's method, and suit for isolated objects. But it needs multiple frames of fringe images which would take much time.

2.3. One shot profilometry using iterative two-step temporal phase-unwrapping

With the widely used of 3D surface measurement, the speed and accuracy of the measurement should be higher and higher. The phase partitions unwrapping algorithm proposed by Ochoa, can improve the measurement speed, but the results are imprecise; and the iterative two-step temporal phase unwrapping algorithm can achieve higher sensitivity and more accurate measurement, but it needs multiple frames of fringe images. We combine these two algorithms and propose a more effective 3D surface measurement method. This method not only retains the speed advantage of Ochoa's algorithm, but also has the accuracy advantage of the iterative two-step temporal phase unwrapping algorithm. The main stages of our algorithm are summarized as follows:

1. Project Eq. (1) onto the object's surface and capture the image by a CCD camera, as shown in Fig. 1**a**;
2. Obtain the wrapped phase $W[C_{x_1} + \varphi^{x_1}]$, $W[C_x + \varphi^x]$, $W[C_y + \varphi^y]$ and $W[C_{xy} + \varphi^{xy}]$, by Fourier transform, filtering operation, and inverse Fourier transform, as shown in Figs. 1**b** and 1**c**;

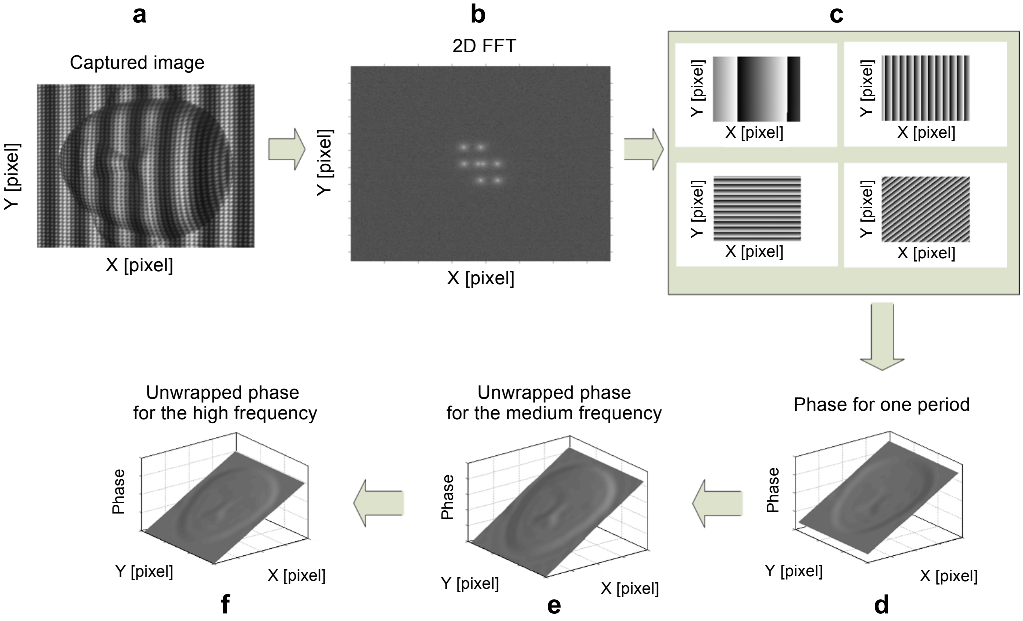

Fig. 1. The specific procedure (see text for explanation).

3. Obtain the unit-frequency phase $2\pi x + \varphi^{\mathrm{eq}}(x, y)$ according to Eqs. (8) and (9) in [17], as shown in Fig. 1**d**;

4. Unwrap the high-frequency phase $W[C_x + \varphi^x]$ with the iterative two-step temporal phase unwrapping algorithm (10), obtain the continuous phase $C_x + \varphi^x$, as shown in Figs. 1**e** and 1**f**;

5. Transform the absolute phase to height information after the system is calibrated.

The specific procedure is shown in Fig. 1. As you might see, the main stages of proposed method are same as that of Ochoa's algorithm except that the partitions unwrapping algorithm is replaced with the iterative two-step temporal phase unwrapping algorithm.

The following experiment is used to verify the proposed algorithm.

3. Experiments

In this section, for evaluating the real performance of our method, we test our method on a series of experiments. Below, we will describe these experiments and practical suggestions for the above procedure.

We develop a 3D shape measurement system, which consists of a DLP projector (Optoma EX762) driven by a computer and a CCD camera (DH-SV401FM). The camera is attached with a 25 mm focal length lens (model ComputarFA M2514-MP2). Figure 2 shows the optical path of phase measuring profilometry, where *P* is the projection center of the projector, *C* is the camera imaging center, and *D* is an arbitrary

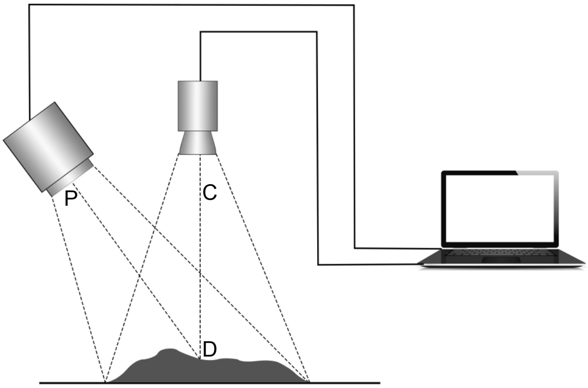

Fig. 2. The optical path of phase measuring profilometry (see text for explanation).

point on the tested object. In our experiments, the distance between the camera and the projector is about 30 cm and the tested object is placed in front of the projector at about 70 cm. The surface measurement software is programmed by Matlab with i5-4570 CPU @ 3.20 GHz.

Firstly, an experiment is provided to demonstrate the feasibility of the proposed algorithm. The tested object is a face model and an isolated cup. As given in [17], there

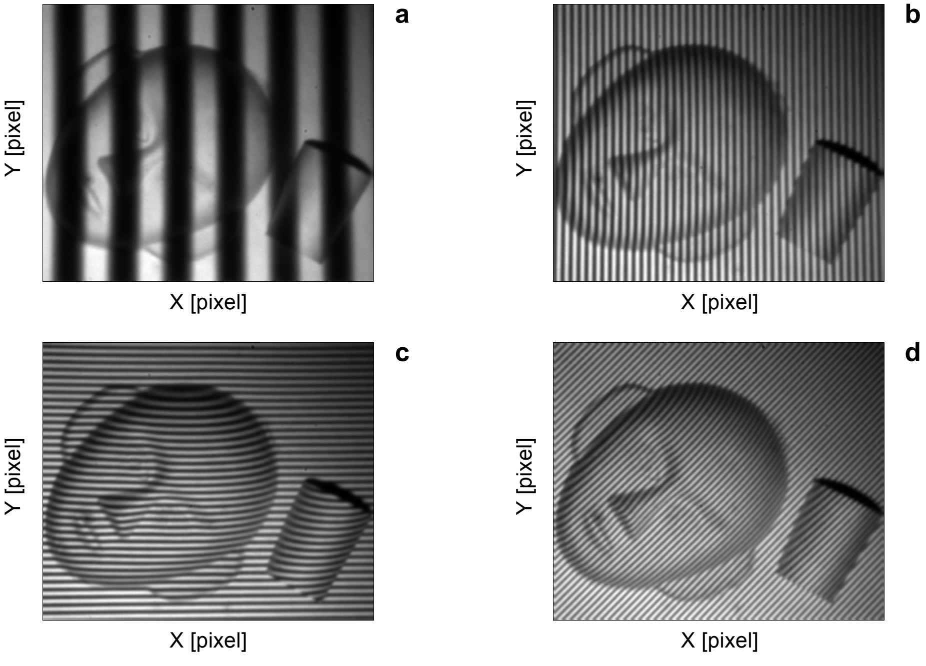

Fig. 3. The captured images. The vertical fringe pattern with 7 periods (**a**). The vertical fringe pattern with 49 periods (**b**). The horizontal fringe pattern with 49 periods (**c**). The tilted fringe pattern whose vertical and horizontal periods are 49 and 50, respectively (**d**).

will be loss of information due to the Fourier process, so we start with phase shift method to prove our approach. In this paper, we use the four-step phase shift method. The four-step phase shift fringes are projected onto the object surface, the captured images are shown in Fig. 3, and the image is 592 pixels wide by 496 pixels high. Figure 3**a** is the vertical fringe pattern with 7 periods, Figs. 3**b** and 3**c** are the vertical and horizontal fringe pattern with 49 periods, respectively, Fig. 3**d** is tilted fringe pattern whose vertical and horizontal periods are 49 and 50, respectively.

Figure 4 is the wrapped phase obtained by the four-step phase shift method. Figures 4**a**–4**d** are the wrapped phases of Figs. 3**a**–3**d**, respectively.

The vertical phase with one period $2\pi x + \varphi^{eq}(x, y)$ is shown in Fig. 5.

First, dealing with the captured images in Fig. 3 by the Ochoa's method using Eqs. (5)–(7), the result is shown in Fig. 6.

Then, we correct the fringe order using the method discussed in [19, 20]. The results are shown in Fig. 7. Figures 7**a** and 7**b** are the corresponding results after two correction methods were used to modify the mistaken fringe order. As one might see, there are still obvious errors in the retrieved phase map.

In contrast, dealing with the captured images in Fig. 3 by the proposed method, we get the result shown in Fig. 8. It is obvious that the proposed method can obtain the precise unwrapped phase while Ochoa's algorithm can introduce large error.

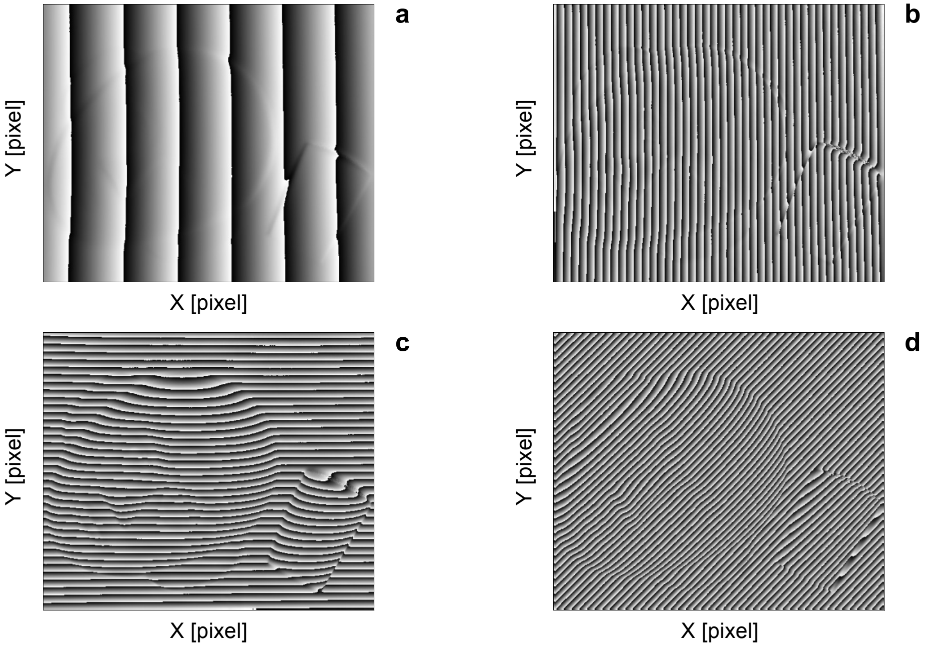

Fig. 4. The wrapped phase obtained by a four-step phase shift method. The wrapped phases of Fig. 3**a** (**a**), Fig. 3**b** (**b**), Fig. 3**c** (**c**), and Fig. 3**d** (**d**).

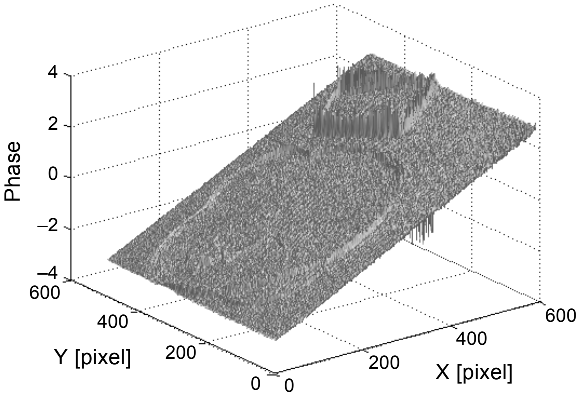

Fig. 5. The vertical phase with one period.

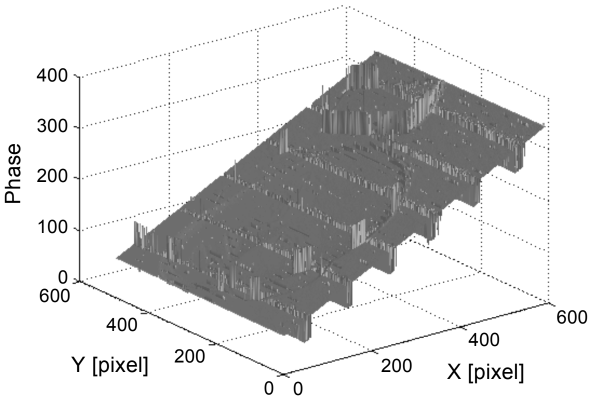

Fig. 6. The unwrapped phase by Ochoa's algorithm.

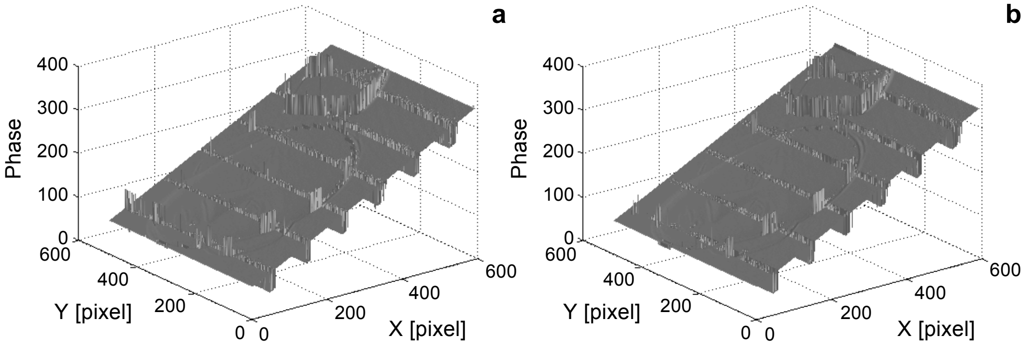

Fig. 7. The unwrapped phase after correction. The unwrapped phase after correction using the method in [19] (**a**), and in [20] (**b**).

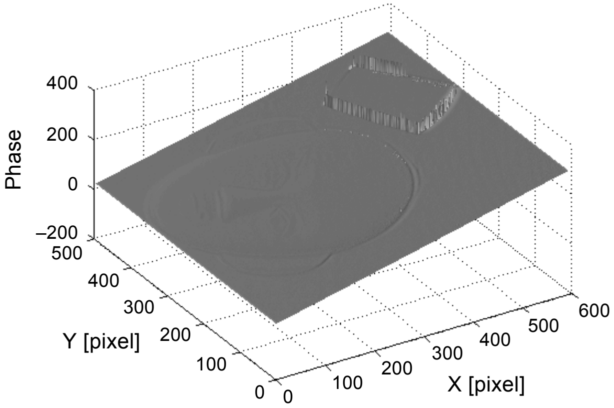

Fig. 8. The unwrapped phase obtain by the proposed method.

For a more complete verification to the proposed method, we do another experiment on a plastic board with a big hole. To validate that the method is applied to objects with hole, we use a black background in the hole area, as shown in Fig. 9. The experiment procedure is similar. But we use Fourier method and composite fringe projection.

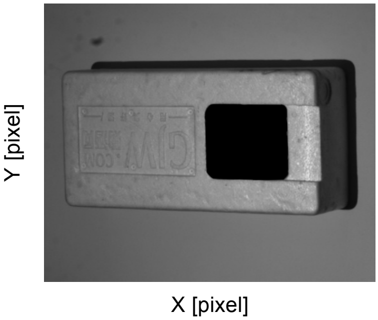

Fig. 9. The tested object.

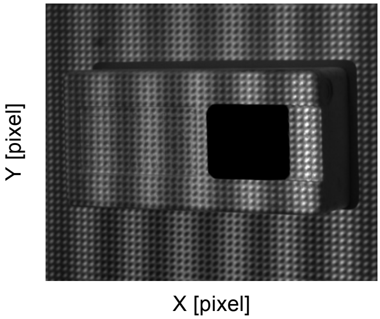

Fig. 10. The captured image.

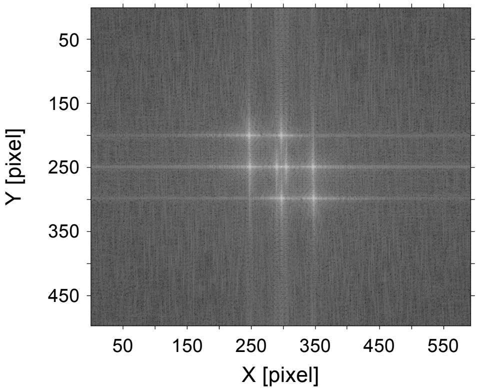

Fig. 11. The spectrum of the captured image.

Project the composite fringe produced according to Eq. (1) onto the object's surface, the deformed fringe pattern is captured by a CCD camera ($f_1 = 7, f_2 = 49$), as shown in Fig. 10. The captured image is 592 pixels wide by 496 pixels high.

Then the image can be analyzed to get the 3D surface shape, as described in Sections 2.2 and 2.3. The spectrum of the captured image by 2D Fourier transform is shown in Fig. 11.

Figure 12 is the wrapped phase obtained by filtering operation. Figure 12**a** is the vertical wrapped phase with 7 periods, Figs. 12**b** and 12**c** are the vertical and horizontal wrapped phase with 49 periods, respectively, Fig. 12**d** is tilted wrapped phase whose vertical and horizontal periods are 49 and 50, respectively.

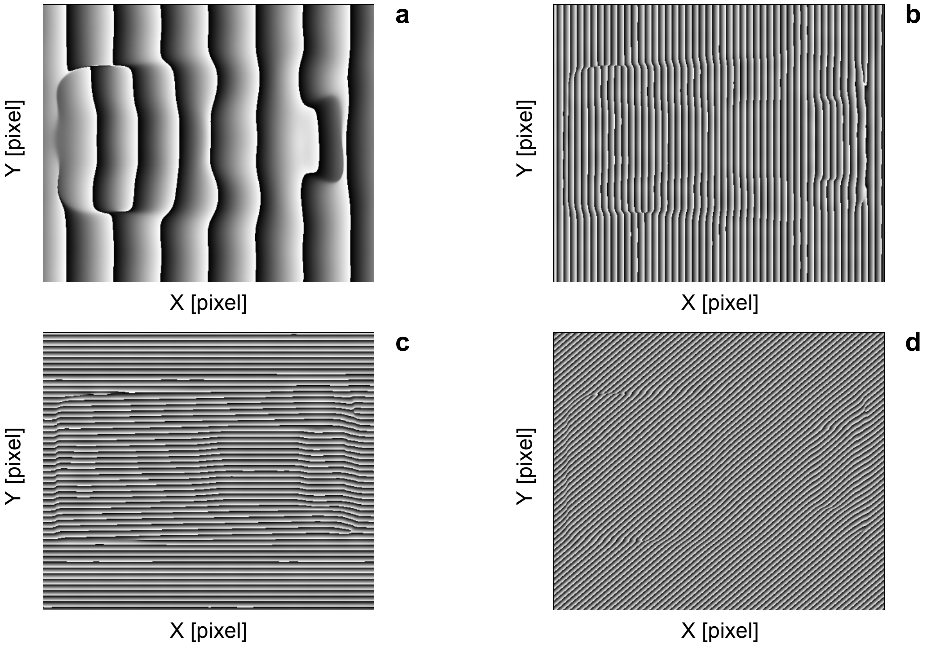

Fig. 12. The wrapped phase obtained by filtering operation. The vertical wrapped phase with 7 periods (**a**). The vertical wrapped phase with 49 periods (**b**). The horizontal wrapped phase with 49 periods (**c**). The tilted wrapped phase whose vertical and horizontal periods are 49 and 50, respectively (**d**).

First, we reconstruct the 3D shape from the wrapped phase shown in Fig. 12 using a phase partitions method. The result is shown in Fig. 13.

Then, the high frequency phase is independently unwrapped from the wrapped phase shown in Fig. 12 by the proposed algorithm. The result is illustrated in Fig. 14. As shown in Figs. 13 and 14, the proposed method can restore 3D surface shape well, while Ochoa's method leads to many errors in the final result.

To have a better comparison, the results of the 200-th row from Figs. 13 and 14 are provided in Fig. 15. Figure 15**a** is the result processed by Ochoa's method. Figure 15**b**

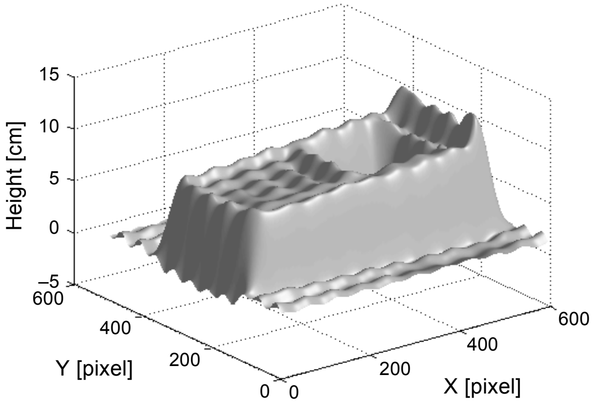

Fig. 13. The 3D reconstructions using Ochoa's algorithm.

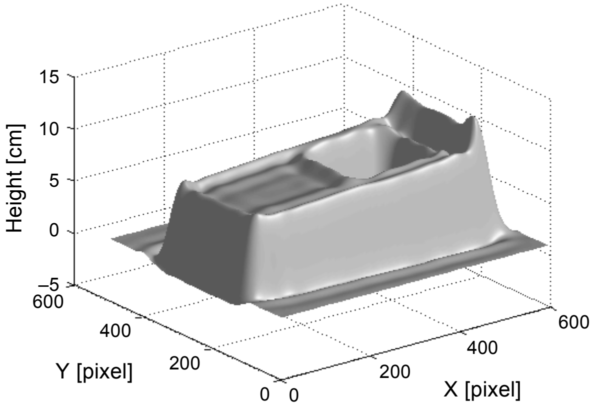

Fig. 14. The 3D reconstructions using the proposed algorithm.

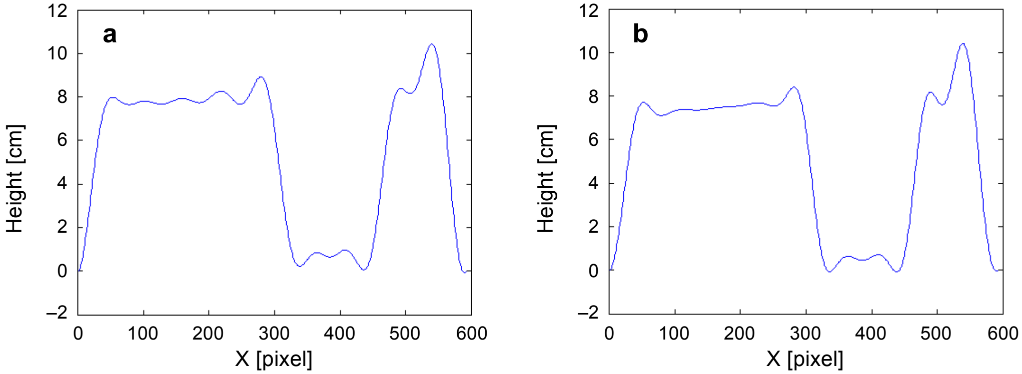

Fig. 15. The results of the 200-th rows of the two methods: from Fig. 13 (**a**), and from Fig. 14 (**b**).

is the result processed by the proposed method. It can be seen from Fig. 15 that the proposed method provides the higher success rate as expected.

It is well-known that some information will be lost due to the spectral leakage, the selection of spectral filtering window and other reasons in the process of Fourier transform. And some erroneous values will be generated especially at the borders. The phase shift method can solve this problem by increasing the number of projected images.

4. Conclusion

In this paper, we propose a one shot profilometry using iterative two-step temporal phase-unwrapping by combining the composite fringe projection and the iterative two-step temporal phase unwrapping algorithm. The proposed method can calculate the unwrapped phase and restore 3D surface shape of isolated and complex objects, meanwhile it needs only one shot. And it not only retains the speed advantage of Ochoa's algorithm, but also greatly improves its measurement accuracy.

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