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Double tunneling induced transparency in the asymmetry quantum dot molecules

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Using the density matrix theory, we have studied the double tunneling induced transparency slow light in the double asymmetry quantum dot molecules. With applied electric field, double tunneling induced transparency occur in the same time. Four absorption peaks are found near the resonance energy level in the absorption spectrum and the absorption peak can be tuned by the applied electric field. The velocity and bandwidth of the multiple-windows slow light can also be controlled by the applied electric field. In our model, with $T_e = 0.1$ meV, we can get about $0.001c$ and 20 GHz bandwidth in each transparency window. Such a property may be applied in all optical buffers, optical switching and filter.

Keywords: quantum dot molecule, slow light, tunneling induced transparency.

1. Introduction

Recently, the slow light in the quantum dots (QDs) systems has been studied widely. Most early work is based on electromagnetically induced transparency (EIT) [1–9]. EIT has also been demonstrated in the atomic system in [10, 11] and references therein, and it has many notable applications in quantum and nonlinear optics such as achieving large nonlinearities, multiwave mixing, optical bistability, optical solitons, and so on. The QDs homogeneous line width [1, 2], inhomogeneous broadening [1, 5, 6, 8], many-body interactions [3], size and geometry [4] dependence on the slow light properties of QDs are studied. The QDs EIT absorption dip is also observed in [7]. The delay -bandwidth product of EIT in gases as well as QDs is studied in [8]. However, the slow factor is greatly reduced by the temperature and inhomogeneous broadening. Simultaneously, slow light based on coherent population oscillations in QDs at room temperature [12–16] is studied due to their three-dimensional confinement potential. However, the delay-bandwidth product is not satisfied in the practical optical communication.

And also, slow light in QD photonic crystal waveguides is examined [17]. The slow and fast light in semiconductor waveguides including QDs is reviewed in [18, 19].

Additionally, slow light based on tunneling induced transparency (TIT) is reported in parallel and vertically coupled QDs [20–23]. Compared with EIT by employing a control laser beam, a laser beam can transmit through a dense medium. In TIT, it is by an external electric field that the inter-dot quantum coupling strength is tuned, and slow light also is found in QDs. However, the QDs usually are not the same and as far as we know, it is less reported that the TIT in the QD molecule.

In this work, we study the double TIT slow light in the asymmetry QD molecules. It shows multiple-windows, wide-bandwidth, and is compatible for the optical communication. It will be applied in the delay line, filter, optical switching, and so on.

2. Model and theory

Figure 1**a** shows a schematic drawing of the QD molecules with a V scheme formed by $|0\rangle$, $|1\rangle$ and $|3\rangle$ energy levels in the QD molecule [24, 25]. It consists of two vertically stacked QDs in the *x* direction, separated by a spacer layer and there are several layers of QDs. Figure 1**b** shows the scheme of the energy levels in our system. $|0\rangle$ is the system in the absence of excitation. $|1\rangle$ ($|3\rangle$) is a pair of electron and hole state bound in the left (right) dot by applying an electromagnetic field which is a direct exciton [24]. Applying a DC voltage in the *y* direction, the electron in the QD can be excited from the valence to the conduction band in the lower QD, which can in turn tunnel to the upper QD in the same layer in the figure. So, the indirect exciton state $|2\rangle$ ($|4\rangle$) of the left (right) QD is formed by one hole in the left (right) QD and an electron in the dot below it in the *y* direction [20].

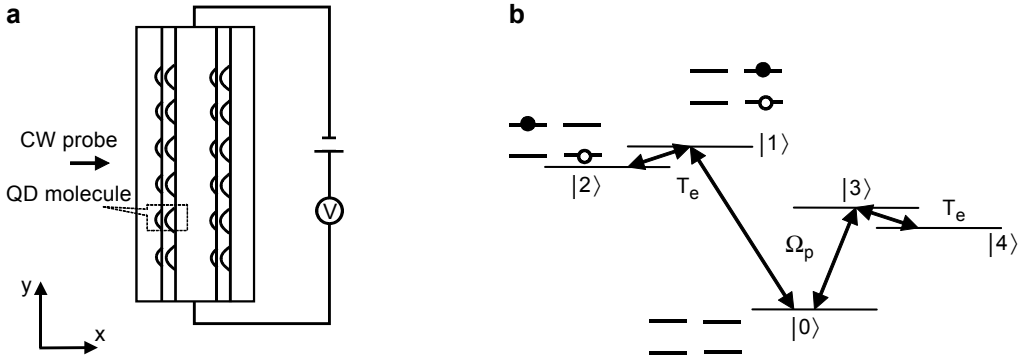

Fig. 1. Schematic drawing of the QD molecules which form a V scheme with $|0\rangle$, $|1\rangle$ and $|3\rangle$ – see part **b** (**a**). Schematic picture of the energy level of the asymmetric double QD molecules with applying an electromagnetic field (**b**). $|0\rangle$ is the system without excitation, $|1\rangle$ ($|3\rangle$) is the direct exciton bound in the left (right) QD, $|2\rangle$ ($|4\rangle$) is the indirect exciton after applying an electromagnetic field. The hole and electron location for different states in the *y* direction for state $|0\rangle$, $|1\rangle$ and $|3\rangle$ is inserted in **b**.

Firstly, the two QDs in the QD molecules are at the equal status. We can investigate one QD at first. For the left QD in the QD molecules, we can write the Hamiltonian as follows:

$$H = E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1| + E_2|2\rangle\langle 2| + T_e|1\rangle\langle 2| + \hbar\Omega_p \exp(i\omega_p t)|0\rangle\langle 1| + \text{h.c.} \tag{1}$$

where $E_i = \hbar\omega_i$ ($i = 0, 1, 2$) is the energy of state $|i\rangle$, T_e is the tunneling coupling, ω_p is the probe laser frequency, and $\Omega_p = \mu E/2\hbar$ is the optical coupling, where μ is the dipole momentum matrix element and E is the electric field amplitude.

The system is described by the Liouville–von Neumman–Lindblad equation,

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + L(\rho) \tag{2}$$

where ρ is the density matrix operator, H is the three-level system Hamiltonian (1), and $L(\rho)$ represents the Liouville operator describing the decoherence process

$$L(\rho) = \frac{1}{2}\sum_i \left\{ \Gamma_i^j \left[2|i\rangle\langle j|\rho|j\rangle\langle i| - \rho|j\rangle\langle j| - |j\rangle\langle j|\rho \right] + \right.$$
$$\left. + \gamma_i \left[2|i\rangle\langle i|\rho|i\rangle\langle i| - \rho|i\rangle\langle i| - |i\rangle\langle i|\rho \right] \right\} \tag{3}$$

where Γ_i^j is the decaying rate from the state $|i\rangle$ to the state $\langle j|$, and γ_i is the pure dephasing rate. With Equations (1), (2) and (3), we can get the complete set of coupled differential equations for the density matrix ρ_{ij} elements as follows:

$$\dot{\rho}_{01} = -\frac{i}{\hbar}\left[(E_0 - E_1)\rho_{01} - T_e\rho_{02} + \hbar\Omega_p \exp(i\omega_p t)(\rho_{11} - \rho_{00}) \right] - \gamma_{01}\rho_{01} \tag{4a}$$

$$\dot{\rho}_{02} = -\frac{i}{\hbar}\left[(E_0 - E_2)\rho_{02} - T_e\rho_{01} + \hbar\Omega_p \exp(i\omega_p t)\rho_{12} \right] - \gamma_{02}\rho_{02} \tag{4b}$$

$$\dot{\rho}_{12} = -\frac{i}{\hbar}\left[(E_1 - E_2)\rho_{12} - (\rho_{22} - \rho_{11})T_e + \hbar\Omega_p \exp(-i\omega_p t)\rho_{02} \right] - \gamma_{12}\rho_{12} \tag{4c}$$

where γ_{ij} is the total off-diagonal decay rates for ρ_{01}, ρ_{02} and ρ_{12} including Γ_i^j and γ_i. In our system, we can adopt: $\tilde{\rho}_{01} = \rho_{01}\exp(-i\omega_p t)$ and $\tilde{\rho}_{02} = \rho_{02}\exp(-i\omega_p t)$. From Equations (4), we can get

$$\dot{\tilde{\rho}}_{01} = i(\omega_1 - \omega_0 - \omega_p)\tilde{\rho}_{01} + i\frac{T_e}{\hbar}\tilde{\rho}_{02} - \Omega_p(\rho_{11} - \rho_{00}) - \gamma_{01}\tilde{\rho}_{01} \tag{5a}$$

$$\dot{\tilde{\rho}}_{02} = i(\omega_2 - \omega_0 - \omega_p)\tilde{\rho}_{02} + i\frac{T_e}{\hbar}\tilde{\rho}_{01} - i\Omega_p\rho_{12} - \gamma_{02}\tilde{\rho}_{02} \tag{5b}$$

$$\dot{\rho}_{12} = -\frac{i}{\hbar}\left[(E_1 - E_2)\rho_{12} + (\rho_{22} - \rho_{11})T_e + \hbar\Omega_p\tilde{\rho}_{02}\right] \tag{5c}$$

As the double QD system is initially in the ground state $|0\rangle$, $\rho_{00} = 1, \rho_{11} = \rho_{22} = 0$. We assume the pump field is much stronger than the signal field. At the steady state, from Eqs. (5), we can get [26, 27]

$$\tilde{\rho}_{01} = -\Omega_p\left[\omega_{10} - \omega_p - \left(\frac{T_e}{\hbar}\right)^2\left(\omega_{20} - \omega_p - \frac{\Omega_p^2}{\omega_{21}} + i\gamma_{02}\right)^{-1} + i\gamma_{01}\right]^{-1} \tag{6}$$

where $\omega_{ij} = \omega_i - \omega_j$. In the weak or strong tunneling regime [25], we adopt $T_e/\hbar \gg \Omega_p$. Equation (6) can be written as

$$\tilde{\rho}_{01} = \Omega_p\left[\omega_p - \omega_{10} + \left(\frac{T_e}{\hbar}\right)^2(\omega_{20} - \omega_p + i\gamma_{02})^{-1} - i\gamma_{01}\right]^{-1} \tag{7}$$

We can get the complex susceptibility of the left QD from $\tilde{\rho}_{01}$. It is as follows [2]:

$$\chi = \frac{\Gamma}{V}\frac{|\mu|^2}{\varepsilon_0\hbar\Omega_p}\tilde{\rho}_{01} \tag{8}$$

where Γ is the optical confinement factor, V is the physical volume of QD, μ is the transition element. So, the complex permittivity can be written as [2]

$$\begin{aligned}\varepsilon_1 &= \varepsilon_{bac} + \chi = \\ &= n_{bac}^2 + U\left[\omega_p - \omega_{10} + \left(\frac{T_e}{\hbar}\right)^2(\omega_{20} - \omega_p + i\gamma_{02})^{-1} - i\gamma_{01}\right]^{-1}\end{aligned} \tag{9}$$

where $U = \frac{\Gamma}{V}\frac{|\mu|^2}{\varepsilon_0\hbar}$. Usually, χ is very small, so the complex refractive index of a QD can be written as

$$\tilde{n}_1 = \varepsilon_1^{1/2} \approx n_{bac} + \frac{U}{2n_{bac}}\left[\omega_p - \omega_{10} + \left(\frac{T_e}{\hbar}\right)^2(\omega_{20} - \omega_p + i\gamma_{02})^{-1} - i\gamma_{01}\right]^{-1} \tag{10}$$

However, for our studied QD molecule system, there are two QDs, and the right QD is the same as the left QD, so the complex refractive index of the QD molecule can be written as [25]

$$\tilde{n} \approx n_{bac} + \frac{U}{2n_{bac}}\left\{ g_1\left[\omega_p - \omega_{10} + \left(\frac{T_e}{\hbar}\right)^2 (\omega_{20} - \omega_p + i\gamma_{02})^{-1} - i\gamma_{01}\right]^{-1} + \right.$$

$$\left. + \; g_2\left[\omega_p - \omega_{30} + \left(\frac{T_e}{\hbar}\right)^2 (\omega_{40} - \omega_p + i\gamma_{02})^{-1} - i\gamma_{01}\right]^{-1} \right\} \tag{11}$$

where g_1 and g_2 account for the possibility of different strengths for the two QDs. In the following, we assume $g_1 = g_2 = 1/2$ for the error introduced is small [25, 28]. By using the definitions above, we can get

$$\tilde{n} \approx n_{bac} + \frac{U}{4n_{bac}}\left\{ \left[\omega_p - \omega_{10} + \left(\frac{T_e}{\hbar}\right)^2 (\omega_{20} - \omega_p + i\gamma_{02})^{-1} - i\gamma_{01}\right]^{-1} + \right.$$

$$\left. + \left[\omega_p - \omega_{30} + \left(\frac{T_e}{\hbar}\right)^2 (\omega_{40} - \omega_p + i\gamma_{02})^{-1} - i\gamma_{01}\right]^{-1} \right\} \tag{12}$$

Finally, we can calculate the group velocity and the absorption coefficient from Eq. (12):

$$\alpha(\omega_p) = \frac{2\omega_p}{c}\mathrm{Im}(\tilde{n}) \tag{13}$$

$$\frac{v_g}{c} = \left[\mathrm{Re}(\tilde{n}) + \omega_p \frac{\mathrm{d}\,\mathrm{Re}(\tilde{n})}{\mathrm{d}\omega_p}\right]^{-1} \tag{14}$$

where v_g is the group velocity, c is the light velocity in vacuum.

3. Discussions

For InGaAs/GaAs double QD molecules, we choose the parameters as follows: $\hbar\gamma_{01}$ = = 1 μeV, $\gamma_{02} = 0.001\gamma_{01}$, $\hbar\omega_{10} = 1.6$ eV, $\hbar\omega_{20} = 1.5995$ eV, $\hbar\omega_{30} = 1.599$ eV, $\hbar\omega_{40} = = 1.5985$ eV, $\Gamma/V = 3 \times 10^{21}$ m^{-3}, and $T_e \sim 0.1$–0.2 meV [21–23].

Firstly, from Equation (14), we can get the absorption coefficient of the asymmetry QD molecules in Fig. 2. We find there are four absorption peaks in the absorption spectrum and there are three transparency windows between the four absorption peaks. The absorption peaks are near the resonance energy of the four energy levels. As a general consequence of the Kramers–Kronig relations, the absorption resonance peaks imply a finite frequency-dependent contribution to the refractive index. We may find multiple-windows and wide-band slow light in the system.

What is more, we find the absorption peaks shift with different T_e. It may be applied in the optical switches and can work at four wavelengths. For example, we can firstly select the wavelengths at absorption peak with $T_e = 0.1$ meV as the working wavelengths. Obviously, all the light will be opaque. It is the off-state of the optical switch-

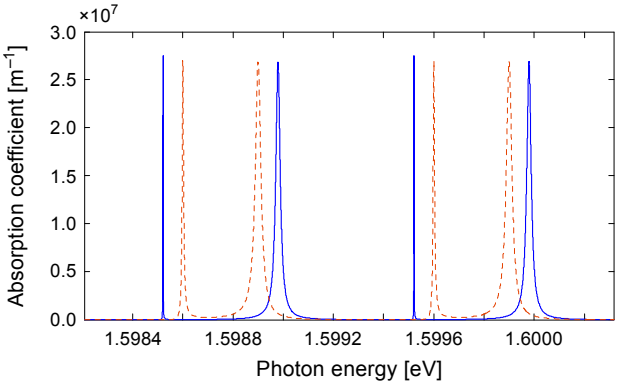

Fig. 2. The absorption coefficient as a function of photon energy with $T_e = 0.1$ meV (solid line) and $T_e = 0.2$ meV (dashed line).

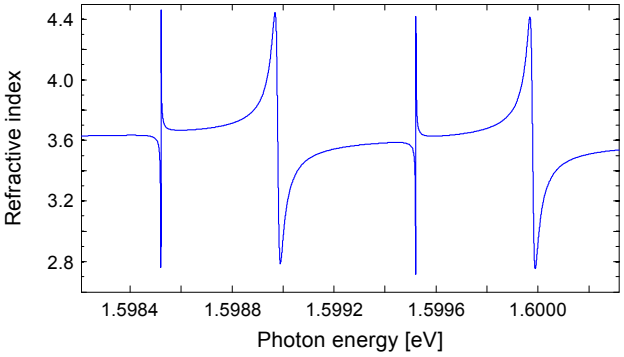

Fig. 3. The refractive index as a function of the photon energy ($T_e = 0.1$ meV).

ing. After altering T_e to 0.2 meV, we can find that the working wavelengths then are in the transparency window and all the light is near transparency. It is the on-state of the optical switching.

Secondly, the refractive index with different $T_e = 0.1$ meV is shown in Fig. 3. We calculate the refractive index by $\mathrm{Re}(\tilde{n})$. It is obvious that the dispersion curve is divided into many normal and anomalous dispersion regimes. In the normal regimes (*i.e.*, $\partial[\mathrm{Re}(\tilde{n})]/\partial\omega_p > 0$), the group velocity $v_g < c$, so the probe field is a slow light; in the anomalous dispersion regimes (*i.e.*, $\partial[\mathrm{Re}(\tilde{n})]/\partial\omega_p < 0$), the group velocity $v_g > c$, so it is superluminal. In Fig. 3, we can find that the normal (anomalous) dispersion regimes correspond to the TIT window (the absorption peaks). As a consequence, the probe field is near transparency (opaque) for the slow light (superluminal) propagation when working in the normal (anomalous) dispersion regimes. What is more, there are several TIT windows and absorption peaks in our two asymmetry QD molecules.

Finally, in Fig. 4, we can plot the variation values of v_g/c as a function of the photon energy which can show the normal and anomalous dispersion regimes in detail. We can find $v_g/c < 0$ (Figs. 4**b** and 4**c**) at the absorption peak and it is in the anomalous dis-

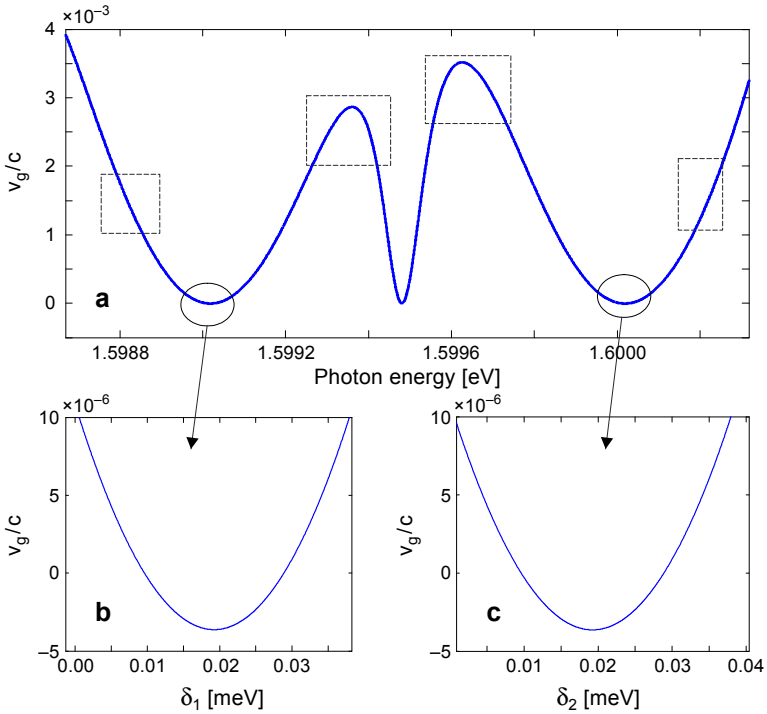

Fig. 4. The v_g/c as a function of photon energy (**a**). Detailed v_g/c as a function of photon energy near $\hbar\omega_{30}$, where $\delta_1 = \hbar\omega_p - \hbar\omega_{30}$ (**b**). Detailed v_g/c as a function of photon energy near $\hbar\omega_{10}$, where $\delta_2 = = \hbar\omega_p - \hbar\omega_{10}$ (**c**).

persion regimes. But near them, we can find that the variation of v_g/c is flat in a wide regime marked by a dashed box which shows large bandwidth property. The bandwidth will be almost up to about 0.1 meV (24 GHz). There are four regimes satisfying the large bandwidth property. And the slow factor c/v_g may reach to about 10^3 in each regime.

Finally, it is interesting that HAMEDI [29] has studied TIT in four QD molecules which can experience very narrow transparency windows accompanied by very steep positive dispersions. Especially, by properly adjusting the electric field and tunneling coupling effects, the number and width of transparency windows can be efficiently controlled in such a way that one-three narrow transparency windows can be established. Here, we have designed a simpler system and adopted different methods, three transparency windows are also found.

4. Conclusion

In conclusion, we have studied the double TIT slow light in the double asymmetry QD molecules. With applied electric field, double TITs occur in the same time. The transparency window is divided into many parts by four absorption peaks, and the absorption peak can be tuned by the applied electric field. The velocity and bandwidth of the

multiple-windows slow light can also be controlled by the applied electric field. In our model, with $T_e = 0.1$ meV, we can get about $0.001c$ and 20 GHz bandwidth in each transparency window. Such a system may be applied in all optical buffers, optical switching and filters.

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