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Electromagnetically induced transparency for Λ-like systems with degenerate autoionizing levels and a broadband coupling laser

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In our previous paper (Doan Quoc K. *et al*., Physica Scripta **T147**, 2012, article 014008), electromagnetically induced transparency for Λ-like systems consisting of two lower bound states and a flat continuum coupled to an autoionization state embedded in it has been considered, in which the laser coupling light is modeled by white noise. In this paper, we investigate a similar scheme, where the continuum involved in the problem is replaced by one with so-called the double-Λ system, when instead of one autoionization state we have two autoionization states of the same energy embedded in the continuum. For such a system containing degenerate autoionization levels we derive a set of coupled stochastic integro-differential equations which can be averaged exactly. This leads to the exact formula determining the stationary solution for the electric susceptibility. Dispersion and absorption spectra for electromagnetically induced transparency are found and compared with those obtained previously by us and other authors.

Keywords: electromagnetically induced transparency, autoionizing states, lambda configuration, white noise.

1. Introduction

The phenomenon of electromagnetically induced transparency (EIT) discovered for the first time by Harris and co-workers [1–3] relies on the destructive quantum interference of the involved transition amplitudes. This process leads to a suppression of absorption, or even to complete transmission of the resonant weak probe beam. This phenomenon arises in the presence of a second strong laser beam coupling coherently to one of the states which participate in the absorption process with some other atomic states.

Laser lights are generally fluctuating in amplitude and phase. Because of the very complicated microscopic nature of all the relevant relaxation mechanisms, we model the laser lights by classical time-dependent random processes. The dynamical equations involved in the considered problem become stochastic differential equations. To obtain an exact solution for such stochastic equations is a very difficult task except for some special cases, for example when the laser light involved in the model is assumed to be a white noise [4].

EIT in a model Λ-like system consisting of two lower bound states and a continuum coupled to an autoionization (AI) state embedded in it has been considered in [5]. The latter state might also be due to an interaction with an additional laser. The authors obtained analytic expressions for the susceptibility in the case of the bound-continuum dipole matrix elements being modeled according to Fano's autoionization theory [6] and examined the shape of the transparency window depending on the amplitude of the control field.

Recently the model studied in [5] has been extended to the case where the continuum involved in the problem is replaced by one with so-called the double-Λ system [7], where instead of one AI state we have two AI states with the same energy embedded in the continuum. It has been shown that the presence of the second AI level leads to the additional EIT window appearance. In this paper we use the same method applied in [8] for modeling the fluctuating control field as a white noise. Then the set of coupled stochastic integro-differential equations involved in the problem can be also solved exactly. The spectra of real and imaginary parts of the medium susceptibility are calculated and compared with the results obtained before by us and other authors. It follows that the structure of the EIT windows changes dramatically when the control field fluctuates.

2. The model of the double-Λ system

In this section we consider the Λ system discussed by THUAN BUI DINH *et al.* [7] which contains two lower states $|b\rangle$ and $|c\rangle$, the bare continuum $|E\rangle$ and two AI levels $|a_1\rangle$ and $|a_2\rangle$ with the same energy. The states $|a_1\rangle$ and $|a_2\rangle$ are coupled with the continuum by two additional couplings U_1 and U_2, respectively. This scheme is so-called the double-Λ system [7]. The AI states and the continuum are coupled by the weak probe and strong control fields of the frequencies ω_p and ω_d, respectively. For simplicity we assume that the frequency ω_d is not large enough to allow for the transition from the state $|b\rangle$ to the continuum and omit level shifts due to nonresonant couplings, which can be taken into account by redefining our detunings. The presence of the AI states may be alternatively taken into account by a prediagonalization procedure which leads to a dressed continuum $|E)$ with a modified density of states [6]. The scheme of the model is shown in Fig. 1.

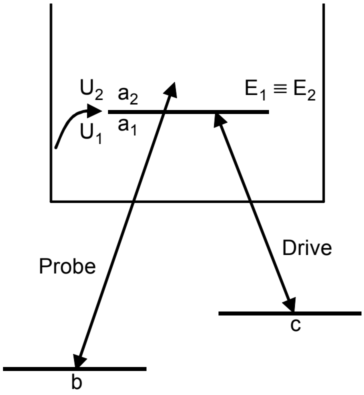

Fig. 1. The levels and coupling scheme.

As usual in EIT, the strong control field, for which the propagation effects are neglected, dresses the atomic medium to create new conditions for the propagation of the probe pulse. Now we assume that the amplitude of the control field has the form

$$\varepsilon_2 = \varepsilon_{02} + \varepsilon(t) \tag{1}$$

where ε_{02} is a deterministic coherent component of the control field and $\varepsilon(t)$ is characterized by a white noise

$$\langle\langle \varepsilon(t)\varepsilon^*(t')\rangle\rangle = a_0^2\delta(t-t') \tag{2}$$

the double brackets in the above equation indicate an average over the ensemble of realisations of the process $\varepsilon(t)$. Then the evolution of the atomic system is described by the von Neumann equation, which after transforming-off the rapidly oscillating terms and after making the rotating wave approximation, reduces in the first order perturbation with respect to the probe field [5] to the set of the following equations for the density matrix $\rho(z, t)$:

$$i\hbar\dot{\rho}_{Eb} = (E - E_b - \hbar\omega_p)\rho_{Eb} - \frac{1}{2}(E|d|b\rangle\varepsilon_1 - \frac{1}{2}(E|d|c\rangle(\varepsilon_{02} + \varepsilon(t))\rho_{cb} \tag{3a}$$

$$\begin{aligned} i\hbar\dot{\rho}_{cb} &= (E_c + \hbar\omega_d - E_b - \hbar\omega_p - i\hbar\gamma_{cb})\rho_{cb} + \\ &- \frac{1}{2}(\varepsilon_{02} + \varepsilon(t))^* \int \langle c|d|E)\rho_{Eb}\mathrm{d}E \end{aligned} \tag{3b}$$

In these equations d is the dipole moment, γ_{cb} is the phenomenological relaxation rate for the coherence ρ_{cb}, $\rho_{Eb} = (E|\rho|b\rangle$ and $\rho_{cb} = (E|\rho|c\rangle$.

The set of Eqs. (3) has the form of the following stochastic differential equation:

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = [D + x(t)F + x^*(t)G]Q + H \tag{4}$$

where Q is a vector function of time and D, F, G and H are constant matrices. As it is known from the multiplicative stochastic process theory, the function $\langle\langle Q\rangle\rangle$ satisfies the nonstochastic equation

$$\frac{\mathrm{d}\langle\langle Q\rangle\rangle}{\mathrm{d}t} = \left[D + \frac{a_0^2\{F, G\}}{2}\right]\langle\langle Q\rangle\rangle + H \tag{5}$$

where $\{F, G\}$ is the anticommutator of F and G.

Next, using Eq. (5) we obtain the system of equations for stochastic averages of the variables (double brackets have been dropped for convenience):

$$i\hbar\dot{\rho}_{Eb} = \left[(E - E_b - \hbar\omega_p) + \frac{a_0^2}{8\Gamma}(E|d|c\rangle\langle c|d|E)\right]\rho_{Eb} + \\ - \frac{1}{2}(E|d|b\rangle\varepsilon_1 - \frac{1}{2}(E|d|c\rangle b_0\rho_{cb} \tag{6a}$$

$$i\hbar\dot{\rho}_{cb} = \left[(E_c + \hbar\omega_d - E_b - \hbar\omega_p - i\hbar\gamma_{cb}) + \frac{a_0^2}{8\Gamma}\langle c|d|E)(E|d|c\rangle\right]\rho_{cb} + \\ - \frac{1}{2}b_0^*\int\langle c|d|E)\rho_{Eb}\mathrm{d}E \tag{6b}$$

where $b_0 = |\varepsilon_{02}|$.

3. The susceptibility spectrum

We can solve analytically the set of Eqs. (6a) and (6b) and get its stationary solutions. The component of the polarization of the medium connected with the b–E coupling is

$$P^+(\omega_p) = N\int d_{bE}\,\rho_{Eb}\,\mathrm{d}E = \varepsilon_0\varepsilon_1\chi(\omega_p) \tag{7}$$

with ε_0 being the vacuum electric permittivity, N is the atom density, and the medium susceptibility χ is given by

$$\chi(\omega_p) = -\frac{N}{\varepsilon_0}\left(A_{bb} + \frac{\frac{1}{4}b_0^2 A'_{bc}A_{cb}}{E_b + \hbar\omega_p - E_c - \hbar\omega_d + i\hbar\gamma_{cb} - \frac{1}{4}b_0^2 A_{cc}}\right) \tag{8}$$

The functions $A_{jk}(\omega_p)$ and $A'_{jk}(\omega_p)$, $j, k = b, c$, are given by

$$A_{jk}(\omega_p) = \lim_{\substack{\eta\to 0^+ \\ E_{21}\to 0^+}}\int\frac{\langle j|d|E)(E|d|k\rangle}{E_b + \hbar\omega_p - E - \frac{a_0^2}{8\Gamma}\langle c|d|E)(E|d|c\rangle + i\eta}\mathrm{d}E \tag{9}$$

$$A'_{jk}(\omega_p) = \lim_{\substack{\eta \to 0^+ \\ E_{21} \to 0^+}} \int \langle j|d\,|\,E\,)(\,E\,|\,d|k\rangle \Big(E_b + \hbar\omega_p - E - \frac{a_0^2}{8\Gamma} \langle c|d\,|E\,)(E\,|\,d\,|c\rangle + i\eta \Big)^{-1}$$
$$\times \left(1 + \frac{a_0^2}{8\Gamma} \frac{\langle c|d\,|E\,)(E\,|\,d\,|c\rangle}{E_c + \hbar\omega_d - E_b - \hbar\omega_p + i\hbar\gamma_{cb} + \frac{1}{4} b_0^2 A_{cc}} \right)^{-1} \mathrm{d}E \tag{10}$$

and the limit $\eta \to 0^+$ assures that the $\mathrm{Im}(\chi) > 0$, whereas $E_{21} = E_2 - E_1$ tends to zero for the degenerate AI levels. The bound-dressed continuum dipole matrix element can be modeled as [6, 9]:

$$\frac{\langle j|d\,|\,E\,)}{\langle j|d\,|E\rangle} = \frac{(E - E_1)(E - E_2) + E(q_{1j}\gamma_1 + q_{2j}\gamma_2) - (E_1 q_{2j}\gamma_2 + E_2 q_{1j}\gamma_1)}{(E - E_1)(E - E_2) - iE(\gamma_1 + \gamma_2) + i(E_1\gamma_2 + E_2\gamma_1)} \tag{11}$$

where the widths $\gamma_1 = \pi\left|\langle a_1|U_1|\,E\rangle\right|^2$ and $\gamma_2 = \pi\left|\langle a_2|U_2|\,E\rangle\right|^2$ are AI widths present in the system. Moreover, similarly as in [8], we have used Fano's asymmetry parameters q_{1j} and q_{2j}. They can be expressed as:

$$q_{1j} = \frac{\langle j|d\,|a_1\rangle}{\pi\langle j|d|\,E\rangle\langle E\,|U|a_1\rangle} \tag{12a}$$

$$q_{2j} = \frac{\langle j|d\,|a_2\rangle}{\pi\langle j|d|\,E\rangle\langle E\,|U|a_2\rangle} \tag{12b}$$

where $j = b$, c. It should be noted that the function inside the integral contains matrix elements corresponding to the transitions to the structured continuum $|E)$. Since such elements are energy dependent, we should apply the formula (11) to get the explicit dependence of the integrand on the energy. Thus, we can write

$$A_{jk}(\omega_p) = \lim_{\substack{\eta \to 0^+ \\ E_{21} \to 0^+}} D_j D_k \int \frac{F_j(E)\,F_k(E)}{E_b + \hbar\omega_p - E - \frac{a_0^2 D_c^2}{8\Gamma}\left|F_c(E)\right|^2 + i\eta} \mathrm{d}E \qquad (j,\, k = b,\, c) \tag{13}$$

$$A'_{jk}(\omega_p) = \lim_{\substack{\eta \to 0^+ \\ E_{21} \to 0^+}} D_j D_k \int \left[F_j(E)\,F_k(E) \Big(E_b + \hbar\omega_p - E - \frac{a_0^2 D_c^2}{8\Gamma}\left|F_c(E)\right|^2 + i\eta \Big)^{-1} \times \right.$$
$$\left. \times \left(1 + \frac{a_0^2 D_c^2}{8\Gamma} \frac{\left|F_c(E)\right|^2}{E_c + \hbar\omega_d - E_b - \hbar\omega_p - i\hbar\gamma_{cb} + \frac{1}{4} b_0^2 A_{cc}} \right)^{-1} \right] \mathrm{d}E \tag{14}$$

where

$$F_j(E) = (Q_j + i)\left(\frac{1}{Q_j + i} + \frac{A_j^+}{E - E_+} + \frac{A_j^-}{E - E_-}\right) \tag{15a}$$

$$F_k(E) = (Q_k - i)\left(\frac{1}{Q_k - i} + \frac{(A_k^+)^*}{E - (E_+)^*} + \frac{(A_k^-)^*}{E - (E_-)^*}\right) \tag{15b}$$

where $E_\pm$ are the complex roots of the denominator of Eq. (11) given by

$$E_\pm = \frac{E_1 + E_2 \pm A_1}{2} + i\frac{\Gamma \pm A_2}{2} \tag{16}$$

with

$$A_1 = \frac{1}{\sqrt{2}}\left\{\left[(E_{21}^2 - \Gamma^2)^2 + 4E_{21}^2(\gamma_2 - \gamma_1)^2\right]^{1/2} + E_{21}^2 - \Gamma^2\right\}^{1/2} \tag{17a}$$

$$A_2 = \frac{1}{\sqrt{2}}\left\{\left[(E_{21}^2 - \Gamma^2)^2 + 4E_{21}^2(\gamma_2 - \gamma_1)^2\right]^{1/2} - E_{21}^2 + \Gamma^2\right\}^{1/2} \tag{17b}$$

The complex amplitudes $A_j^\pm$ are given by the following expression:

$$A_j^\pm = \frac{\Gamma}{2}\left(1 \pm \frac{E_{21}K_j + i\Gamma}{A_1 + iA_2}\right) \tag{18}$$

where

$$K_j = \frac{Q_{j21} + i\Gamma_{21}}{Q_j + i} \tag{19}$$

The effective asymmetry parameters Q_j, Q_{j21}, Γ_{21} and AI width Γ are defined as:

$$\Gamma = \gamma_1 + \gamma_2, \quad Q_j = \frac{q_{1j}\gamma_1 + q_{2j}\gamma_2}{\Gamma}, \quad j = b, c \tag{20}$$

$$\Gamma_{21} = \frac{\gamma_2 - \gamma_1}{\Gamma}, \quad Q_{j21} = \frac{q_{2j}\gamma_2 - q_{1j}\gamma_1}{\Gamma}, \quad j = b, c \tag{21}$$

Moreover, we denote the matrix elements of the dipole moment transition $\langle j|d|E\rangle$ and $\langle E|d|k\rangle$ by D_j and D_k, respectively.

As it was mentioned earlier, we neglect threshold effects, so we extend the integration limits for $A_{jk}(\omega_p)$ and $A'_{jk}(\omega_p)$ from minus to plus infinities. Thanks to this assumption, we can find the analytical solution for this parameter and hence, for the medium susceptibility $\chi(\omega_p)$.

The susceptibility $\chi(\omega_p)$ can be computed completely numerically from the above formulas. We have found it in the stationary regime, assuming that the time-derivatives appearing in (3) are equal to zero. However the form of the final solution is very complicated and unreadable, and therefore, we do not present it here. The results will be presented in a graphical form in successive figures.

To compare our results with those from [7, 8] we assume the same values for the parameters describing atomic system and its interaction with external fields. Thus, we have assumed that $\Gamma = 10^{-9}$ a. u., and that the values of the coherent part of the field amplitude b_0 ranged from 10^{-9} to 10^{-6} a. u. The coupling constants that are the bound-bare continuum dipole matrix elements are equal to $D_b = 2$ a. u. and $D_c = 3$ a. u. (all parameters used here are in atomic units). Moreover, the asymmetry parameters are of the order of 10–100, whereas the atomic density is assumed to be equal to $N = 0.33\times10^{12}$ cm^{-3}. The relaxation rate γ_{cb} is neglected, and detuning is $\omega = \omega_p + (E_b - E_1)/\hbar$.

The spectra of real and imaginary parts of the medium susceptibility for various values of the parameters involved in the problem are shown in Figs. 2–4.

When coherent part of the light dominates over the fluctuations, we can assume that the fluctuation part of the field amplitude vanishes ($a_0 = 0$) and then, our result becomes exactly the same as that obtained by Bui Dinh Thuan *et al*. [7]. The dispersion and absorption parts of the medium susceptibility for these cases are shown in Fig. 2, when the additional EIT window appears.

The coherent part of the laser light of the strong control field is negligible in comparison with the chaotic component. As a consequence, due to the disappearance of

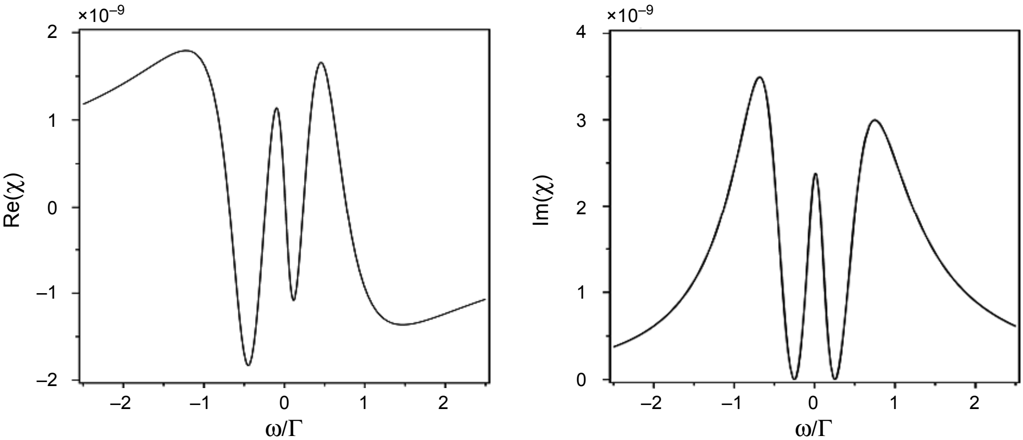

Fig. 2. The dispersion (real) and absorption (imaginary) parts of the susceptibility as a function of the ω for the value of $b_0 = 4\times10^{-7}$ a. u., $\Gamma_{21} = 0$, $Q_b = Q_c = 20$, $Q_{b21} = 1$, $Q_{c21} = 8$ and $a_0 = 0$.

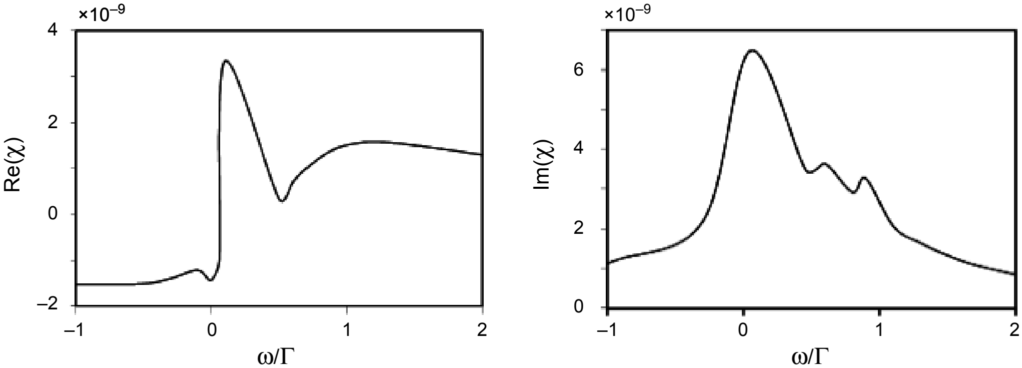

Fig. 3. The dispersion (real) and absorption (imaginary) parts of the susceptibility as a function of the ω for the value of $b_0 = 0$, $\Gamma_{21} = 0$, $Q_b = Q_c = 20$, $Q_{b21} = 1$, $Q_{c21} = 8$ and $a_0 = 0.02\Gamma$.

the coherence part, for the pure noisy light case ($b_0 = 0$), the susceptibility $\chi(\omega_p)$ can be written as

$$\chi(\omega_p) = -\frac{N}{\varepsilon_0} A_{bb} \tag{22}$$

The dispersion and absorption parts of the medium susceptibility for these cases are shown in Fig. 3. We find that the left and right peaks of dispersion and absorption profiles drop fast. The slope of the dispersion curves and the depth of the transparency windows decrease fast. Moreover, the zero point shifts to the right when the chaotic component exists in comparison with the case when the white noise is absent. This effect is observed already in the case with a single AI level, discussed by DOAN QUOC *et al.* [8].

However, for the general case, both the coherence and fluctuation parts of the control field amplitude are present. These results presented in Fig. 4 show that the left peaks of dispersion and left absorption profiles drop faster than others. Moreover, the transparency window is also shifted to the right from the zero frequency. Furthermore,

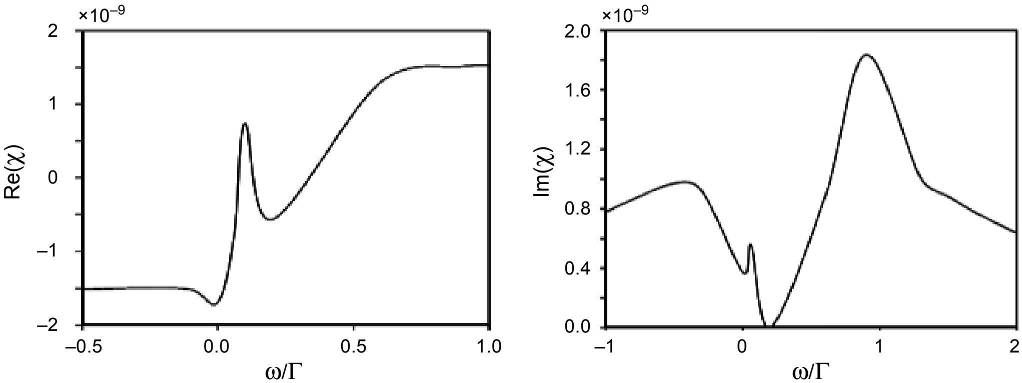

Fig. 4. The dispersion (real) and absorption (imaginary) parts of the susceptibility as a function of the ω for the value of $b_0 = 4\times10^{-7}$ a. u., $\Gamma_{21} = 0$, $Q_b = Q_c = 20$, $Q_{b21} = 1$, $Q_{c21} = 8$ and $a_0 = 0.02\Gamma$.

the slope of the dispersion curves and the depth of the transparency windows decrease slower as compared with the case of the pure noisy light.

The group velocity of the probe beam depends on the refractive index of the medium and its changes are related to the derivative of Re(χ) with respect to the probe beam frequency (the slope of the dispersion curves). This fact is expressed by the following formula:

$$n_g = 1 + \frac{\omega_p}{2}\frac{\mathrm{d}}{\mathrm{d}\omega_p}\mathrm{Re}[\chi(\omega_p)] \tag{23}$$

Thus, when the slope of the dispersion curves decreases, the group velocity of light will increase. As a consequence, the parameter a_0 related to the chaotic component is an important parameter which controls the propagation group velocity of light in medium.

4. Conclusions

In this paper we discussed the atomic model of Λ-configuration involving two AI states of the same energy as proposed in [7]. We assumed that, as in [4], the laser coupling light applied in the system is decomposed into two parts: coherent part and white noise. For such a system, the stationary solution for the electric susceptibility was found by solving a set of coupled stochastic integro-differential equations involved in the problem. Next, we derived the exact formulas determining the dispersion and absorption spectra of the medium susceptibility and compared these results with those obtained in [7]. We have shown that, similarly as in [7], the EIT effect appears for the system discussed here. Moreover, both the position and the width of the transparency window change dramatically as we compare them with those discussed for the case when the noise of the control laser field is absent. We have pointed that the parameter a_0 related to the chaotic component can be treated as an important parameter for controlling the propagation group velocity of light in the medium.

Similarly to the case considered in [8], we believe that our model is more realistic than that discussed in [7], because the amplitudes of the real laser light used in experimental setups always contain some fluctuating component.

Recently the model of Λ-configuration involving two non-degenerate AI states has been considered in [10]. We will generalize our formalism to this case in a future paper.

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