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Dark and singular optical solitons with competing nonlocal nonlinearities

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In this work, we study the dynamics of optical solitons in a synthetic nonlocal nonlinear media. The nonlinear dynamical model which describes the propagation of optical solitons in the weakly nonlocal nonlinear media with parabolic law nonlinearity is investigated analytically. The tool of integration that is the Riccati equation mapping approach is introduced to extract exact traveling wave solutions. As a result, an explicit dark soliton, singular soliton and periodic solutions are derived.

Keywords: solitons, parabolic law nonlinearity, competing nonlinearities.

1. Introduction

The study of dynamical behaviours of optical solitons in nonlocal nonlinear media has greatly attracted researchers' attention in recent years [1–30]. Nonlocality of nonlinearity, which includes weak nonlocality, general nonlocality and strong nonlocality, exists in nematic liquid crystals, plasmas and many other nonlinear systems [4, 9, 13, 17]. In the paraxial approximation, the propagation of optical solitons in a nonlocal nonlinear medium is ruled by the well-known nonlocal nonlinear Schrödinger equation (NNLSE) in the form [7, 10, 17, 18]

$$i\frac{\partial u}{\partial z} + \frac{1}{2}\frac{\partial^2 u}{\partial z^2} + \Delta n u = 0 \tag{1a}$$

where $u(x, z)$ represents the normalized complex wave envelope that is the function of transverse coordinate x and longitudinal coordinate z;

$$\Delta n(x, z) = s\int_{-\infty}^{+\infty} R(x' - x)|u(x', z)|^2 \mathrm{d}x' \tag{1b}$$

refers to the refractive index change that depends on optical pulse intensity $|u(x, z)|^2$, in which $R(x)$ is the nonlocal response function, and $s = \pm 1$ determines the types of nonlinearity, specifically, the plus (minus) sign corresponds to the self-focusing (self-defocusing) nonlinearity.

For strongly nonlocal medium, Δn is proportional to x^2, then NNLSE (Eq. (1a)) reduces to a linear differential equation, also known as the Snyder–Mitchell model.

For weakly nonlocal medium $\Delta n = s[|u|^2 + \gamma\partial_x^2(|u|^2)]$ in which γ is a small constant. Then, in this case, NNLSE becomes [10]

$$i\frac{\partial u}{\partial z} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} + s|u|^2 u + \gamma s\frac{\partial^2(|u|^2)}{\partial x^2}u = 0 \tag{2}$$

Recently, there has been a growing interest in the study of optical solutions in the nonlocal nonlinear media with competing nonlinearities where the nonlinear response is the result of contributions from nonlocal nonlinearity and other nonlinear effects. CHEN *et al.* [4] reported the dynamical behaviours of dark solitons in a medium with competing nonlocal nonlinearity and quintic nonlinearity. Bang's research team [7, 9, 13, 16] studied the properties of optical solitons in a medium with competing nonlocal self-focusing and self-defocusing nonlinearities. In our previous studies [5, 10, 12, 15, 17], exact solitons in a medium with competing weakly nonlocal nonlinearity and parabolic law nonlinearity, as well as a medium with competing nonlocal self-focusing nonlinearity, self-defocusing nonlinearity and quintic nonlinearity, are derived.

The aim of the present work is to study the dynamics of optical solitons in a medium with competing weakly nonlocal nonlinearity and parabolic law nonlinearity. Without loss of generality, the nonlinear dynamical model is given by [5, 10]

$$i\frac{\partial u}{\partial z} + a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^2(|u|^2)}{\partial x^2}u + c|u|^2 u + \mu|u|^4 u = 0 \tag{3}$$

where a represents the coefficients of diffraction, while b gives the coefficients of weakly nonlocal nonlinearity, and finally the terms with c and μ account for the parabolic law (cubic-quintic) nonlinearity [5, 10, 31, 32].

It should be noted that Eq. (3) has been investigated analytically by traveling wave hypothesis, Jacobi's elliptic equations expansion approach and ansatz scheme [5, 10]. This work is an extension of earlier studies. We will use a different algorithm to integrate Eq. (3). It is the generalized Riccati equation expansion technique [33–41]. Finally, explicit traveling wave solutions, along with the integrability conditions, are reported.

2. Exact solitons

Assume that Eq. (3) admits the following stationary solutions:

$$u(x, z) = \psi(x)\exp(i\lambda z) \tag{4}$$

where λ is the propagation constant.

Substituting the above hypothesis (4) into Eq. (3) yields

$$(a + 2b\psi^2)\frac{d^2\psi}{dx^2} + 2b\psi\left(\frac{d\psi}{dx}\right)^2 - \lambda\psi + c\psi^3 + \mu\psi^5 = 0 \tag{5}$$

Below, we will perform the Riccati equation mapping scheme to construct explicit traveling wave solutions to Eq. (5). Based on the homogeneous balance principle, Eq. (5) has the solution in the form

$$\psi(x) = l_0 + l_1\varphi(x) \tag{6}$$

where l_0 and l_1 are real constants to be determined later, and $\varphi(x)$ is the solution of the following second-order nonlinear ordinary differential equation:

$$\varphi'(x) = \alpha + \beta\varphi(x) + \delta\varphi^2(x) \tag{7}$$

with α, β and δ being real constants. Equation (7) is the famous generalized Riccati equation, exact solutions of which are given in [33–41].

Substituting Eqs. (6) and (7) into Eq. (5), and vanishing the coefficients of φ^i ($i = 0$, 1, 2, 3, 4, 5) yield a system of nonlinear algebraic equations which solves to:

$$l_0 = 0 \tag{8}$$

$$l_1 = \pm\sqrt{-\frac{6b\delta^2}{\mu}} \tag{9}$$

$$\alpha = \frac{a\mu - 3bc}{24b^2\delta} \tag{10}$$

$$\beta = 0 \tag{11}$$

$$\lambda = \frac{a^2\mu - 3abc}{12b^2} - \frac{(a\mu - 3bc)^2}{48\mu b^2} \tag{12}$$

where a, b, c, μ and δ are arbitrary constants.

Equation (9) naturally introduces the constraint

$$b\mu < 0 \tag{13}$$

which means that the coefficients of weakly nonlocal nonlinearity and cubic-quintic nonlinearity have the opposite sign.

Equation (12) gives the expression of propagation constant.

Finally, we can obtain analytical traveling wave solutions to Eq. (3), which are listed as follows:

Case 1: periodic solutions and periodic singular solutions. Equation (3) admits 10 periodic traveling wave solutions, which are given by

$$u_1(x, z) = \pm\sqrt{-\frac{6b\alpha\delta}{\mu}}\tan(\sqrt{\alpha\delta}\,x)\exp(i\lambda z) \tag{14}$$

$$u_2(x, z) = \mp\sqrt{-\frac{6b\alpha\delta}{\mu}}\cot(\sqrt{\alpha\delta}\,x)\exp(i\lambda z) \tag{15}$$

$$u_3(x, z) = \pm\sqrt{-\frac{6b\alpha\delta}{\mu}}\left[\tan(\sqrt{4\alpha\delta}\,x) \pm \sec(\sqrt{4\alpha\delta}\,x)\right]\exp(i\lambda z) \tag{16}$$

$$u_4(x, z) = \mp\sqrt{-\frac{6b\alpha\delta}{\mu}}\left[\cot(\sqrt{4\alpha\delta}\,x) \pm \csc(\sqrt{4\alpha\delta}\,x)\right]\exp(i\lambda z) \tag{17}$$

$$u_5(x, z) = \pm\sqrt{-\frac{3b\alpha\delta}{2\mu}}\left[\tan\left(\frac{\sqrt{\alpha\delta}}{2}x\right) - \coth\left(\frac{\sqrt{\alpha\delta}}{2}x\right)\right]\exp(i\lambda z) \tag{18}$$

$$u_6(x, z) = \pm\sqrt{-\frac{6b\alpha\delta}{\mu}}\,\frac{\pm\sqrt{A^2 - B^2} - A\cos(\sqrt{4\alpha\delta}\,x)}{A\sinh(\sqrt{4\alpha\delta}\,) + B}\exp(i\lambda z) \tag{19}$$

$$u_7(x, z) = \pm\sqrt{-\frac{6b\alpha\delta}{\mu}}\,\frac{\mp\sqrt{A^2 - B^2} - A\cos(\sqrt{4\alpha\delta}\,x)}{A\sinh(\sqrt{4\alpha\delta}\,) + B}\exp(i\lambda z) \tag{20}$$

$$u_8(x, z) = \mp\sqrt{-\frac{6b\alpha\delta}{\mu}}\frac{\cos(\sqrt{\alpha\delta}\,x)}{\sin(\sqrt{4\alpha\delta}\,) \pm 1}\exp(i\lambda z) \tag{21}$$

$$u_9(x, z) = \pm\sqrt{-\frac{6b\alpha\delta}{\mu}}\frac{\sin(\sqrt{\alpha\delta}\,x)}{\cos(\sqrt{4\alpha\delta}\,) \pm 1}\exp(i\lambda z) \tag{22}$$

$$u_{10}(x, z) = \pm 2\sqrt{-\frac{6b\alpha\delta}{\mu}}\frac{\sin\left(\frac{\sqrt{\alpha\delta}}{2}\right)\cos\left(\frac{\sqrt{\alpha\delta}}{2}\right)}{2\cos^2\left(\frac{\sqrt{\alpha\delta}}{2}\right) - 1}\exp(i\lambda z) \tag{23}$$

which naturally pose the constraint conditions

$$b\alpha\delta\mu < 0 \tag{24}$$

$$A^2 - B^2 > 0 \tag{25}$$

where both A and B are constants.

Here, we noted that the periodic solutions and periodic singular solutions are not necessary because these periodic solutions are of no interest in optical fibers at all.

Case 2: soliton solutions. Equation (3) admits a dark soliton solution

$$u_{11}(x, z) = \mp\sqrt{\frac{6b\alpha\delta}{\mu}}\tanh(\sqrt{-\alpha\delta}\,x)\exp(i\lambda z) \tag{26}$$

and singular soliton solutions

$$u_{12}(x, z) = \mp\sqrt{\frac{6b\alpha\delta}{\mu}}\coth(\sqrt{-\alpha\delta}\,x)\exp(i\lambda z) \tag{27}$$

which naturally introduce the restriction

$$b\alpha\delta\mu > 0 \tag{28}$$

3. Conclusion

The dynamics of optical solitons in a medium with competing weakly nonlocal nonlinearity and parabolic law nonlinearity has been studied analytically. The integration tool that is the Riccati equation mapping approach is adopted to retrieve exact solutions. Finally, ten periodic traveling wave solutions and two soliton solutions are reported. The presented results show that the first restriction for the existence of those explicit traveling wave solutions is $b\mu < 0$, *i.e.*, the coefficients of weakly nonlocal nonlinearity and cubic-quintic nonlinearity have the opposite sign.

In the future studies, we plan to present the analytic study on optical solitons in a medium with competing weakly nonlocal nonlinearity and other types of non-Kerr law nonlinearity. They are power law and dual-power law. The results will be published in other papers.

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