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Dynamic evolution and classification of coherent vortices in atmospheric turbulence

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Based on the extended Huygens–Fresnel principle, the analytical expressions for the cross-spectral density function of partially coherent sine-Gaussian vortex beams and partially coherent sine-Gaussian non-vortex beams propagating through free space and atmospheric turbulence are derived, and used to study the dynamic evolution behavior of coherent vortices in free space and atmospheric turbulence. According to the creation, the coherent vortices are grouped into three classes: the first is their inherent coherent vortices of the vortex beam, the second is created by the vortex beam itself during transmission process in free space, and the third is created by the atmospheric turbulence inducing the vortex beam.

Keywords: vortex beam, coherent vortices, atmospheric turbulence.

1. Introduction

Vortex beams have attracted much attention and become one of current research focuses because of their important applications, such as optical tweezers, optical data storage, quantum cryptography systems, optical communications and astronomy, *etc*. [1–7]. As pointed out by GBUR and TYSON, the topological charge of the vortex beams propagation through atmospheric turbulence is a robust quantity that could be used as an information carrier in optical communications [8]. YALONG GU and GBUR have suggested a possible method for measuring atmospheric turbulence strength using vortex beam [9]. The scintillation index of vortex beam in atmospheric turbulence is experimentally simulated by CHEN *et al*. [10]. FANG GUI-JUAN and PU JI-XIONG studied the degree of polarization of stochastic electromagnetic vortex beams in atmospheric turbulence [11]. Trajectory of an optical vortex has been identified in atmospheric turbulence using numerical simulations by DIPANKAR *et al*. [12]. LÜ BAI-DA and LI JIN-HONG analyzed the influence of atmospheric turbulence along an uplink path and a downlink path on coherence vortex; it is shown that the infulence of atmospheric turbulence on Gaussian –Schell model (GSM) vortex beam propagation is smaller along a downlink path than that along an uplink path, and the distance of topological charge conservation for

GSM vortex beam is longer along a downlink path than that along an uplink path [13]. However, the work mentioned above did not refer to the creation classification of coherent vortices. In this paper, taking the partially coherent sine-Gaussian (SiG) vortex beams as an example of partially coherent vortex beams, we have studied the dynamic evolution and classification of coherent vortices in the free space and atmospheric turbulence.

2. Theoretical formulation

The initial field of a SiG vortex beam at the plane $z = 0$ reads as

$$E(\mathbf{s}, 0) = \exp\left(-\frac{s_x^2 + s_y^2}{w_0^2}\right) \sin[\Omega_0(s_x + s_y)][s_x + i\,\mathrm{sgn}(m)s_y]^{|m|} \tag{1}$$

where $\mathbf{s} \equiv (s_x, s_y)$ is the two-dimensional (2D) position vector, w_0 denotes the waist width of the Gaussian part, Ω_0 is the parameter associated with the sine-part, where $\Omega_0 \neq 0$ because $E(\mathbf{s}, z = 0) = 0$ if $\Omega_0 = 0$, sgn$(\cdot)$ specifies the sign function, m is the topological charge, and in following we take $m = \pm 1$.

By introducing a Schell-correlator [14], the cross-spectral density function of the partially coherent SiG vortex beams at the source plane $z = 0$ is expressed as

$$\begin{aligned} W_0(\mathbf{s}_1, \mathbf{s}_2, 0) = {} & [s_{1x}s_{2x} + s_{1y}s_{2y} \pm i(s_{1x}s_{2y} - s_{2x}s_{1y})]\exp\left(-\frac{s_1^2 + s_2^2}{w_0^2}\right) \times \\ & \times \sin[\Omega_0(s_{1x} + s_{1y})]\sin[\Omega_0(s_{2x} + s_{2y})]\exp\left(-\frac{(\mathbf{s}_1 - \mathbf{s}_2)^2}{2\sigma_0^2}\right) \end{aligned} \tag{2}$$

where $\mathbf{s}_i \equiv (s_{ix}, s_{iy})$ $(i = 1, 2)$ is the 2D position vector at the source plane $z = 0$, σ_0 denotes the spatial correlation length, and $\pm$ corresponds to $m = \pm 1$.

In accordance with the extended Huygens–Fresnel principle [15], the cross-spectral density function of partially coherent SiG vortex beams propagating through atmospheric turbulence is given by

$$\begin{aligned} W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = {} & \left(\frac{k}{2z\pi}\right)^2 \iint \mathrm{d}^2\mathbf{s}_1 \iint \mathrm{d}^2\mathbf{s}_2 \times \\ & \times W_0(\mathbf{s}_1, \mathbf{s}_2, 0)\exp\left\{-\frac{ik}{2z}\left[(\boldsymbol{\rho}_1 - \mathbf{s}_1)^2 - (\boldsymbol{\rho}_2 - \mathbf{s}_2)^2\right]\right\} \times \\ & \times \langle \exp[\psi^*(\boldsymbol{\rho}_1, \mathbf{s}_1) + \psi(\boldsymbol{\rho}_2, \mathbf{s}_2)]\rangle \end{aligned} \tag{3}$$

where $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ denote the position vector at the z plane, k is the wave number related to the wavelength λ by $k = 2\pi/\lambda$, $\langle\cdot\rangle$ denotes the average over the ensemble of the turbulence medium. It is worth mentioning that a quadratic approximation of Rytov's phase structure function [16] is used in Eq. (3), which can be written as

$$\langle \exp[\psi^*(\boldsymbol{\rho}_1, \mathbf{s}_1) + \psi(\boldsymbol{\rho}_2, \mathbf{s}_2)]\rangle = \\ = \exp\left\{-\frac{1}{\rho_0^2}\left[(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)^2 + (\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)(\mathbf{s}_1 - \mathbf{s}_2) + (\mathbf{s}_1 - \mathbf{s}_2)^2\right]\right\} \quad (4)$$

where $\rho_0 = (0.545 C_n^2 k^2 z)^{-3/5}$ denotes the spatial coherence radius of a spherical wave propagating through turbulence and C_n^2 denotes the structure constant, Eq. (4) is accepted to be valid not only for weak fluctuations but also for strong ones [17–19].

To simplify the calculation, introducing two variables of integration $\boldsymbol{\mu} = (\mathbf{s}_1 + \mathbf{s}_2)/2$, $\mathbf{v} = \mathbf{s}_1 - \mathbf{s}_2$, and recalling integral formula [20]

$$\int \exp(-px^2 + 2qx)\mathrm{d}x = \sqrt{\frac{\pi}{p}}\exp\left(\frac{q^2}{p}\right) \quad (5a)$$

$$\int x\exp(-px^2 + 2qx)\mathrm{d}x = \sqrt{\frac{\pi}{p}}\frac{q}{p}\exp\left(\frac{q^2}{p}\right) \quad (5b)$$

$$\int x^2\exp(-px^2 + 2qx)\mathrm{d}x = \frac{1}{2p}\sqrt{\frac{\pi}{p}}\left(1 + \frac{2q^2}{p}\right)\exp\left(\frac{q^2}{p}\right) \quad (5c)$$

substituting Eqs. (2) and (4) into Eq. (3), we obtain the cross-spectral density function of partially coherent SiG vortex beams propagating through atmospheric turbulence as follows:

$$W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = -\frac{k^2}{16ACz^2}\exp\left[-\frac{ik}{2z}(\boldsymbol{\rho}_1^2 - \boldsymbol{\rho}_2^2)\right]\exp\left[-\frac{1}{\rho_0^2}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)^2\right] \times \\ \times (M_1 + M_2 - M_3 - M_4) \quad (6)$$

where

$$M_1 = \left(\frac{E_x^2 + E_y^2 + C}{C^2} - \frac{I_x^2 + I_y^2 + H}{4H^2} \pm i\,\frac{I_x E_y - E_x I_y}{CH}\right)\exp\left(\frac{B_x^2 + B_y^2}{4A} + \frac{E_x^2 + E_y^2}{C}\right) \quad (7a)$$

$$M_3 = \left(\frac{G_x^2 + G_y^2 + C}{C^2} - \frac{J_x^2 + J_y^2 + H}{4H^2} \pm i\,\frac{J_x G_y - G_x J_y}{CH}\right)\exp\left(\frac{F_x^2 + F_y^2}{4A} + \frac{G_x^2 + G_y^2}{C}\right) \quad (7b)$$

$$A = \frac{1}{2w_0^2} + \frac{1}{2\sigma_0^2} + \frac{1}{\rho_0^2} \tag{7c}$$

$$B_x = \frac{ik}{2z}(\rho_{1x} + \rho_{2x}) - \frac{1}{\rho_0^2}(\rho_{1x} - \rho_{2x}) \tag{7d}$$

$$C = \frac{2}{w_0^2} + \frac{k^2}{4Az^2} \tag{7e}$$

$$D_x = \frac{ik}{z}(\rho_{1x} - \rho_{2x}) + 2i\Omega_0 \tag{7f}$$

$$E_x = \frac{1}{2}\left(D_x - \frac{ik}{2Az}B_x\right) \tag{7g}$$

$$F_x = B_x + i\Omega_0 \tag{7h}$$

$$G_x = \frac{1}{2}\left[\frac{ik}{z}(\rho_{1x} - \rho_{2x}) - \frac{ik}{2Az}F_x\right] \tag{7i}$$

$$H = A + \frac{k^2 w_0^2}{8z^2} \tag{7j}$$

$$I_x = \frac{1}{2}\left(B_x - \frac{ikw_0^2}{4z}D_x\right) \tag{7k}$$

$$J_x = \frac{1}{2}\left[F_x + \frac{k^2 w_0^2(\rho_{1x} - \rho_{2x})}{4z^2}\right] \tag{7l}$$

According to the symmetry, B_y, D_y, E_y, I_y, F_y, G_y and J_y can be obtained by replacement of ρ_{1x} and ρ_{2x} in B_x, D_x, E_x, I_x, F_x, G_x and J_x with ρ_{1y} and ρ_{2y}; M_2 and M_4 can be obtained by replacement of Ω_0 in M_1 and M_3 with $-\Omega_0$; $\pm$ in M_1 and M_3 corresponds to $m = \pm 1$, respectively.

For $C_n^2 = 0$, Eq. (6) reduces to the cross-spectral density function of partially coherent SiG vortex beams in free space, which is expressed as

$$W_{\text{free}}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = -\frac{k^2}{16A_0C_0z^2}\exp\left[-\frac{ik}{2z}(\boldsymbol{\rho}_1^2 - \boldsymbol{\rho}_2^2)\right](M_{10} + M_{20} - M_{30} - M_{40}) \tag{8}$$

By letting $C_n^2 = 0$ in A, C, M_1, M_2, M_3 and M_4, we can obtain A_0, C_0, M_{10}, M_{20}, M_{30} and M_{40}, respectively.

For $m = 0$, Eq. (1) reduces to the filed of an SiG non-vortex beam at the $z = 0$, which is written as

$$E'(\mathbf{s}, 0) = \exp\left(-\frac{s_x^2 + s_y^2}{w_0^2}\right)\sin[\Omega_0(s_x + s_y)] \tag{9}$$

Similarly, we can obtain the cross-spectral density function of partially coherent SiG non-vortex beams propagating through atmospheric turbulence as follows:

$$W'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = -\frac{k^2}{16ACz^2}\exp\left[-\frac{ik}{2z}(\boldsymbol{\rho}_1^2 - \boldsymbol{\rho}_2^2)\right]\exp\left[-\frac{1}{\rho_0^2}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)^2\right] \times (M'_1 + M'_2 - M'_3 - M'_4) \tag{10}$$

where

$$M'_1 = \exp\left(\frac{B_x^2 + B_y^2}{4A} + \frac{E_x^2 + E_y^2}{C}\right) \tag{11a}$$

$$M'_3 = \exp\left(\frac{F_x^2 + F_y^2}{4A} + \frac{G_x^2 + G_y^2}{C}\right) \tag{11b}$$

parameters M'_2 and M'_4 can be obtained by replacing Ω_0 in M'_1 and M'_3 with $-\Omega_0$. Obviously, for $C_n^2 = 0$, Eq. (10) reduces to the cross-spectral density function of partially coherent SiG non-vortex beams propagating through free space.

The spectral degree of coherence is defined as [21]

$$\mu(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = \frac{W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)}{\sqrt{I(\boldsymbol{\rho}_1, z)I(\boldsymbol{\rho}_2, z)}} \tag{12}$$

where $I(\boldsymbol{\rho}_i, z) = W(\boldsymbol{\rho}_i, \boldsymbol{\rho}_i, z)$ $(i = 1, 2)$ stands for the spectral intensity of the point $(\boldsymbol{\rho}_i, z)$. The position of coherent vortices is determined by [22]

$$\mathrm{Re}[\mu(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)] = 0 \tag{13a}$$

$$\mathrm{Im}[\mu(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)] = 0 \tag{13b}$$

where Re and Im denote the real and imaginary parts of $\mu(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)$, respectively. The sign of coherent vortices are determined by the vorticity of phase contours around singularities [23].

3. Numerical calculations and analyses

Figure 1 gives curves of $\mathrm{Re}(\mu) = 0$ (solid curves) and $\mathrm{Im}(\mu) = 0$ (dashed curves) of partially coherent SiG non-vortex beams (**a**–**c**) and partially coherent SiG vortex

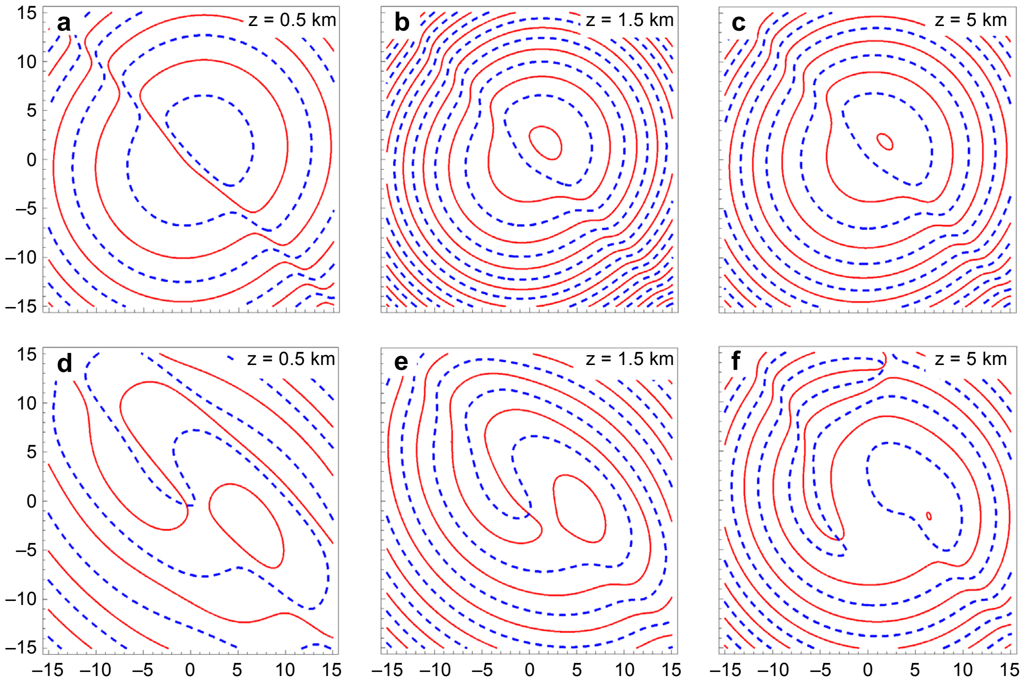

Fig. 1. Curves of Re(μ) = 0 and Im(μ) = 0 of partially coherent SiG non-vortex beams (**a**–**c**) and partially coherent SiG vortex beams (**d**–**f**) in free space at different propagation distance z.

beams (**d**–**f**) propagating through free space at the propagation distance $z = 0.5$, 1.5 and 5 km, the abscissa represents ρ_{2x} direction, ordinate represents ρ_{2y} direction, and their units are cm. The calculation parameters are $\lambda = 1.06$ μm, $w_0 = 5$ cm, $\sigma_0 = 3$ cm, $m = +1$, $\Omega_0 = 30$. Figures 1**a**–1**c** indicate that there never exists an intersection point for curves of Re and Im of partially coherent SiG non-vortex beams propagating through free space, namely no coherent vortex occurs. Figures 1**d**–1**f** show that there exist intersection points (coherent vortices) for curves of Re and Im of partially coherent SiG vortex beams propagating through free space. For example, there is a coherent vortex at $z = 0.5$ and 1.5 km, respectively, whose positions are (–0.399 cm, –0.484 cm) and (–0.025 cm, –1.309 cm), and two coherent vortices take place at $z = 5$ km which are located at (–3.078 cm, –3.959 cm) and (1.594 cm, 13.611 cm). Therefore, coherent vortices are created when partially coherent SiG vortex beams propagate through free space.

Figure 2 gives curves of Re(μ) = 0 (solid curves) and Im(μ) = 0 (dashed curves) of partially coherent SiG non-vortex beams (**a**–**c**) and partially coherent SiG vortex beams (**d**–**f**) propagating through atmospheric turbulence at the propagation distance $z = 0.5$, 1.5 and 5 km, where $C_n^2 = 10^{-14}$ m$^{-2/3}$, and the other calculation parameters are the same as in Fig. 1. From Figs. 2**a**–2**c** we can see that there exists no coherent

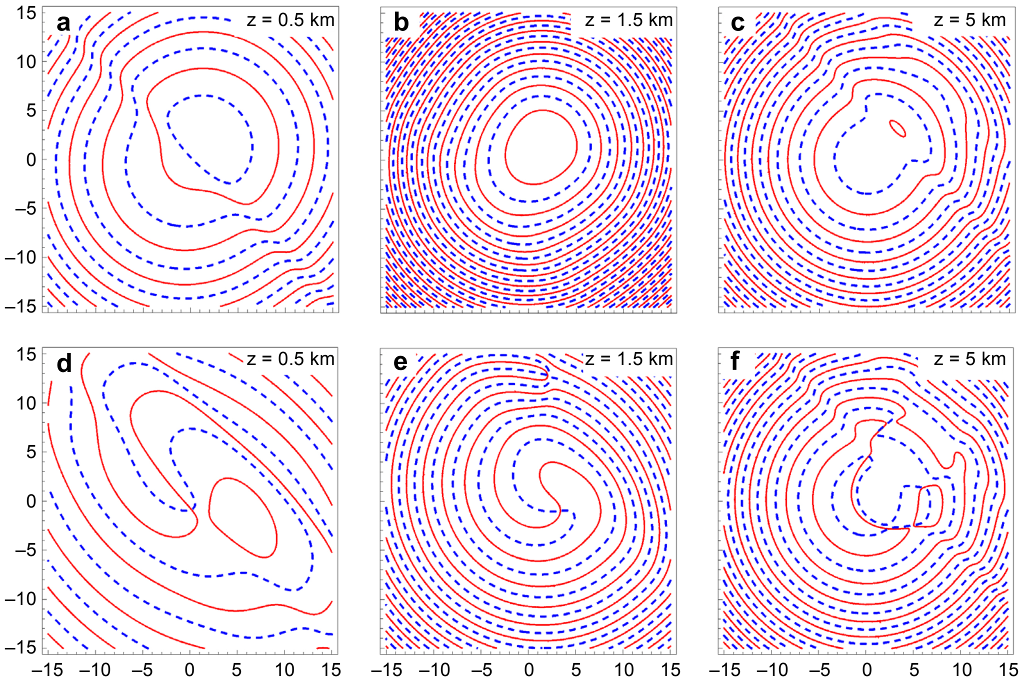

Fig. 2. Curves of Re(μ) = 0 and Im(μ) = 0 of partially coherent SiG non-vortex beams (**a**–**c**) and partially coherent SiG vortex beams (**d**–**f**) in atmospheric turbulence at different propagation distance z.

vortex for partially coherent SiG non-vortex beams propagating through atmospheric turbulence. Figures 2**d**–2**f** indicate that coherent vortices are created when partially coherent SiG vortex beams propagate through atmospheric turbulence, and the numbers of coherent vortices will increase with the gradual increment of the propagation distance. From Figs. 1 and 2 we find that the propagation of partially coherent SiG non-vortex beams will not create coherent vortices in both free space and atmospheric turbulence. In contrast, for partially coherent SiG vortex beams, coherent vortices are observed in both free space and atmospheric turbulence, thus we come to a conclusion that the creation of the coherent vortex depends on the vortex beams.

Figure 3 gives the position of coherent vortices of partially coherent SiG vortex beams propagating through free space and atmospheric turbulence *versus* the propagation distance z, and the illustrations give the contour lines of the phase which corresponds to the coherent vortices. The other calculation parameters are the same as in Fig. 2. From Figs. 3**a** and 3**b**, we can see that at the beginning there is only one coherent vortex (being marked as no. 1) whose topological charge is +1 for partially coherent SiG vortex beams propagating through free space, then a new coherent vortex (no. 2) occurs at z = 1.75 km whose topological charge is –1. The difference between the two coherent vortices is that no.1 is inherent coherent vortex of the vortex beam which

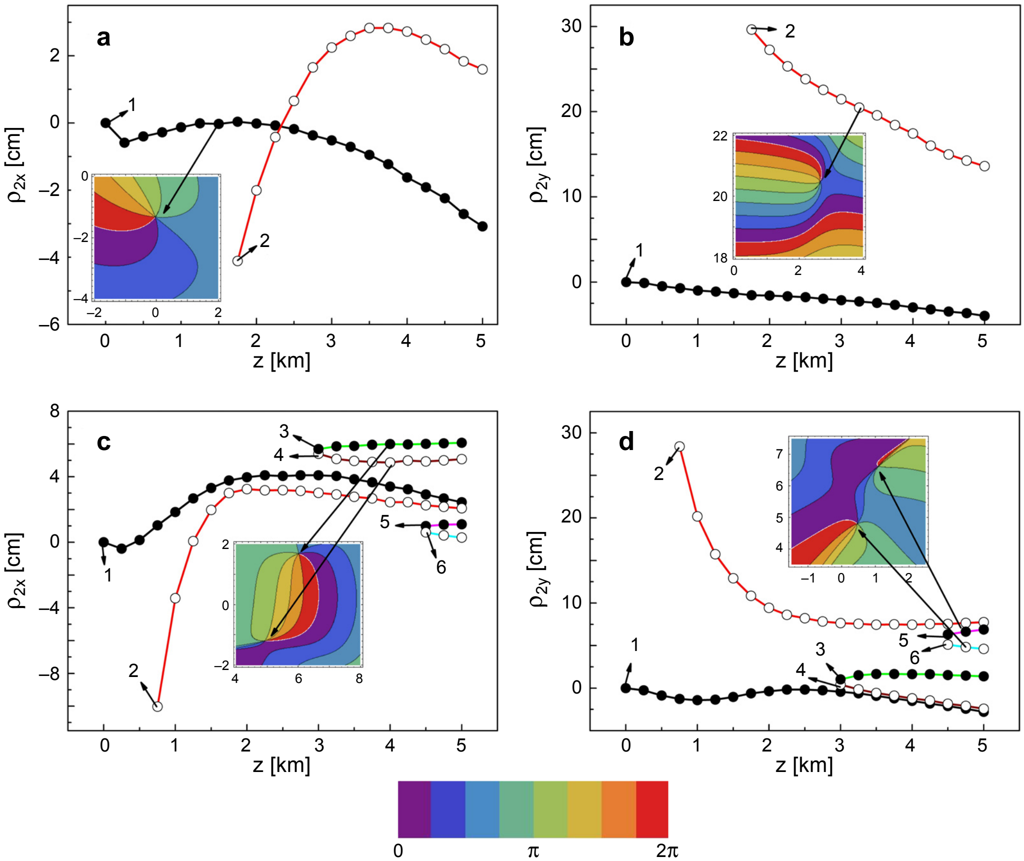

Fig. 3. The position of coherent vortices of partially coherent SiG vortex beams *versus* the propagation distance z when propagating through free space (**a**–**b**) and atmospheric turbulence (**c**–**d**); $m = +1$ (●) and $m = -1$ (○).

exists at the source plane $z = 0$, whereas, no. 2 is created when partially coherent SiG vortex beams propagate through free space. From Figs. 3**c** and 3**d** we see that there also exist no. 1 and no. 2 coherent vortices which are similar to that in Figs. 3**a** and 3**b** for partially coherent SiG vortex beams propagating through atmospheric turbulence. In addition, two pairs of coherent vortices marked as nos. 3, 4, 5 and 6 are created at $z = 3$ and 4.5 km.

From Figures 1 to 3 it is shown that the coherent vortices are grouped into three classes according to the creation: the first are their inherent coherent vortices of the vortex beam, as suggests no.1 coherent vortex in Fig. 3; the second is created by the vortex beam itself during transmission process in free space, as show no. 2 coherent vortices in Fig. 3; the third is created by the atmospheric turbulence inducing the vortex beam, namely, the coherent vortex is the outcome of the combination of the vortex beam with the atmospheric turbulence, as show nos. 3–6 coherent vortices in Figs. 3**c** and 3**d**.

4. Conclusion

In this paper, by using the extended Huygens–Fresnel principle, the analytical expressions for the cross-spectral density function of partially coherent SiG vortex beams and partially coherent SiG non-vortex beams propagating through free space and atmospheric turbulence have been derived, and used to study the evolution behavior of coherent vortices through free space and atmospheric turbulence, which is useful for us to classify the coherent vortices. We find that the creation of the coherent vortex depends on the vortex beams, and the coherent vortices are grouped into three classes according to the creation: the first is their inherent coherent vortices of the vortex beam; the second is created by the vortex beam itself during transmission process in free space; and the third is created by the atmospheric turbulence inducing the vortex beam. These results have potential applications in optical vortex communication systems.

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References

[1] LEE W.M., YUAN X.-C., CHEONG W.C., *Optical vortex beam shaping by use of highly efficient irregular spiral phase plates for optical micromanipulation*, Optics Letters **29**(15), 2004, pp. 1796–1798.

[2] THIDÉ B., THEN H., SJÖHOLM J., PALMER K., BERGMAN J., CAROZZI T.D., ISTOMIN YA.N., IBRAGIMOV N.H., KHAMITOVA R., *Utilization of photon orbital angular momentum in the low-frequency radio domain*, Physical Review Letters **99**(8), 2007, article 087701.

[3] SWARTZLANDER G.A., JR., GBUR G., *Singular optical phenomena in nature*, Proceedings of SPIE **7057**, 2008, article 705703.

[4] KURZYNOWSKI P., BORWIŃSKA M., MASAJADA J., *Optical vortex sign determination using self-interference methods*, Optica Applicata **40**(1), 2010, pp. 165–175.

[5] XINLUN CAI, JIANWEI WANG, STRAIN M.J., JOHNSON-MORRIS B., JIANGBO ZHU, SOREL M., O'BRIEN J.L., THOMPSON M.G., SIYUAN YU, *Integrated compact optical vortex beam emitters*, Science **338**(6105), 2012, pp. 363–366.

[6] HUIYUN WU, SHEN SHENG, ZHISONG HUANG, HUA WANG, SIQING ZHAO, XIEGU XU, ZHENHAI SUN, RUI XIAO, *Study on power efficiency of vortex beam propagation through an optical system with phase optimization*, Optica Applicata **42**(3), 2012, pp. 597–611.

[7] YUANJIE YANG, YUAN DONG, CHENGLIANG ZHAO, YANGJIAN CAI, *Generation and propagation of an anomalous vortex beam*, Optics Letters **38**(24), 2013, pp. 5418–5421.

[8] GBUR G., TYSON R.K., *Vortex beam propagation through atmospheric turbulence and topological charge conservation*, Journal of the Optical Society of America A **25**(1), 2008, pp. 225–230.

[9] YALONG GU, GBUR G., *Measurement of atmospheric turbulence strength by vortex beam*, Optics Communications **283**(7), 2010, pp. 1209–1212.

[10] CHEN Z., LI C., DING P., PU J., ZHAO D., *Experimental investigation on the scintillation index of vortex beams propagating in simulated atmospheric turbulence*, Applied Physics B: Lasers and Optics **107**(2), 2012, pp. 469–472.

[11] FANG GUI-JUAN, PU JI-XIONG, *Propagation properties of stochastic electromagnetic double-vortex beams in a turbulent atmosphere*, Chinese Physics B **21**(8), 2012, article 084203.

[12] DIPANKAR A., MARCHIANO R., SAGAUT P., *Trajectory of an optical vortex in atmospheric turbulence*, Physical Review E **80**(4), 2009, article 046609.

[13] LÜ BAI-DA, LI JIN-HONG, *Comparative study of partially coherent vortex beam propagation through atmospheric turbulence along a uplink path and a downlink path*, Acta Physica Sinica **60**(7), 2011, article 074205.
[14] ZAHID M., ZUBAIRY M.S., *Directionality of partially coherent Bessel–Gauss beams*, Optics Communications **70**(5), 1989, pp. 361–364.
[15] ANDREWS L.C., PHILLIPS R.L., *Laser Beam Propagation through Random Media*, SPIE Press, Bellingham, 2005.
[16] WANG S.C.H., PLONUS M.A., *Optical beam propagation for a partially coherent source in the turbulent atmosphere*, Journal of the Optical Society of America **69**(9), 1979, pp. 1297–1304.
[17] EYYUBOGLU H.T., BAYKAL Y., *Analysis of reciprocity of cos-Gaussian and cosh-Gaussian laser beams in a turbulent atmosphere*, Optics Express **12**(20), 2004, pp. 4659–4674.
[18] ISHIMARU A., *Phase fluctuations in a turbulent medium*, Applied Optics **16**(12), 1977, pp. 3190–3192.
[19] LI J., DING C., LÜ B., *Generalized Stokes parameters of random electromagnetic vortex beams propagating through atmospheric turbulence*, Applied Physics B: Lasers and Optics **103**(1), 2011, pp. 245–255.
[20] GRADSHTEYN I.S., RYZHIK I.M., *Table of Integrals, Series and Products*, Academic, New York, 2007.
[21] MANDEL L., WOLF E., *Optical Coherence and Quantum Optics*, Cambridge University Press, Cambridge, 1995.
[22] GBUR G., VISSER T.D., *Coherence vortices in partially coherent beams*, Optics Communications **222**(1–6), 2003, pp. 117–125.
[23] FREUND I., SHVARTSMAN N., *Wave-field phase singularities: the sign principle*, Physical Review A **50**(6), 1994, pp. 5164–5172.

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