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Different approaches to phase restoration of distant complex optical fields

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The paper presents principal approaches to diagnosing the structure forming skeleton of the complex optical field. An analysis of optical field singularity algorithms depending on intensity discretization and image resolution has been carried out. An optimal approach is chosen, which allows to bring much closer the solution of the phase problem of localization of speckle-field special points. A possible approach to diagnosing the signs of zero amplitudes has been offered.

Keywords: phase skeleton, saddle points, gradient lines.

1. Introduction

Great attention has been paid to solving the phase problem in optics, particularly to tackling the problems of diagnosing the structure of objects in microscopy, astrophysics and biomedicine [1–4]. The phase problem involves restoring spatial (coordinate) phase distribution in complex fields of the speckle-field type [5, 6]. The state of these problems, the achievements and prospects in this field are set out in a number of reviews [7–9] and original publications [10–12]. A separate new and promising line of solving the phase task, *i.e.*, finding spatial (coordinate) phase distribution in complex fields of the speckle type, is offered by singular optics. The approaches [13] in the framework of singular optics are based on the concepts of:

– the so-called field "skeleton" with "reference" structure-forming points of the field amplitude zeros;

– the visualization of the sign principal in the coordinate (spatial) distribution of the wave dislocation, *i.e.* of the field amplitude zeros;

– the interconnection of intensity spatial distribution and field phase distribution.

This paper deals with the elaboration of these concepts and presents different methods for tackling the problem of diagnosing spatial (coordinate) phase distribution using registered intensity distribution in complex optical fields. The results of the computer simulation of the experiment, fulfilled within the framework of scalar approximation, form the basis of the investigation. The algorithm of solving the phase problem in the approximation of scalar coherent optical fields can be divided into several stages. The first one is to obtain complex amplitude-phase transformations in fields, which are generally called developed speckle fields with statistically distributed zeros of the field amplitude (phase dislocations). The second one is a search for optimal algorithms for diagnosing the zeros of speckle fields (of complex spatial intensity distribution), registered with the help of a CCD camera. The third one is searching for approaches and methods of restoring the phase coordinate distribution of the field by defining the principles and algorithm, which can make it possible to restore the phase grid (the coordinate phase distribution grid) in a complex speckle field in a justified optimal way.

As a rule, phase problems of the above described type are solved rather successfully in the concrete case of using a reference field (*i.e.*, holography methods) [14–16]. Holographic techniques are among the most popular methods that have been proposed to measure the phase of the optical wave. While holographic techniques have been successfully applied in certain areas of optical imaging, they are generally difficult to implement in practice [17]. In our case of remote diagnosing the possibility of using a concrete reference field is excluded.

The existing methods of phase retrieval rely on all kinds of *a priori* information about the signal, such as positivity, atomicity, support constraints, real-valuedness, and so on [18–20]. Direct methods are limited in their applicability to small-scale problems due to their large computational complexity. There is a number of mathematical and experimental approaches which bring us closer to tackling the problem of restoring the optical field distribution phase, including fully coherent or practically coherent sources. But neither of the existing approaches allows to reproduce the phase distribution of the distant complex speckle-field.

Hence, the development of algorithms for signal restorations using magnitude measurements is still a very active field of research. The given work offers a new approach to restoring the phase of an optical field with complex intensity distribution, including zero amplitudes.

2. Algorithms of phase distribution and singularity line reconstruction

Let the CCD camera enable us to change the spatial intensity picture, which can be presented by a set of pixels. The figure presents a set of pixels. In the general case, each of the pixels under consideration is surrounded by eight other neighbouring pixels (Fig. 1).

The arrows in Fig. 1 point the direction of the gradient vectors of intensity. They are defined at each point by the equation

$$\mathrm{grad}(I) = \frac{\partial I}{\partial x}\mathbf{i} + \frac{\partial I}{\partial y}\mathbf{j} \tag{1}$$

where **i**, **j** – the unit vectors along the x- and the y-axis, respectively, I – the intensity.

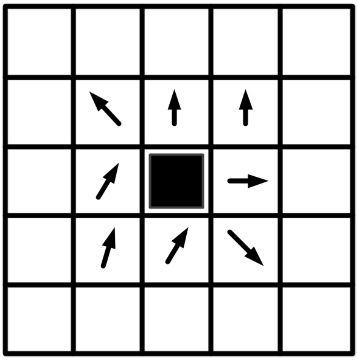

Fig. 1. Schematic sketch of a set of pixels.

If a standard camera is used, the intensity of each pixel varies from 0 to 255, *i.e.* it can take 256 different values. Three possible algorithms of defining the localization places of the field amplitude zeros were investigated.

3. Possible approximations of optical field skeleton restoration

The first approach to finding the optical field phase singularities is based on the assumption: the point under review is marked as a potential point of field phase singularity if there are two intensity gradient vectors in the vicinity of the point and the angle between them is 180°, *i.e.*, two gradient vectors are oppositely directed. The effect of changing the image resolution (of the pixel size) upon the potentialities of diagnosing the points of amplitude zeros (singularities) is considered. Pictures of the size of 200×200, 600×600 pixels were studied. The influence of intensity discretizations (the number of possible intensity values for each pixel) is also investigated. The number of values is 256 (a standard camera), 500 and 1000. Corresponding pictures are shown in Fig. 2. The white and black lines on them are the zero lines of the real and imaginary part of the field complex amplitude, which have been obtained by calculation. The intersection points of these lines correspond to the field singularity (a well-known classic diagnostic indicator). The areas marked in bold type and points marked by triangles have been obtained using the offered algorithm.

To understand better the results obtained by restoring phase singularities, proposed by the algorithm, the statistics of exactness of optical field skeleton restoration is shown in Fig. 3.

The given algorithm can be useful in searching for amplitude zero points in the field, but only if intensity discretization is absent. In this case many "excessive" points are diagnosed.

At discretization the possibility to find singularity grows worse, particularly with the increase of the size of pictures. The accuracy of restoring singularity points essentially depends on the size of pictures and the discretization value (Fig. 4). In general, being not high, the given algorithm cannot be used as an estimating one for constructing

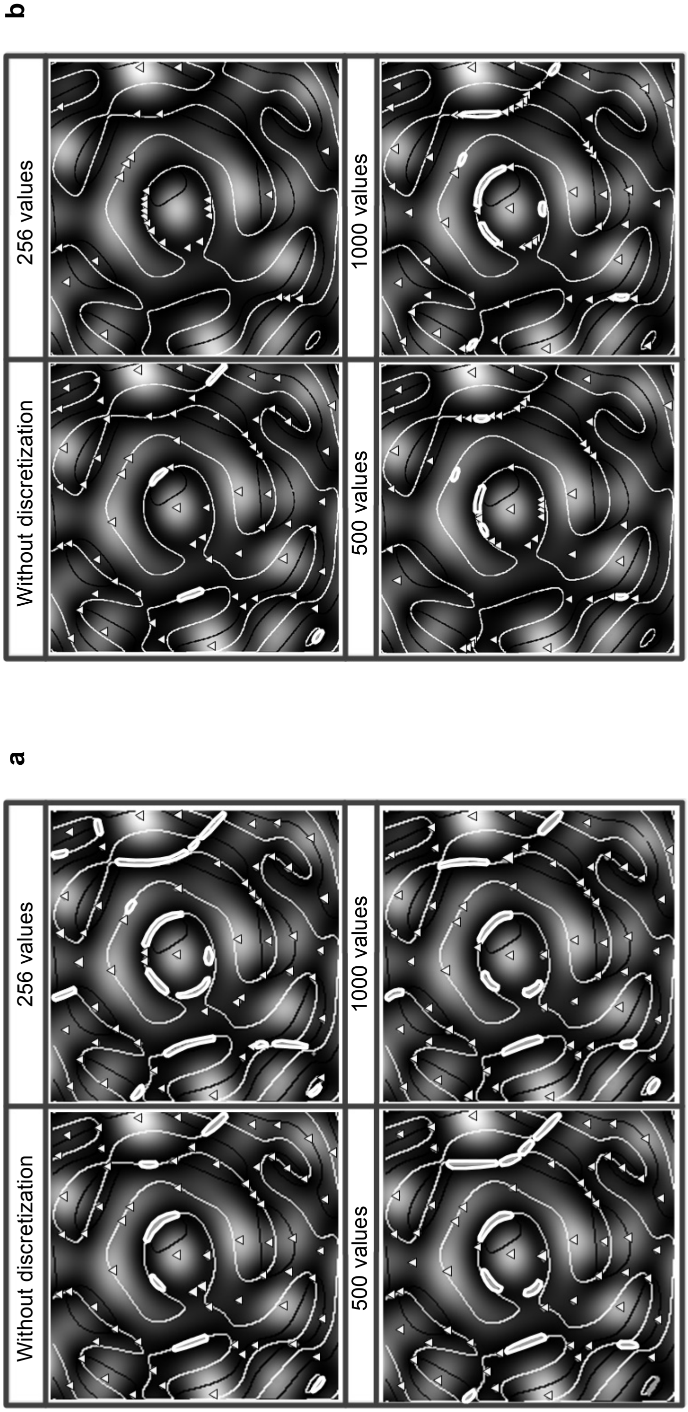

Fig. 2. Identification of singularity with the image size 200×200 (**a**), and 600×600 (**b**).

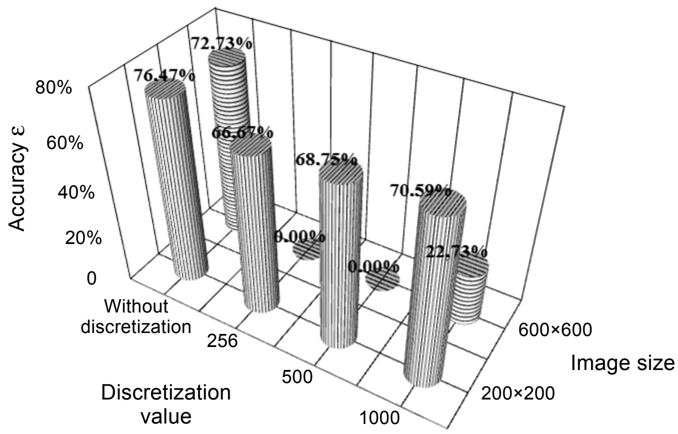

Fig. 3. The accuracy of singularity points restoration.

the optical field skeleton. The opportunity to find singularity worsens, particularly with the increase both of the size of pictures and the discretization value (Fig. 3).

The second step in solving the phase field restoring problem is searching for points with a zero intensity, *i.e.*, searching for pixels informing that the intensity in the given point is equal to zero. The effect caused by the change in the image resolution (the values are in pixels) was considered. The pictures with the size of 200×200, 600×600 with different intensity discretization – 256 (a standard camera), 500, 1000 were considered. This algorithm makes it possible to search for fields containing singularity. The algorithm is practically insensitive to the size of the image, but dependent on the discretization. The increase in the discretization makes it possible to predict a greater number of singular points. But in this case, regions without singularity are found as well. It is quite possible to use it in combination with other algorithms.

The possibility of obtaining more sensitive and tolerant methods for tacking the problem of diagnosing structure-forming points in a complex speckle-field was considered at the next stage. The productivity and efficiency of using the procedure of smoothing was investigated.

The third step considered here is a logical continuation of the first approach, and is the most promising one of all approaches discussed earlier. Two alternative procedures of smoothing – "cross" smoothing by two points and "circle" smoothing by two points were proposed (Fig. 4).

The point under consideration was marked off if there were two gradient vectors in the vicinity with an angle of 180° between them and the point intensity was zero. The procedure of "smoothing" was performed in addition. The effect of changing the image resolution was studied (the sizes are given in pixels). The pictures with the size of 200×200 and 600×600 were chosen. The effect of the intensity discretion (the number of possible intensity values for each pixel) was studied as well. The number of values was 256 (standard camera) and 1000. The obtained results for the pictures with the size of 600×600 are shown in Fig. 5.

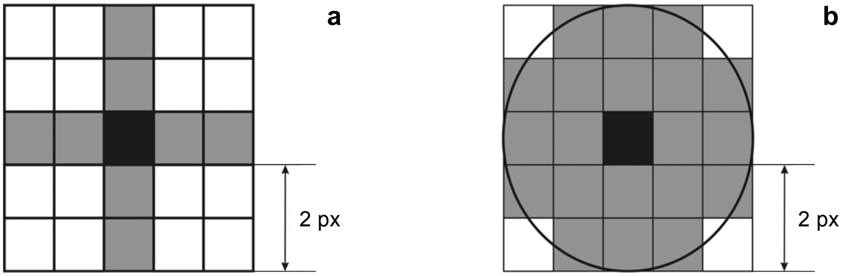

Fig. 4. "Cross" (**a**) and "circle" (**b**) smoothing by two points. The black point is the point for which the smoothing is carried out. The grey points are the points which take part in the process of smoothing. The result is the arithmetical mean of the sum of intensities of all marked off points.

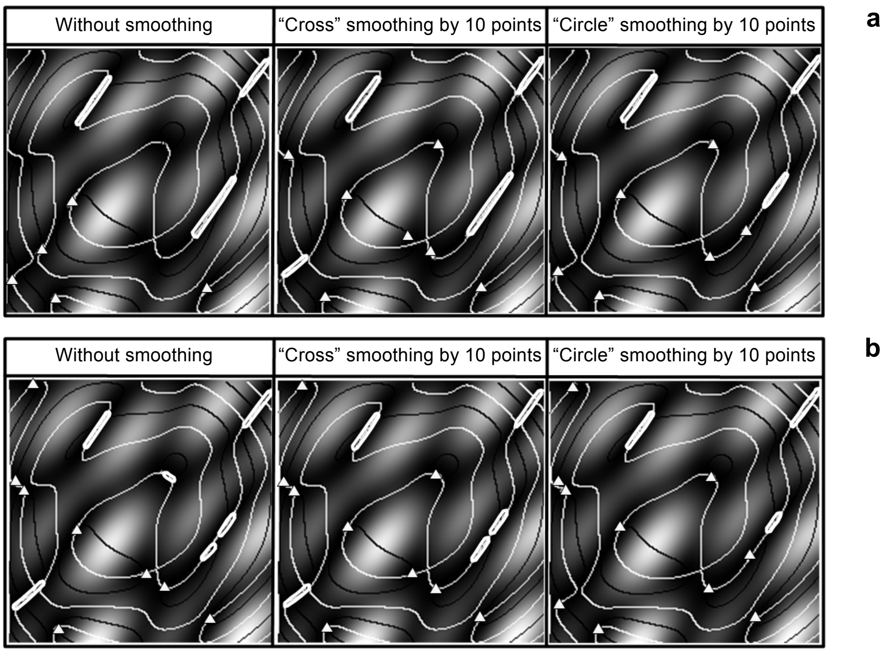

Fig. 5. Identification of singularities for different smoothing processes: the size of images 600×600 and intensity discretization 256 (**a**) and the size of images 600×600 and intensity discretization 1000 (**b**).

The white and black lines on them are the zero lines of the real and imaginary parts of the complex amplitude. Their intersection points are the points of singularity. The areas marked in bold type and points marked by triangles are defined by the proposed algorithm. Smoothing provides greater scope for finding "zero points" for discrete image, especially in the case of "circle" smoothing by 10 points.

Optimum smoothing parameters depend upon the half-width of the image correlation function (in pixels) and the intensity discretization of the image pixels. In the given

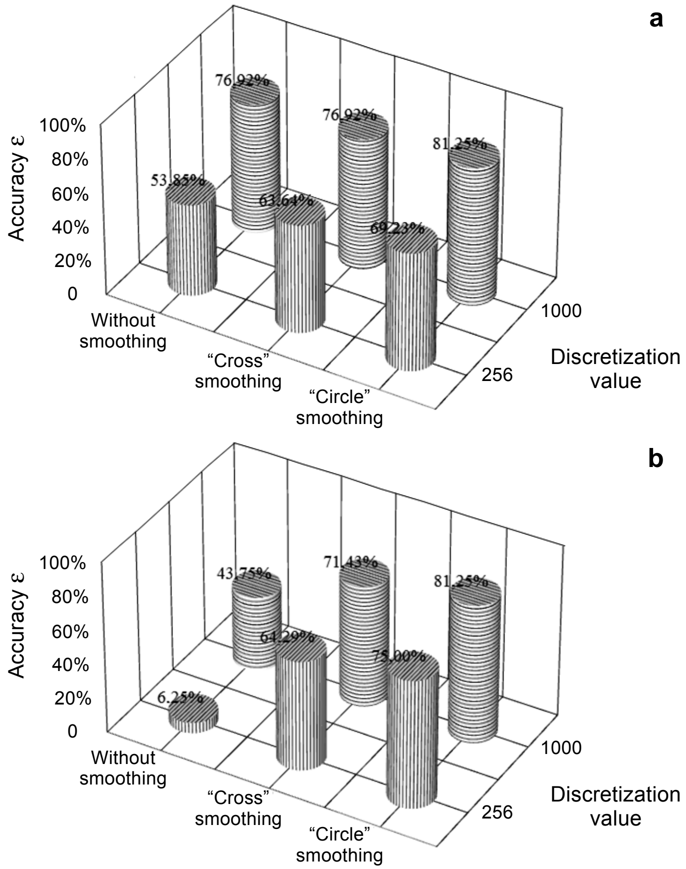

Fig. 6. Exactness of singularity reconstruction for images with the size of 200×200 (**a**) and 600×600 (**b**) at different intensity discretization and different smoothing procedures.

situation, the probability of faultless diagnostics of the image pixel intensities can be 82% (Fig. 6).

However the question of defining the sign of the field amplitude zero remains open and, correspondingly, the question of determining the trajectory of phase lines. In connection with this, the following algorithm was proposed.

4. Approximations by bicubic splines

The intensity distribution image was smoothed and interpolated by bicubic splines [21]. It is a standard independent method, which makes it possible to interpolate the digital image. The given splines must satisfy several conditions:

– equality of calculated values at nodal points,
– continuity of partial derivatives of the first order,
– continuity of mixed derivatives of the second order.

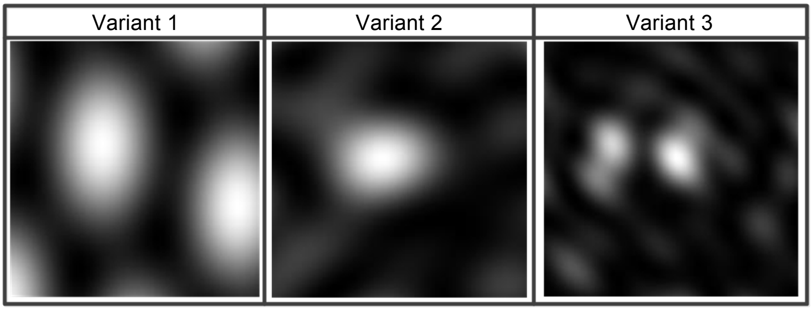

Fig. 7. Starting intensities.

The spline coefficients are calculated. It permits to perform interpolation. Intensity values in each pixel of the initial image are chosen as nodal points. Then lines are constructed: $\mathrm{d}I/\mathrm{d}x$ and $\mathrm{d}I/\mathrm{d}y$, where I is the intensity. The intersection points of these lines are singularity points, maxima and saddle points [22]. The effect of the image resolution change (sizes in pixels) upon the potentiality and exactness of diagnosing the localization places of intensity extrema and field phase singularity has been considered. The sizes of pictures are 200×200 and 600×600. The influence of the intensity discretization (the number of possible intensity values for each pixel) upon the feasibility of diagnosing the same field parameters has been investigated as well. The number of intensity values is 256 (a standard camera), 1000 and 10000 (the maximum possible, *i.e.* the practically continuous one). Three different intensity distributions have been considered (Fig. 7).

The exactness of reconstructing singularity points using the given algorithm for different sizes of images is shown in Fig. 8.

The interpolation of bicubic splines ensures about 90% probability for zero amplitude reconstruction. It allows to assert that it is advisable to use precisely this algorithm for optical field phase retrieval.

A distinguished feature of the lines $\mathrm{d}I/\mathrm{d}x = 0$ and $\mathrm{d}I/\mathrm{d}y = 0$ is their connection with the field "skeleton". The physical meaning of the given lines is the following: they form the intensity gradient if the gradient components x and y are equal to 0. A schematic sketch of it is offered in Fig. 9.

Figure 10 shows that the given lines are not invariant with respect to the system of coordinates. In other words, the position of the given lines will change with the rotation of the figure about the system of coordinates. At the same time, the points formed by the intersection of such lines, *i.e.*, the ones which meet the following conditions:

$$\begin{cases} \mathrm{d}I/\mathrm{d}x = 0 \\ \mathrm{d}I/\mathrm{d}y = 0 \end{cases} \tag{2}$$

create a system of points invariant to the change of the system of coordinates. The given invariance has a quite definite physical meaning. The thing is that special points in

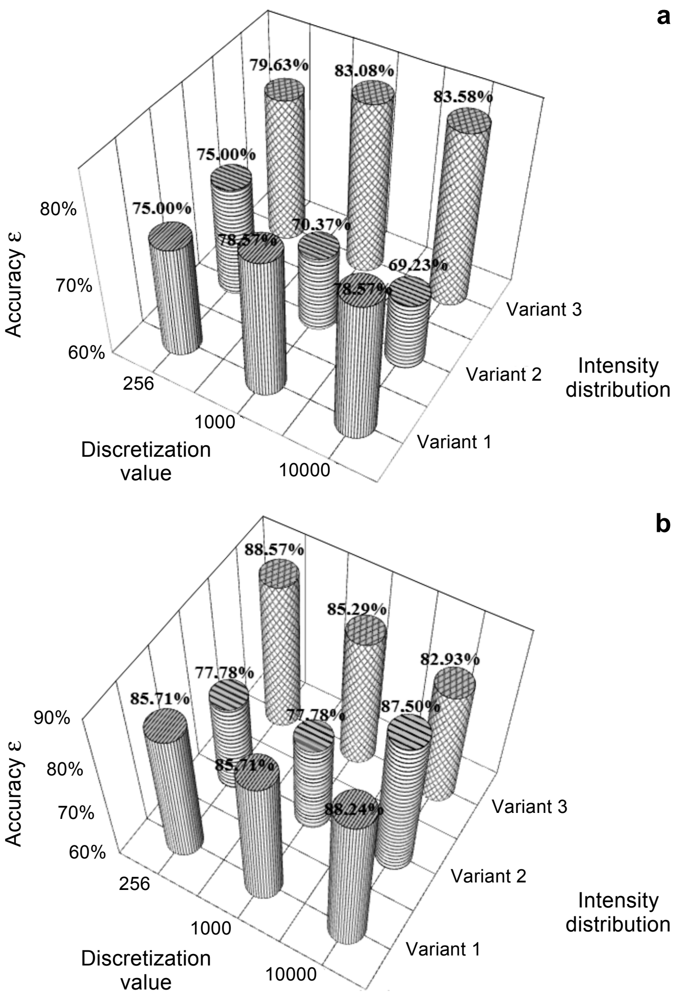

Fig. 8. Exactness of singularity reconstruction for images with the size of 200×200 (**a**) and 600×600 (**b**).

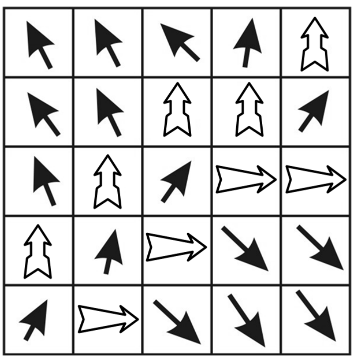

Fig. 9. Schematic representation of "zero" derivatives line.

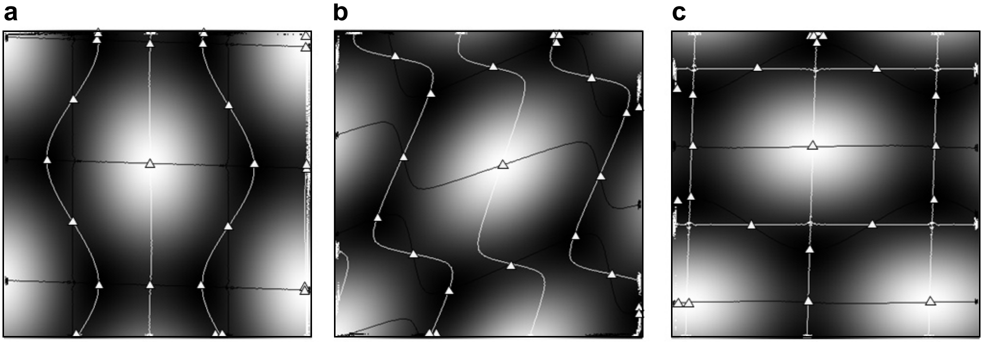

Fig.10. An example of figure rotation by 0° (**a**), 45° (**b**), 90° (**c**) and the behavior of lines of "zero" derivatives.

the intensity distribution, *i.e.*, focal maxima, local minima (including "singular" points) and saddle points are reference points of the structure forming skeleton of the field under review. The position of these points does not depend upon the position of the figure relative the coordinate system. Condition (2) is a criterion for defining similar "special" points in mathematics.

Correspondingly, the line system $dI/dx = 0$ and $dI/dy = 0$ is connected with the "singular" points and the field skeleton. An example of figure rotation by 0°, 45°, 90° is shown in Fig. 10.

The considered behavior of structure formation in the irradiating optical field, has a quite definite real "life" analogy if a coaxial plane reference wave with a smoothly changing phase is used for diagnosing complex speckle fields. At the same time we have to state that the diagnostics of the zero signs of the field amplitude and the signs of vortices still present some difficulties. Originally the sign principle was proposed by FREUND and SHVARTSMAN [16]. It was assumed that when moving along the connective line from one singularity to the other, the second singularity "changes" its sign after meeting with the "saddle point". It is absolutely clear that the availability of one

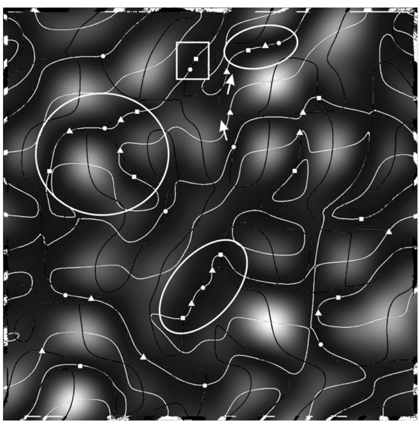

Fig. 11. Examples of intensity distribution and localization of "reference" points by the bicubic splines algorithm: image size 600×600, without discretization.

saddle point between singularities assures the difference of their signs. The cases are marked off by white ovals (Fig. 11). Correspondingly, in this case, which is confirmed by the results of the simulation, it can be stated with confidence that: *i*) the sign of one arbitrary phase singularity automatically determines the signs of all other wave field singularities, *ii*) if the sign of one field singularity is changed. the signs of all other singularities change automatically. It is obtained from the sign principle: if an even number of saddle points is found along the line, the singularity sign will be identical to that of the point from which we began. If the number of the saddle points is not even, the sign will be opposite.

The figures below illustrate both lines, singularities (small white squares and circulars – the form of the point corresponds to the singularity sign) and saddle points (marked off by white triangles) (Fig. 11).

But the results of simulation show that there can be situations when the sign principle and the conclusions following immediately from it fail. It is observed when moving along the line marked off by white arrows (the presence of an even number of saddle points between singularities of different signs). There are situations when it is impossible to diagnose the saddle point because there are cases when it does not exist at all (as it is in the case marked off by the big white square, Fig. 11.

5. Conclusions

The analysis of different approaches to restoring phase distribution of the complex optical field allows to single out the best method for speckle-field skeleton reconstruction. The suggested method allows not only to diagnose singularity and saddle points, but to define the signs of singularity as well. An optimal skeleton modeling algorithm of a random optical field with all possible intensity values has been proposed.

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