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NEAR POVERTY – DEFINITION, FACTORS, PREDICTIONS

SFERA BLISKO UBÓSTWA – DEFINICJA, CZYNNIKI, PROGNOZY

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Summary: The aim of the paper was to analyse near poverty in Poland. The first specific aim was to analyse the transitions into and out of near poverty in Poland using the Markov transition matrix. Three poverty states were considered: poverty, near poverty (an income of between 100 and 125 per cent of the poverty threshold is assumed in the paper) and above near poverty. The analysis was conducted for Poland based on the balanced panel from 2009 to 2015, the framework of the "Social Diagnosis" project. The second specific aim was to determine the factors that increase and decrease the odds of being in near poverty using binomial logistic regression.

Keywords: near poverty, poverty transitions, the Markov chain model, Shorrocks' mobility index, logistic regression.

Streszczenie: Celem artykułu była analiza sfery blisko ubóstwa w Polsce. Pierwszym celem szczegółowym była analiza przejść do i ze sfery ubóstwa w Polsce z użyciem macierzy przejścia Markowa. Rozważane były trzy stany: ubóstwo, blisko ubóstwa (dochody gospodarstw domowych od 100% do 125% przyjętej granicy ubóstwa) oraz poza zagrożeniem ubóstwem (dochody wyższe niż 125% przyjętej granicy ubóstwa). Analiza została przeprowadzona na podstawie zbilansowanego panelu 2009-2015 w ramach projektu „Diagnoza społeczna". Drugim szczegółowym celem było określenie czynników zwiększających i zmniejszających szanse pobytu w sferze blisko ubóstwa z wykorzystaniem dwumianowej regresji logistycznej.

Słowa kluczowe: sfera blisko ubóstwa, przejścia pomiędzy stanami ubóstwa, łańcuchy Markowa, indeks mobilności Shorrocka, regresja logistyczna.

1. Introduction

Poverty is a phenomenon which concerns, to a greater or lesser extent, individuals, families or households in all parts of the world. The actions of governments and social organizations are focused on helping the poor, especially households. There is a group of households living near the poverty line. Their incomes are little higher than the poverty line and these households usually do not receive any help. Actions should be focused on this group of households and to prevent their entry into poverty. The aim of this paper was to analyse near poverty in Poland. The first specific purpose was to analyse the transitions into and out of the state of near poverty in Poland. We estimated the probabilities of remaining in near poverty in two years, and the probabilities of exit to above near poverty and into poverty in two years. It can be assumed that the households stay in near poverty for a short time because it is hard to keep the income of the household in such a narrow range, only a little higher than poverty line.

The second purpose was to determine the factors that increase and decrease the odds of being in near poverty. These included the personal and household's characteristics in the model. It can be assumed that near poverty is related to the education of the household's head, which is a factor strongly changing the odds of being in poverty [Sączewska-Piotrowska 2016b] and in wealth [Sączewska-Piotrowska 2015].

Poverty is a wide category covering monetary and nonmonetary aspects. Attention will be focused on the economic aspects of poverty considered through the prism of income.

2. Near poverty in literature

The income of a household living in near poverty is close to, but not below, the poverty threshold. The term "close to" is not clearly defined. The first idea of near poverty was proposed by Mollie Orshansky [1966]. She defined the near poor as those living from 100 to 133 percent of the poverty threshold. Since 1971, the U.S. Census Bureau reports have contained information about near poor persons defined as those living from 100 to 125 percent of the poverty threshold. Other authors proposed different solutions concerning the definition of near poverty. For example, Ben-Shalom, Moffitt and Scholz [2011], defined the near poor as those living from 100 to 150 percent of the poverty threshold. Short and Smeeding [2012], defined the near poor very widely – as those living between 100 and 200 percent of the poverty threshold. In one of the most recent studies, Hokayem and Heggeness [2014], returned to the idea of the 125 percent definition. The first analysis of near poverty in Poland was conducted by Sączewska-Piotrowska [2016a]. The near poor were defined as those living from 100 to 125 percent of the poverty threshold.

It should be noted that poverty thresholds may be defined in a different way. The poverty threshold in the United States is absolute (i.e. it does not depend on the standard of living of the other members of society and is defined as the absolute needs standard remaining constant over time). The level of poverty in the European Union (EU) is measured using the relative poverty threshold (i.e. it depends on the standard of living of the other members of society and changes over time), mainly 60% of the national median income. There are also used thresholds set at 40% and 50% of the national median income. The Luxembourg Income Study literature and the analyses conducted by the Organization for Economic Cooperation and Development (OECD), often used the 50% threshold. In the EU the 40% threshold is referred to as "severe poverty" and the 60% threshold is sometimes called "near poverty" [Gornick, Jäntti 2009].

In the analysis the poverty threshold was set at 60% of the national income and the term "near poverty" was referred to as income between 100 and 125 percent of the 60% threshold. Therefore we used the European poverty threshold, but the U.S. way of defining near poverty.

3. Data and methods

The analysis of the transitions into and out of near poverty was conducted for Poland based on the balanced panel 2009-2015 (four waves: 2009, 2011, 2013, 2015) in the framework of the "Social Diagnosis" project [Council for Social Monitoring 2015]. The panel contains 3653 households. The analysis of the factors changing the odds of being in near poverty was conducted on the basis of data from the whole panel in 2015 (more than ten thousand households).

The poverty analysis adopted the economic definition of poverty. It was assumed that the indicator for poverty measurement is the net income of households in Poland in March/June 2009, 2011, 2013, and 2015. In order to take into account the differences in a household's size and its composition we calculated an equivalised income by dividing the household's income by its equivalent size using the modified OECD equivalence scale. This scale assigns 1 to the first adult of the household, 0.5 to each subsequent adult aged 14 or more and 0.3 to children (each person under 14). The poverty threshold was set at 60% of the median equivalised income.

The near poverty transitions were analysed using discrete-time Markov chains (also called Markov chains), which permitted to model the transition probabilities between discrete states with the aid of matrices.

Markov chain with the state space $S = \{1,2,...,r\}$ is [Bhat 2000, pp. 98-99] a stochastic process $\{X_n, n \geq 0\}$ with memoryless property

$$P(X_n = j \mid X_0 = i_0, X_1 = i_1, \ldots, X_{n-1} = i) = P(X_n = j \mid X_{n-1} = i) \qquad (1)$$

for all $j, i, i_0, ..., i_{n-2} \in S$.

The probability

$$P(X_n = j \mid X_{n-1} = i) = p_{ij}(n), \tag{2}$$

is called the (one-step) transition probability at step *n*.

There will be considered only time homogeneous (or having stationary transition probabilities) Markov chains, i.e. chains in which the transition probabilities from state i to state j are independent of time index *n*.

The collection of all one-step transition probabilities forms a matrix:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1r} \\ p_{21} & p_{22} & \cdots & p_{2r} \\ \cdots & \cdots & \cdots & \cdots \\ p_{r1} & p_{r2} & \cdots & p_{rr} \end{bmatrix}. \tag{3}$$

The matrix $\mathbf{P}$ is therefore a stochastic matrix ($\underset{i,j}{\forall} p_{ij} \geq 0$ and $\underset{i}{\forall} \sum_j p_{ij} = 1$).

With a known transition probability matrix it is possible to predict future distribution according to the formula:

$$\mathbf{d}_n = \mathbf{d}_{n-1}\mathbf{P}, \tag{4}$$

hence

$$\mathbf{d}_n = \mathbf{d}_0 \mathbf{P}^n, \tag{5}$$

where $\mathbf{d}_n = \begin{bmatrix} d_{n1} & d_{n2} & \cdots & d_{nr} \end{bmatrix}$ denotes distribution at time *n*, $d_{nj} = P(X_n = j)$.

A Markov chain with initial distribution $\mathbf{d}_0$ and with transition matrix $\mathbf{P}$ is ergodic if a limit [Podgórska et al. 2002, pp. 15-16]

$$\lim_{n\to\infty} \mathbf{d}_n = \lim_{n\to\infty} \mathbf{d}_0 \mathbf{P}^n = \mathbf{d}_0 \mathbf{E} = \mathbf{e} \tag{6}$$

exists independently from initial distribution $\mathbf{d}_0$. Matrix $\mathbf{E}$ is an ergodic stochastic matrix (has identical rows), vector $\mathbf{e}$ is a row of matrix $\mathbf{E}$.

Two two types of data were used to estimate the parameters of Markov chains: micro-data and macro-data. In the conducted analysis there were available detailed micro-data for several time periods. In this case the maximum likelihood estimators of the stationary transition probabilities over the entire sample period are given by [Anderson, Goodman 1957; Podgórska et al. 2002, p. 63]:

$$\hat{p}_{ij} = \frac{\sum_{t=1}^{m} v_{ij}(t)}{\sum_{t=1}^{m}\sum_{j=1}^{r} v_{ij}(t)}, \tag{7}$$

with $v_{ij}(t)$ denoting the number of individuals transitioning from state *i* to state *j* in all periods $\langle t-1, t\rangle, t = 1,2,...,m$.

Transition probability matrices are often used to measure income mobility. The commonly used measure of mobility is the Shorrocks' mobility index defined as [Shorrocks 1978]:

$$M_S = \frac{r - \mathrm{Trace}(\mathbf{P})}{r-1}, \tag{8}$$

where r is the number of states. Prais [1955] has shown that the mean exit time from state *i* (or the average length of stay in state *i*) is given by:

$$\mu_i = \frac{1}{1-p_{ii}}, \tag{9}$$

where p_{ii} is the probability that an individual will remain in state *i* for one period to the next period. Shorrocks' mobility index is the reciprocal of the harmonic mean of the mean exit times, normalized by the factor $\frac{r}{r-1}$.

In the analysis we used binomial logistic regression to determine the factors that change the odds of being in near poverty. The response variable *Y* had two categories: 1 if the household was near poor, 0 if the household was not near poor. Then [Jackowska, Wycinka 2011; Bieszk-Stolorz, Markowicz 2013]:

$$P(Y=1 \mid x_1, x_2, ..., x_s) = \frac{\exp\left(\alpha_0 + \sum_{l=1}^{s}\alpha_l x_l\right)}{1+\exp\left(\alpha_0 + \sum_{l=1}^{s}\alpha_l x_l\right)}, \tag{10}$$

where α_0, α_l are the parameters of the model $(l = 1,2,...,s)$. In the conducted analysis all the independent variables $X_1, X_2, ..., X_S$ were dummy variables (dummy coding uses only ones and zeros to convey all of the necessary information on group membership). To interpret the results of estimation an odds ratio was used. In the case of dummy variables, the odds ratio expresses how many times the odds that a given event will take place changes for the household, for which $X_l = 1$ compared to household, for which $X_l = 0$.

To test the statistical significance of the whole model likelihood ratio the (LR) test was used, and to test the statistical significance of each parameter, z statistics with $N(0,1)$ was used. The quality of the model was evaluated using $R^2_{McFadden}$. We also used the ROC (receiver operating characteristics) curve and the AUC (area under ROC curve) to evaluate the predictive power of the estimated model. The quality of classification was evaluated using sensitivity, specificity and accuracy[1].

All figures and calculations are performed in R program [R Development Core Team, 2016] using `gmodels` [Warnes et al. 2015], `msm` [Jackson 2016], `markovchain` [Spedicato et al. 2016], `OptimalCutpoints` [Lopez-Raton, Rodriguez-Alvarez 2015], and `verification` [NCAR – Research Applications Laboratory 2015] packages.

4. Transitions into and out of the near poverty

In the first step we calculated the near poverty rates and poverty rates from 2009 to 2015 in Poland. The results are shown in Figure 1.

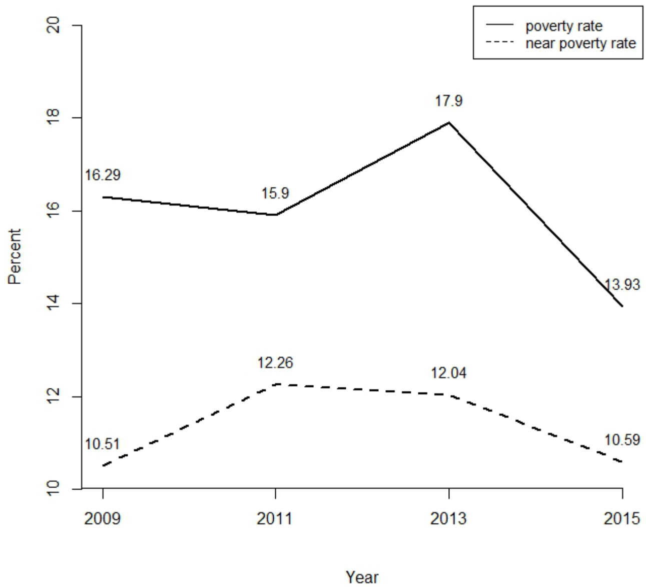

Fig. 1. Poverty and near poverty rates in Poland 2009-2015

Source: own study based on [Council for Social Monitoring 2015].

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[1] More about LR test and z statistics in Gruszczyński [2012], about $R^2_{McFadden}$ in McFadden [1977], about ROC curves, AUC, sensitivity, specificity and accuracy (also called count-R^2) in Sompolska-Rzechuła et al. [2014] and Sączewska-Piotrowska [2016b].

In 2009-2015 the percentage of near poor households was lower than the percentage of poor households. It should be noted that in the observation period the changes in poverty and near poverty rates were different. Only in 2015 the changes were the same – a decrease in the percentages of poor and near poor households. That was an echo of the improvement in the economic situation after the slight recession which took place in Poland in late 2012 and early 2013.

The tables show the near poverty entry rates from poverty and from above near poverty (Table 1), and near poverty exit rates to poverty and to above near poverty (Table 2).

Table 1. Near poverty entry rates (%)

Near poverty entry rate	2009-2011	2011-2013	2013-2015
from poverty	15.97	12.74	18.50
from above near poverty	7.55	7.39	4.10

Source: own study based on [Council for Social Monitoring 2015].

Table 2. Near poverty exit rates (%)

Near poverty exit rate	2009-2011	2011-2013	2013-2015
to poverty	22.66	24.55	16.14
to above near poverty	38.02	37.05	47.27

Source: own study based on [Council for Social Monitoring 2015].

It is evident that between 2013 and 2015 there were positive changes in the economic situation – on the one hand, the highest percentages of households entered from poverty to near poverty and exited from poverty to above near poverty; on the other hand, the lowest percentages of households entered into near poverty from above near poverty and exited from near poverty to poverty. The most negative changes in poverty states were between 2011 and 2013. The exception was the entry rate from above near poverty (the highest value between 2009 and 2011).

In the next step we estimated the transition probability matrix:

$$\hat{\mathbf{P}} = \begin{bmatrix} 0.5983 & 0.1585 & 0.2432 \\ 0.2107 & 0.3805 & 0.4088 \\ 0.0485 & 0.0638 & 0.8877 \end{bmatrix}.$$

The elements of the estimated matrix represent the transition probabilities from one poverty state to another. The main diagonal elements of the matrix indicate the probability that the household will remain in the same poverty state in the next period (two years). It can be seen that the highest probability of remaining in the same state for two years is in the case of above near poverty, and the lowest – in the case of near poverty. The probability of the worsening economic situation of the near poor households in two years is lower than the probability of an improvement of the situation (0.21 and 0.41, respectively).

From the transition matrix there was computed the value of overall mobility measure – Shorrocks' mobility index. It measures the average probability across all poverty states that the household will leave its initial state in two years. The computed value of 0.567 means that the mobility of households due to belonging to a poverty state is quite high.

Based on the transition matrix we computed the mean exit times from the poverty states. The average length of stay is the longest in the case of above near poverty (18 years). Much shorter is the mean exit time from poverty (5 years) and definitely the shortest is the mean exit time from near poverty (3.2 years).

Based on the actual distribution of households by poverty state and based on transition probabilities we calculated the predicted distributions of poverty states (Table 3).

Table 3. Actual and predicted distributions of poverty states

State	Distribution			
	2015 (actual)	2017	2019	Ergodic
Poverty	0.1393	0.1423	0.1447	0.1488
Near poverty	0.1059	0.1105	0.1122	0.1139
Above near poverty	0.7547	0.7472	0.7431	0.7373

Source: own study based on [Council for Social Monitoring 2015].

The predicted distributions in 2017 and in 2019 are very similar to the actual distribution. The Markov chains are ergodic and reach their limit distributions. This means that in the long run the poverty state will not depend on the current poverty state. The difference between actual and ergodic distribution is not high and indicates a small modification among the poverty states.

5. Factors of near poverty

We estimated the parameters of the binomial logistic regression for near poverty. The independent variables were related to the personal characteristics of the household's head and to the household's characteristics. The results are shown in Table 4.

The model as a whole was statistically significant ($\chi^2 = 513.192, p < 0.001$). Attention should be paid to the low value of $R^2_{McFadden}$, which indicates the poor goodness of fit (values from 0.2 to 0.4 represent an excellent fit [McFadden, 1977]). The

Table 4. Results of logistic regression model for near poverty in Poland

Variable	Parameter's estimation	Odds ratio
Intercept	–3.286***	x
Gender of household's head:		
Male	Ref.	
Female	0.430***	1.538
Age of household's head:		
<35	Ref.	
35-44	–0.001	0.999
45-59	–0.287	0.751
60 and more	–0.203	0.816
Education of the household's head:		
Low education (lower secondary, primary and incomplete primary)	1.767***	5.852
Basic vocational education	1.536***	4.645
Secondary education	1.083***	2.953
Tertiary education	Ref.	
Place of residence:		
Urban	Ref.	
Rural	0.185*	1.203
Number of household's members:		
1	Ref.	
2	–0.755***	0.816
3	–0.328**	0.470
4	–0.236	0.720
5	0.090	0.789
6 and more	–0.024	1.094
Socio-economic group:		
Household of employees	Ref.	
Household of farmers	–0.019	0.981
Household of the self-employed	–0.281	0.755
Household of retirees and pensioners	0.233*	1.262
Household of living on unearned sources	–0.333	0.717
Labour-force status:		
Household with unemployed person	0.196*	1.216
Household without unemployed person	Ref.	
Disabled person in household:		
Household with disabled person	0.211**	1.235
Household without disabled person	Ref.	
Region:		
Centralny	Ref.	
Południowy	–0.390***	0.677
Wschodni	0.143	1.154
Północno-zachodni	–0.063	0.939
Południowo-zachodni	–0.106	0.890
Północny	–0.136	0.873
LR	513.192***	
$R^2_{McFadden}$	0.072	
AUC	0.698***	

Ref. – reference category
Significance codes: $*p < 0.05; ** p < 0.01; *** p < 0.001$.

Source: own study based on [Council for Social Monitoring 2015].

clearest impact no belonging to near poverty came from the education of the household's head. The odds of being in near poverty were almost three times higher, 4.6 times higher and almost six times higher in households with the head having secondary, basic vocational and low education relative to households with a highly educated head. Two-person households had about 18% lower odds and three-person households had 53% lower odds of being in near poverty than single households. The odds were also lower in households from Południowy Region relative to Centralny Region. The odds were higher in households with a female head (relative to households with a male head), in households in rural areas (relative to households in urban areas), in households of retirees and pensioners (relative to households of employees), in households with an unemployed and disabled person (relative to households without an unemployed and disabled person, respectively).

The AUC estimate for the model was significantly higher than 0.5 and the value of 0.698 means almost acceptable classification obtained by the model [Hosmer, Lemeshow 2000, p. 162; Kumari, Rajnish 2015][2]. The AUC higher than 0.5 is also shown in Figure 2 (the ROC curve is located higher than straight line Sensitivity= 1-Specificity and thereby AUC is higher than 0.5).

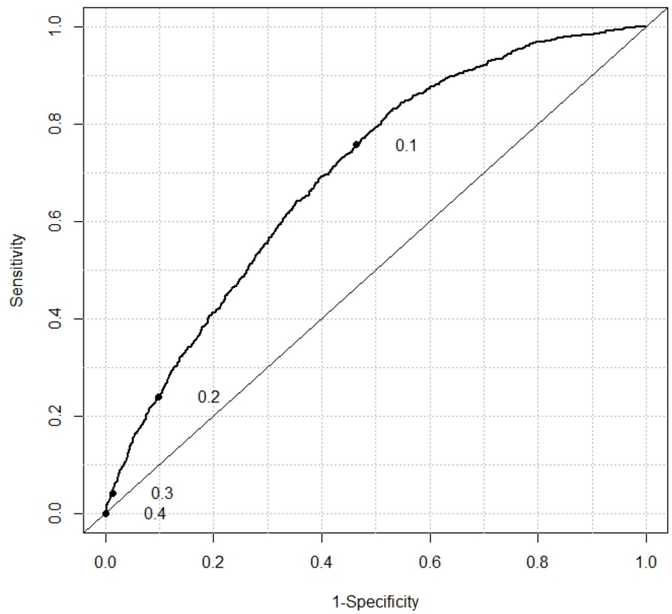

Fig. 2. The ROC curve

Source: own study based on [Council for Social Monitoring 2015].

[2] Fair classification: $0.6 \le AUC < 0.7$, acceptable classification: $0.7 \le AUC < 0.8$.

The optimum cut-off point (value 0.110) was identified as the closest point on the ROC curve to (0.1). For the optimal cut-off point we calculated sensitivity, specificity and accuracy (Table 5).

Table 5. Classification for cut-off point 0.110

Observed values	Expected values		
	$\hat{y}_i = 0$	$\hat{y}_i = 1$	Total
$y_i = 0$	5 467	3 588	9 055
$y_i = 1$	352	785	1 137
Total	4 443	1 737	10 192
Specificity 60.38%, sensitivity 69.04%, accuracy 61.34%			

Source: own study based on [Council for Social Monitoring 2015].

The proportion of near poor households classified by the model as near poor was equal to 69.04% (sensitivity), the proportion of not near poor households classified as not near poor was equal to 60.38% (specificity) and the proportion of the total number of households that were correctly classified was equal to 61.34% (accuracy). Therefore the predictions determined on the basis of the considered model should be treated very carefully. There is a strong possibility of classifying the near poor household as not near poor, and vice versa.

6. Conclusions

Based on the conducted analysis it can be concluded that in 2009-2015 the near poverty rate was lower than the poverty rate. The most positive changes in the economic situation of the near poor took place between 2013 and 2015. The probability that a household will remain in near poverty for two years is lower than the probability of remaining in poverty and above near poverty. The probability that a household will exit in two years from near poverty to poverty is lower than the probability of an exit from near poverty to above near poverty. The households are mobile due to belonging to poverty state.

The aim of the paper was also to determine the factors that increase and decrease the odds of being in near poverty using logistic regression. It should be noted that in 2015 all the extracted groups of the households due to the education of the household's head had higher odds to be in near poverty than households with a highly educated head. Therefore the influence of the age of the household's head is just as clear as in the case of poverty and wealth.

In further research there will be used ordinal logistic regression with response variable having three categories: poor household, near poor household and above near poor household. This approach will allow to evaluate the odds of being in higher poverty categories versus being in lower poverty categories. There will be also used event history models which allow to estimate the survival and hazard functions, and to determine the factors changing the odds for exiting from near poverty and for entering near poverty.

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