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VISUALIZATION OF LINEAR ORDERING RESULTS FOR METRIC DATA WITH THE APPLICATION OF MULTIDIMENSIONAL SCALING

WIZUALIZACJA WYNIKÓW PORZĄDKOWANIA LINIOWEGO DLA DANYCH METRYCZNYCH Z WYKORZYSTANIEM SKALOWANIA WIELOWYMIAROWEGO

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Summary: The article discusses the two-step research procedure allowing the visualization of linear ordering results for metric data. In the first step, as a result of the application of multidimensional scaling (see [Borg, Groenen 2005; Mair et al. 2016]) the visualization of objects in two-dimensional space is obtained. In the next step, the linear ordering of a set of objects is carried out based on the Euclidean distance from the pattern (ideal) object. The suggested approach expanded the possibilities for the interpretation of the linear ordering results of a set of objects. The article applies the concept of isoquants and the path of development (the shortest way connecting a pattern and an anti-pattern object) proposed by [Hellwig 1981]. The graphical presentation of the linear ordering results based on this concept was possible for two variables only. The application of multidimensional scaling expanded the applicability of the results of linear ordering visualization for *m* variables. The suggested approach is illustrated by an empirical example with the application of R environment script.

Keywords: linear ordering, multidimensional scaling, distance measures, composite measures, R environment.

Streszczenie: W artykule zaproponowano dwukrokową procedurę badawczą pozwalającą na wizualizację wyników porządkowania liniowego. W pierwszym kroku w wyniku zastosowania skalowania wielowymiarowego (zob. [Borg, Groenen, 2005; Mair i in. 2016]) otrzymuje się wizualizację obiektów w przestrzeni dwuwymiarowej. W następnym kroku przeprowadza się porządkowanie liniowe zbioru obiektów na podstawie odległości Euklidesa od wzorca rozwoju. Zaproponowane podejście rozszerzyło możliwości interpretacyjne wyników porządkowania liniowego zbioru obiektów. W artykule wykorzystano koncepcję izokwant i ścieżki rozwoju (osi zbioru – najkrótszej drogi łączącej wzorzec i antywzorzec rozwoju) zaproponowaną w pracy [Hellwig 1981]. Graficzna prezentacja wyników porządkowania liniowego w tej koncepcji możliwa była dla dwóch zmiennych. Zastosowanie skalowania wielowymiarowego rozszerzyło możliwości zastosowania wizualizacji wyników porządkowania liniowego dla *m* zmiennych. Zaproponowane podejście zilustrowano przykładem empirycznym z zastosowaniem skryptu przygotowanego w środowisku R.

Słowa kluczowe: porządkowanie liniowe, skalowanie wielowymiarowe, miary odległości, miary agregatowe, program R.

1. Introduction

The article presents the proposal of the application of multidimensional scaling [Borg, Groenen 2005] in linear ordering of a set of objects based on the pattern of development [Hellwig 1968]. A two-step research procedure was suggested, which allows the visualization of linear ordering results for metric data. First, following the application of multidimensional scaling, the visualization of objects in two-dimensional space is obtained. Next, the linear ordering of a set of objects is carried out based on the Euclidean distance from the pattern of development. The suggested approach is illustrated by an empirical example.

The article applies the concept of isoquant and the path of development (the axis of the set – the shortest way connecting a pattern and an anti-pattern object[1]) proposed by [Hellwig 1981]. The graphical presentation of the linear ordering results, based on this concept, was possible for two variables. The application of multidimensional scaling expanded the applicability of linear ordering visualization results for *m* variables.

2. The genesis of the concept of the pattern of development and measure of development

The first research paper discussing the concept of the pattern of development and the measure of development in English was presented by Professor Zdzisław Hellwig at the UNESCO conference in Warsaw in 1967 [Hellwig 1967]. The study was published in English in a monograph edited by Z. Gostkowski [Hellwig 1972]. The first article analyzing the pattern of development and the measure of development in Polish was published in the journal “Przegląd Statystyczny” (The Statistical Review) in 1968 [Hellwig 1968]. These studies introduced the following terms:

- stimulants and destimulants,
- pattern of development,
- measure of development (distance from the pattern of development).

[1] There are two types of pattern objects: a pattern object (upper pattern object, ideal object, upper pole) and an anti-pattern object (lower pattern object, anti-ideal object, lower pole). The coordinates of a pattern object cover the most preferred preference variable (stimulants, destimulants, nominants) values. The coordinates of an anti-pattern object cover the least preferred preference variable values.

It can be stated, without any exaggeration, that Hellwig's idea initiated an avalanche of proposals for the development of linear ordering methods. These modifications aimed at (see [Borys, Strahl, Walesiak 1990; Pociecha, Zając 1990]):

a) differentiating the method for the normalization of variable values,
b) introducing nominant variables in a set,
c) determining the pattern of development (comparative base) in a different way,
d) applying various constructions of the composite measure,
e) applying fuzzy sets in the construction of the composite measure.

Recently the concepts using fuzzy numbers were developed in linear ordering based on the pattern of development (see e.g. [Chen 2000; Wysocki 2010; Jefmański, Dudek 2016]) and taking into account spatial dependencies (see [Antczak 2013; Pietrzak 2014]) and interval symbolic data (see [Młodak 2014]).

3. Linear ordering for metric data based on a pattern object – general procedure

The general procedure in linear ordering of the set of objects based on a pattern object (or an anti-pattern object) and metric data takes the following form:

$$P \to A \to X \to SDN \to T_w \to N \to d_i \to R\,, \quad (1)$$

where: P – choice of a complex phenomenon – the overriding phenomenon for ordering A set objects, which is not subject to direct measurement; A – choice of objects; X – selection of variables. Collecting data and the construction of data matrix $[x_{ij}]$ (x_{ij} – the value of the j-th variable on i-th object); SDN – identifying preferential variables (stimulants, destimulants, nominants). M_j variable is a stimulant (see [Hellwig 1981, p. 48]), when for every two of its observations x_{ij}^S, x_{kj}^S referring to objects A_i, A_k take $x_{ij}^S > x_{kj}^S \Rightarrow A_i \succ A_k$ ($\succ$ means A_i object domination over A_k object). M_j variable is a destimulant (see [Hellwig 1981, p. 48]), when for every two of its observations x_{ij}^D, x_{kj}^D referring to objects A_i, A_k take $x_{ij}^D > x_{kj}^D \Rightarrow A_i \prec A_k$ ($\prec$ means A_k object domination over A_i object). Therefore, M_j variable represents a unimodal nominant (see [Borys 1984, p. 118]), when for every two of its observations x_{ij}^N, x_{kj}^N referring to objects A_i, A_k (nom_j means the nominal level of j-th variable): if $x_{ij}^N, x_{kj}^N \le nom_j$, then $x_{ij}^N > x_{kj}^N \Rightarrow A_i \succ A_k$; if $x_{ij}^N, x_{kj}^N > nom_j$, then $x_{ij}^N > x_{kj}^N \Rightarrow A_i \prec A_k$, T_w – transformation of nominants into stimulants (required for an anti-pattern object only). Transformation formulas can be found for example in the study by [Walesiak 2011, p. 18]; N – normalization of variable values. The review of methods for the normalization of variable values is presented in the study [Walesiak 2014a]; d_i – aggregated measure (composite measure) calculation for i-th object – the application of distance measures from a pattern object using weights; R – ordering of objects in accordance with d_i value.

Table 1. Selected distance measures from a pattern object for metric data

Name	Distance d_i	Interval
Measure of development I [Hellwig 1968]	$1-\dfrac{d_i^+}{\bar{d}_\bullet^+ + 2s_d}$	$(-\infty; 1]$
Measure of development II [Hellwig 1981]	$1-\dfrac{d_i^+}{\sqrt{\sum_{j=1}^m \alpha_j^2 \left(z_{+j}-z_{-j}\right)^2}}$	[0; 1]
TOPSIS measure [Hwang, Yoon 1981]	$\dfrac{d_i^-}{d_i^- + d_i^+}$	[0; 1]
GDM1 distance [Walesiak, 2002]	$1-GDM1_i^+ = \dfrac{1}{2}+\dfrac{\sum\limits_{j=1}^m \alpha_j (z_{ij}-z_{wj})(z_{wj}-z_{ij}) + \sum\limits_{j=1}^m \sum\limits_{\substack{l=1 \\ l\neq i,w}}^n \alpha_j (z_{ij}-z_{lj})(z_{wj}-z_{lj})}{2\left[\sum\limits_{j=1}^m \sum\limits_{l=1}^n \alpha_j (z_{ij}-z_{lj})^2 \times \sum\limits_{j=1}^m \sum\limits_{l=1}^n \alpha_j (z_{wj}-z_{lj})^2\right]^{\frac{1}{2}}}$	[0; 1]
GDM1_TOPSIS – TOPSIS measure with GDM1 distance [Walesiak, 2014b]	$\dfrac{GDM1_i^-}{GDM1_i^- + GDM1_i^+}$	[0; 1]

$i, l = 1,..., n$ – object number, $j = 1,..., m$ – variable number, $z_{ij}(z_{lj}, z_{wj})$ – the normalized value of j-th variable for the i-th (l-th, w-th) object, $z_{+j}(z_{-j})$ – the normalized j-th coordinate of pattern object (anti-pattern object), $d_i^+ = \sqrt{\sum_{j=1}^m \alpha_j^2 \left(z_{ij}-z_{+j}\right)^2}$ – weighted Euclidean distance between i-th object and pattern object, $d_i^- = \sqrt{\sum_{j=1}^m \alpha_j^2 \left(z_{ij}-z_{-j}\right)^2}$ – weighted Euclidean distance between i-th object and anti-pattern object, $\bar{d}_\bullet^+ = \frac{1}{n}\sum_{i=1}^n d_i^+$, $s_d = \sqrt{\frac{1}{n}\sum_{i=1}^n (d_i^+ - \bar{d}_\bullet^+)^2}$, α_j – weight of j-th variable ($\alpha_j \in [0; 1]$ and $\sum_{j=1}^m \alpha_j = 1$ or $\alpha_j \in [0; m]$ and $\sum_{j=1}^m \alpha_j = m$), $GDM1_i^-$ ($GDM1_i^+$) – GDM1 distance between i-th object and anti-pattern object (pattern object), $z_{wj} = z_{+j}$ ($z_{wj} = z_{-j}$) for $GDM1_i^+$ ($GDM1_i^-$).

Source: author's compilation.

Table 1 presents the chosen distance measures from the pattern of development characterized by the normalized variability interval. The subject literature also offers other distance measures from the pattern (ideal) object (see e.g. [Grabiński 1984; Pawełek 2008]).

4. Research procedure allowing the visualization of linear ordering results for the set of objects for metric data

The research procedure allowing the visualization of linear ordering results for a set of objects covers the following steps:

1. The choice of a complex phenomenon in linear ordering which is not subject to an immediate measurement (e.g. the economic development level of countries worldwide, tourism attractiveness' level of counties).

2. Determining the set of objects and the set of variables substantively related to the analyzed complex phenomenon. The variables used to describe the objects are measured on metric scale. Following data collection a data matrix is constructed $[x_{ij}]_{n'xm}$ (x_{ij} – j-th variable value for i-th object; $i = 1,\ldots,n'$ – object number, $j = 1,\ldots,m$ – variable number).

3. Among the variables the following preferential variables are distinguished: stimulants, destimulants, nominants. Nominants are transformed into stimulants.

4. A pattern object (upper pole of development) and an anti-pattern object (lower pole of development) are added to the set of objects and result in a data matrix $[x_{ij}]_{nxm}$ ($n = n' + 2$).

5. If the variables describing objects are measured on an interval or ratio scale, they should be comparable using normalization (see [Walesiak, 2014a]) and receive a normalized data matrix $[z_{ij}]_{nxm}$.

6. The distances between objects are calculated and arranged in a distance matrix $[\delta_{ik}]$. The following distance measures can be applied in this case (measures taking into account weights of variables) e.g. city-block, Euclidean, GDM1 (see [Walesiak 2011, pp. 23–24]).

Multidimensional scaling: $f : \delta_{ik} \rightarrow d_{ik}$ is carried out. Multidimensional scaling is the method representing the distance matrix between the objects in m-dimensional space $[\delta_{ik}]$ into the distance matrix between the objects in q-dimensional space $[d_{ik}]$ ($q < m$) for the purposes of the graphical visualization of relations occurring between the analyzed objects and to specify (interpret) the content of q dimensions. The dimensions cannot be observed directly. They represent latent type of variables, which allow explaining the similarities and differences between the analyzed objects. Due to the possibility for the graphical presentation of linear ordering results, q equals 2. The iterative procedure in the **smacof** algorithm was presented in the study [Borg, Groenen 2005, pp. 204-205]. Finally, the data matrix in two-dimensional space $[v_{ij}]_{nx2}$ is obtained.

7. The graphical presentation and the interpretation of the results in a two-dimensional (multidimensional scaling results) and one-dimensional space (linear ordering results):

- in the figure a straight line connects the points determining a pattern and an anti-pattern object in the so-called axis of the set in a two-dimensional space (multidimensional scaling results). Isoquants of development are determined based on a pattern object, e.g. dividing the set axis into four parts allows determining four isoquants. The objects between isoquants present a similar development level. The same development level can be achieved by the objects placed in different points on the same isoquant of development (due to a different configuration of variable values). Such a presentation of the results expands the interpretation of the linear ordering results;
- normalized d_i^+ distances of *i*-th object from the pattern of development are calculated in accordance with the formula (cf. [Hellwig 1981, p. 62]):

$$d_i^+ = \frac{\sqrt{\sum_{j=1}^{2}\left(v_{ij} - v_{+j}\right)^2}}{\sqrt{\sum_{j=1}^{2}\left(v_{+j} - v_{-j}\right)^2}}, \ d_i^+ \in [0;1], \tag{2}$$

where: $\sqrt{\sum_{j=1}^{2}\left(v_{ij} - v_{+j}\right)^2}$ – Euclidean distance between *i*-th object and pattern object (ideal point co-ordinates),

$\sqrt{\sum_{j=1}^{2}\left(v_{+j} - v_{-j}\right)^2}$ – Euclidean distance between pattern object and anti-pattern object (anti-ideal point co-ordinates).

The objects of the study are ordered by the growing values of distance measure (2). The linear ordering results are graphically presented in the figure.

5. Empirical results

The empirical study uses the statistical data presented in the article [Gryszel, Walesiak 2014], referring to the attractiveness level of 29 Lower Silesian counties. The evaluation of the touristic attractiveness of Lower Silesian counties was performed using 16 metric variables (measured on a ratio scale):

x_1 – beds in hotels per 1 km^2 of a county area,

x_2 – number of nights spent daily by resident tourists (Poles) per 1000 inhabitants of a county,

x_3 – number of nights spent daily by foreign tourists per 1000 inhabitants of a county,

x_4 – gas pollution emission in tons per 1 km^2 of a county area,

x_5 – number of criminal offences and crimes against life and health per 1000 inhabitants of a county,

x_6 – number of property crimes per 1000 inhabitants of a county,

x_7 – number of historical buildings per 100 km^2 of a county area,

x_8 – % of a county forest cover,

x_9 – % share of legally protected areas within a county area,

x_{10} – number of events as well as cultural and tourist ventures in a county,

x_{11} – number of natural monuments calculated per 1 km^2 of a county area,

x_{12} – number of tourist economic entities per 1000 inhabitants of a county (natural and legal persons),

x_{13} – expenditure of municipalities and counties on tourism, culture and national heritage protection as well as physical culture per 1 inhabitant of a county in PLN,

x_{14} – cinema goers per 1000 inhabitants of a county,

x_{15} – museum visitors per 1000 inhabitants of a county,

x_{16} – number of construction permits (hotels and accommodation buildings, commercial and service buildings, transport and communication buildings, civil and water engineering constructions) issued in a county in 2011-2012 per 1 km^2 of a county area.

The statistical data were collected in 2012 and come from the Local Data Bank of the Central Statistical Office of Poland, the data for x_7 variable only were obtained from the Regional Conservation Officer.

R environment script was used in this article and prepared in accordance with the research procedure discussed in section 4, which applied the following methodology:

- x_4, x_5 and x_6 variables take the form of destimulants, x_9 is a nominant (50% level was adopted as the optimal one). The other variables represent stimulants, whereas x_9 nominant was transformed into a stimulant.
- a pattern and an anti-pattern object were added to the set of 29 counties. Therefore, the data matrix covers 31 objects described by 16 variables.
- due to the fact that all variables are metrical, the normalization of variable values was performed using the following method (see [Walesiak 2014a; Walesiak, Dudek 2016]):

$$z_{ij} = \frac{x_{ij} - med_j}{\sqrt{\sum_{i=1}^{n} (x_{ij} - med_j)^2}}, \tag{3}$$

where: $x_{ij}(z_{ij})$ – value (normalized value) of j-th variable for i-th object, $med_j = \underset{i}{med}(x_{ij})$ – median for j-th variable.

- distance matrix $[\delta_{ik}]$ between objects was calculated using GDM1, for which the same weights were used (see [Walesiak 2011, p. 47]):

$$\delta_{ik} = \frac{1}{2} - \frac{\sum_{j=1}^{m} \alpha_j a_{ikj} b_{kij} + \sum_{j=1}^{m} \sum_{\substack{l=1 \\ l \neq i,k}}^{n} \alpha_j a_{ilj} b_{klj}}{2 \left[\sum_{j=1}^{m} \sum_{l=1}^{n} \alpha_j a_{ilj}^2 \cdot \sum_{j=1}^{m} \sum_{l=1}^{n} \alpha_j b_{klj}^2 \right]^{\frac{1}{2}}}, \tag{4}$$

where: δ_{ik} – GDM1 distance measure,
$i,k,l = 1,\ldots,n$ – object number, $j = 1,\ldots,m$ – variable number,
$a_{ipj} = z_{ij} - z_{pj} \quad for \quad p = k,l$
$b_{krj} = z_{kj} - z_{rj} \quad for \quad r = i,l$,
z_{ij} (z_{kj}, z_{lj}) – normalized i-th (k-th, l-th) observation for j-th variable,
α_j – weight of j-th variable ($\alpha_j \in [0; m]$ and $\sum_{j=1}^{m} \alpha_j = m$ or $\alpha_j \in [0; 1]$ and $\sum_{j=1}^{m} \alpha_j = 1$).

- The multidimensional scaling of 31 objects was carried out (29 Lower Silesian counties plus the pattern and the anti-pattern object) in terms of tourist attractiveness level using the **smacofSym** function of the **smacof** package [Mair et al. 2016], as a result of which the configuration of 31 objects (points) in a two-dimensional space was obtained.
- Figure 1 illustrates the graphical presentation of the multidimensional scaling results for 31 objects. The pattern object (30) and the anti-pattern object (31) were connected by a straight line and the so-called axis of the set was obtained. Four isoquants of development were determined by dividing the axis into four equal parts.
- The distances of each object (county) from a pattern object were calculated in accordance with formula (2). Counties were ordered by the growing measure values (2) and the next four classes of similar counties, regarding their tourist attractiveness, were distinguished. The ordering of 31 objects referring to 29 counties,

the pattern object (30) and the anti-pattern object (31) regarding tourist attractiveness, presented by the growing measure values (2) are as follows (see Table 2).

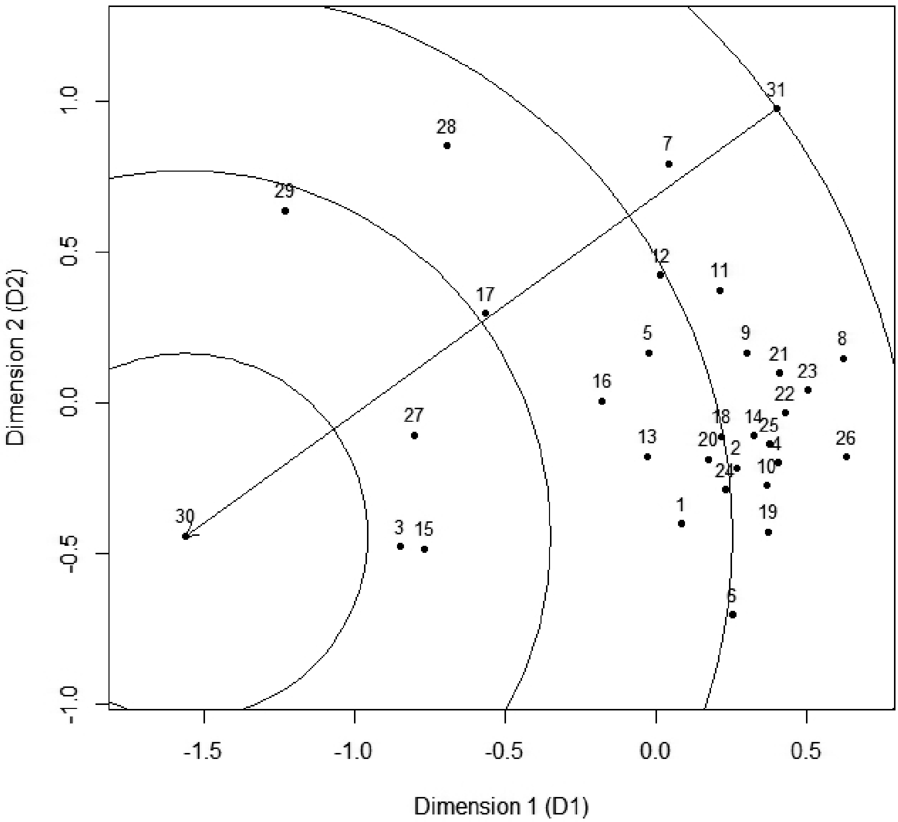

Figure 1. Graphical presentation of multidimensional scaling results in a two-dimensional space of 31 objects containing 29 counties, pattern object (30) and anti-pattern object (31) referring to the Lower Silesian counties' tourist attractiveness

Source: author's compilation using R program.

Table 2. The ordering of 31 objects regarding tourist attractiveness

Object	Name	Distance
1	2	3
30	Pattern	0.0000000
3	Jeleniogórski	0.2941865
15	Kłodzki	0.3275781
27	Jelenia Góra	0.3434270
29	Wrocław	0.4662754

Table 2, cont.

1	2	3
17	Wałbrzyski	0.5112598
16	Świdnicki	0.6000163
13	Polkowicki	0.6427173
28	Legnica	0.6428797
1	Bolesławiecki	0.6805073
5	Lubański	0.6826672
20	Oleśnicki	0.7237595
12	Lubiński	0.7419232
24	Trzebnicki	0.7436534
18	Ząbkowicki	0.7476896
6	Lwówecki	0.7568931
2	Jaworski	0.7606505
14	Dzierżoniowski	0.7918136
19	Milicki	0.7980240
10	Górowski	0.7997443
11	Legnicki	0.8066449
9	Głogowski	0.8089235
25	Wołowski	0.8102554
4	Kamiennogórski	0.8181317
7	Zgorzelecki	0.8365424
22	Strzeliński	0.8381159
21	Oławski	0.8438694
23	Średzki	0.8767444
26	Wrocławski	0.9125158
8	Złotoryjski	0.9340235
31	Anti-pattern	1.0000000

Source: author's compilation using R program.

The graphical results of the linear ordering for 31 objects covering 29 counties, the pattern object (30) and the anti-pattern object (31), in terms of tourist attractiveness, presented by the growing measure values (2) are presented in Figure 2.

Such a form of presenting the results allows for:

- the presentation of counties' ordering in terms of their tourist attractiveness in accordance with measure (2) values and in the form of graphical presentation in Figure 2,

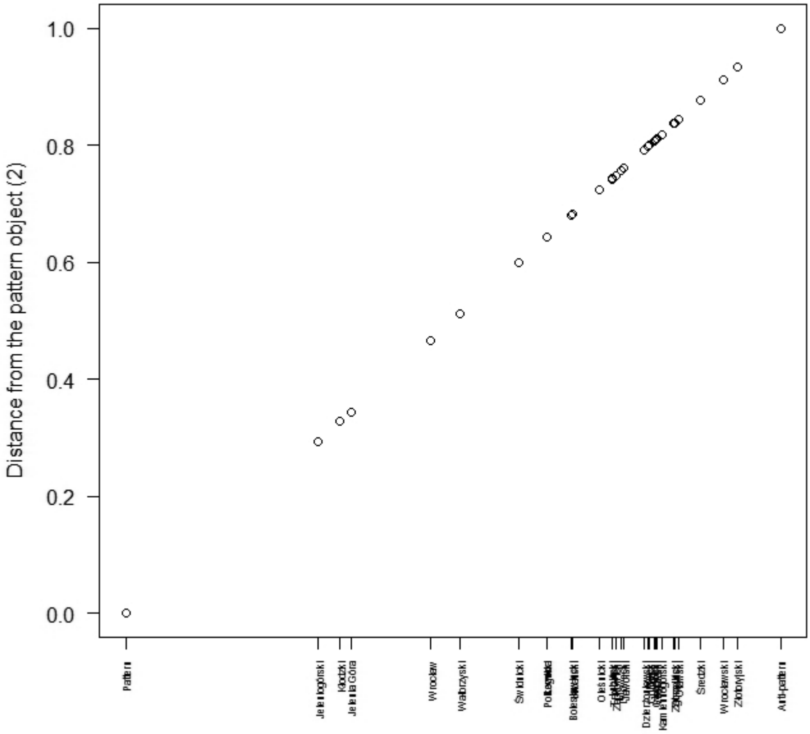

Figure 2. Graphical presentation of linear ordering of 31 objects containing 29 counties, pattern object (30) and anti-pattern object (31) referring to the Lower Silesian counties' tourist attractiveness by the growing measure values (2)

Source: author's compilation using R program.

- distinguishing the classes of counties (counties between isoquants) presenting the similar level of tourist attractiveness (see Figure 1),
- identifying counties characterized by a similar level of tourist attractiveness, but different regarding their location on the isoquant of development (see Figure 1). For example, Zgorzelecki County (7) and Strzeliński County (22) have a similar level of touristic attractiveness, but a different location on the isoquant of development. A similar situation occurs for Polkowicki County (13) and Legnica County (28). Therefore these counties achieved a similar level of development, however they are characterized by quite different configurations of variable values.

6. Final remarks

The article presents the proposal of a research procedure allowing the visualization of linear ordering results for the set of objects by applying multidimensional scaling for this purpose.

The concept of isoquants and the path of development suggested in the study [Hellwig 1981], allow for the graphical presentation of linear ordering results for two variables only. The application of multidimensional scaling extends the possibilities of visualizing linear ordering results for *m* variables. Following such a solution, the interpretation of linear ordering results was expanded.

The proposed approach was illustrated by an empirical example using R environment script [R Development Core Team 2016].

It should be borne in mind that the application of multidimensional scaling results in a partial loss of information about the objects. A set of objects is initially presented in the space of *m* variables. As a result of multidimensional scaling application, the graphical presentation of objects in a two-dimensional space is obtained. In the **smacofSym** function of the **smacof** package STRESS-1 Kruskal's fit measure is used [Borg, Groenen 2005, pp. 250–254].

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