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Spis treści

Wstęp

Z wielką przyjemnością oddajemy w Państwa ręce publikację pt. *Polityka ekonomiczna*, wydaną w ramach Prac Naukowych Uniwersytetu Ekonomicznego we Wrocławiu. Opracowanie składa się z 58 artykułów (w tym 5 w języku angielskim), w których Autorzy prezentują wyniki badań dotyczących zagadnień związanych z funkcjonowaniem współczesnych systemów gospodarczych w zakresie polityki gospodarczej. Tematyka podjęta w artykułach jest stosunkowo szeroka – mieści się w czterech obszarach problemowych. Pierwszy przedstawia rozważania związane z polityką innowacyjną, wolnością prowadzenia działalności gospodarczej oraz formami współpracy przedsiębiorstw. Drugi obszar dotyczy polityki transportowej, w tym infrastruktury i konkurencji. Trzeci obejmuje opracowania z zakresu polityki społecznej i zdrowotnej państwa – na poziomie zarówno krajowym, jak i lokalnym. Czwartą grupę stanowią artykuły dotyczące rolnictwa, w tym szczególnie wspólnej polityki rolnej i przemian w strukturze agrarnej.

Publikacja przeznaczona jest dla pracowników naukowych szkół wyższych, specjalistów zajmujących się w praktyce problematyką ekonomiczną, studentów studiów ekonomicznych oraz słuchaczy studiów podyplomowych i doktoranckich.

Artykuły składające się na niniejszy zbiór były recenzowane przez samodzielnych pracowników naukowych uniwersytetów, w większości kierowników katedr polityki ekonomicznej. W tym miejscu chcielibyśmy serdecznie podziękować za wnikliwe i rzetelne recenzje, często inspirujące do dalszych badań. Oddając powyższą publikację do rąk naszych Czytelników, wyrażamy nadzieję, że ze względu na jej wszechstronny charakter spotka się ona z zainteresowaniem i przyczyni do rozpoczęcia inspirujących dyskusji naukowych.

Jerzy Sokołowski, Grażyna Węgrzyn

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THE FOSTER-HART MEASURE AS A TOOL FOR DETERMINING THE SET OF RISKY PORTFOLIOS THAT DO NOT EXPOSE THE INVESTOR TO THE BANKRUPTCY

MIARA FOSTERA-HARTA JAKO NARZĘDZIE DO WYZNACZANIA ZBIORU RYZYKOWNYCH PORTFELI, KTÓRE NIE NARAŻAJĄ INWESTORA NA BANKRUCTWO

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Summary: The aim of this publication is to present the Foster-Hart measure in the process of building the portfolios, which in theory do not expose the investors to the bankruptcy. This measure, besides the fact that it is monotonic, objective, universal, and independent from the parameters of an 'ad hoc', includes the possibility of the investor's bankruptcy. In the context of this objective, it was considered appropriate to develop a basic set of equations, which are required for calculating the share of risky assets in these portfolios. In the later part of the study, a theoretical portfolio was presented, built through using the measures of Foster-Hart. In addition, the consideration was enriched by the opportunity to invest in risk-free assets. As a result, it was shown that the characterized measure allowed for the construction of a set of investments, which in theory should not expose the investor to go bankrupt, and which may offer a combination of attractive investments for different types of investors.

Keywords: portfolio management, risk, measures, investment, wealth management.

Streszczenie: Celem niniejszej publikacji jest prezentacja miary Fostera-Harta w procesie budowy portfeli, które w teorii nie narażają inwestora na bankructwo. Miara ta, oprócz faktu, że jest monotoniczna, obiektywna i uniwersalna, a także niezależna od parametrów typu ad hoc, uwzględnia możliwość bankructwa inwestora. W kontekście realizacji powyższego celu, uznano za zasadne opracowanie podstawowego zestawu równań, które są niezbędne dla obliczenia udziałów ryzykownych aktywów w takich portfelach. W dalszej części opracowania zaprezentowano teoretyczny portfel zbudowany przy wykorzystaniu miary Fostera-Harta. Ponadto, rozważania wzbogacono o możliwość inwestowania w aktywa wolne od ryzyka. W rezultacie wykazano, iż charakteryzowana miara pozwala na budowę zbioru inwestycyjnego, który w teorii nie powinien narażać inwestora na bankructwo, a który może oferować kombinacje inwestycyjne atrakcyjne dla różnych typów inwestorów.

Słowna kluczowe: zarządzanie portfelami, ryzyko, miary, inwestycje, *wealth management.*

1. Introduction

The Markowitz's model [Markowitz 1952] is the most famous tool to assist the process of portfolio management. It allows the determination of the effective border formed on the basis of portfolios with the lowest variance for each present rates of return. It presents the same height of the achieved rates of return, depending on the risk of portfolios built on the basis of pre-selected risky shares. In addition to this model, we developed many others that have to support this process of portfolio management. Significant as it may seem usually the common denominator of most economic models developed in the context of the support to invest cash assets, is that they are not universal and objective but more dependable on specific ad hoc parameters.

Referring to the Markowitz's model, it is essential to mention its two important drawbacks, which are not often mentioned in the literature. These drawbacks have limited its usefulness in the process of portfolio management and in the area of wealth management. The first is the way of measuring portfolio risk, because in the classical Markowitz's approach, it is measured by the variance of returns, because the model is based on the assumption that the risk of the shares should be measured by the standard deviation. As itis known, the standard deviation is not an appropriate measure for the risk measurement, because it is not monotonic [Foster, Hart 2009]. Secondly, the Markowitz's model does not take into account the danger of the bankruptcy of the investor, who is interested in investing in risky financial instruments characterized by the specific expected rates of return. All this means that its theoretical design restricts its practical usefulness.

With all this in mind, one should try to find such a risk measure which will fulfill a number of important conditions, making it a desirable tool to assist the process of portfolio management and improving the quality of services type of wealth management. These conditions require that such a measure:

- would be universal and objective,
- takes into account the size of the required assets to protect investors from potential bankruptcy, which can occur in case of selection of certain financial instruments whose value in the future will decrease,
- prescinds from objectives and preferences of investors,
- would be independent from the parameters of an ad hoc,
- would be monotonic.

On the grounds of the literature research, it was concluded that these conditions were fulfilled with the Hart Foster measure [Foster, Hart 2009], whose biggest advantage is that it can be used as a tool for determining the set of risky portfolios that do not expose an investor to the bankruptcy. It is worth noting that in the literature, such sample collections cannot be found. Therefore, the aim of this paper is to present how to use the Foster-Hart risk measures in the process of building a theoretical set of portfolios that will not in theory expose the investor to

the bankruptcy. It should be noted that the publication uses research literature and own calculations based on hypothetical data. It has also adopted the conventional assumption [Foster, Hart 2013] that the risk relates to the possibility of gains or losses, with certain probabilities.

2. General presentation of the key measures of the investment risk

The area of portfolio management uses a number of risk measures. The most important and the most popular are:

- portfolio risk calculated by using the model of Markowitz [Markowitz 1952],
- the risk rate of return of a financial instrument and portfolio (measured variance) within the single index model [Sharpe 1963],
- coherent risk measures (that meet certain four axioms in order to properly measure the risk [Artzner et al. 1999]). They include the so-called indicator ES (Expected Shortfall) [Acerbi, Tasche 2001],
- VaR (Value at Risk) which is a tool that allows investors to specify what number of resources should be allocated to secure investments in risky assets [Linsmeier, Pearson 2000] (the measure is not coherent),
- other known risk measurement models that include, for example: the variance-covariance method [Deutsch 2002], the historical simulation method [Butler, Schachter 1998] and the Monte Carlo Simulation [Haralambides 1991].

Apart from the exact presentation of disadvantages of each of these measures, one can, for a better understanding of the leading reasoning, focus on the issue only superficially. However, VaR, which depends on the parameters of an ad hoc, from a mathematical point of view, is not a measure [Artzner et al. 1999], and therefore cannot be considered useful from the point of view of the portfolio management. Other well-known single index model assumes that the residual components of the individual shares are not correlated, and therefore it allows only an approximation of the risk. The Markowitz's model, which has already been mentioned, is based on the assumption that the risk of shares should be measured by the standard deviation. This assumption can be considered as a basis for denying the usefulness of this model, because this measure is not monotonic, thus inappropriate for the risk measurement. Coherent risk measures, that meet certain assumptions, do not take into account the possible bankruptcy of the investor, and therefore cannot be regarded as an important element to support the portfolio management process and the provision of services of wealth management type. The reasons presented above suggest that it is worth focusing on the analysis of the Foster-Hart measurement, because it meets the criteria presented in the introduction.

3. Presentation of the concept of the Foster-Hart measure

The measure of Foster-Hart, fulfilling the expectations placed in front of the most important measure of the risk, seems to be an appropriate tool to support the portfolio management. Its main advantage is the slope, objectivity and universality [Foster, Hart 2009]. It allows investors to identify too risky investments that can lead to the bankruptcy of the investor. The detailed mathematical analysis [Foster, Hart 2009] showed that it ignores the objectives and preferences of investors and does not depend on any ad hoc parameters (eg. confidence level, as is the case with VaR). In addition, this measure brings important details in the theory of portfolio management for two reasons. First, it takes into account the possibility of the investor's bankruptcy. Secondly, it can be seen as a dynamic measurement of risk [Hellmann, Riedel 2015]. Thirdly, this measure is useful as a theoretical and analytical tool for a lot of investments with a large number of values [Riedel, Hellmann 2015]. Its properties make that it will arouse a growing interest among researchers.

The measure of Foster-Hart takes the form of the following formula (which is the measure of the risk for the "g" investment):

$$E\left[\log\left(1+\frac{1}{R(g)}g\right)\right]=0,$$

where g is the result of investments (cash flow generated by the investment with a certain probability in a given period of time), $R(g)$ is a critical wealth level, for the investment with known distribution. It is also the measure of investment risk g.

The presented formula treats $R(g)$ as unknown, while its value is the minimum level of wealth, which is assigned to investments designated as and it "(…) may also be viewed as a sort of minimal 'reserve' needed for g" [Foster, Hart 2009]. Therefore, the aim to calculate the ratio $R(g)$ it is to demonstrate the critical value of the current wealth of the investor, who is considering an investment of specific investment performance. The result of Foster-Hart equation is interpreted as follows: if the current level of investor wealth is lower than the value of $R(g)$ in a given period of time, then the g investment is too risky, and in the long run may lead to its bankruptcy. But if this level is equal to, or higher than the value of $R(g)$, the investment is considered to be acceptable and causes an increase in the investor's wealth in the long term.

The idea of this model should be illustrated by a theoretical example. Let us assume that the investment in the g share is associated with the issuance of USD 120, and that this share with equal probability can be sold in the future for USD 255 or USD 7.5. Accordingly, the investment will after some time get a payment of the value of USD 135 or USD -112.5. The critical level of wealth in a given period of time is the only positive solution of the Foster-Hart equation. In the case of the present embodiment, the equation takes the following form:

$$\frac{1}{2}\log\left(1+\frac{135}{R(g)}\right)+\frac{1}{2}\log\left(1+\frac{-112,5}{R(g)}\right)=0, \text{ hence}$$

$$R(g)=675.$$

As it can be seen, in the case of the analysis of the *g* share, the formula of Foster-Hart suggests that the minimum level of the investor's wealth, necessary for the acceptance of investment in this share, should be at least USD 675. With such wealth of values, according to the theory of Foster-Hart [Foster, Hart 2009], the investor is inert with respect to such investments and therefore will be willing to accept it, or reject it. Therefore, each higher value of the investor's wealth will make them be willing to accept it. The investor, therefore, in order to avoid bankruptcy, in the acquisition of one *g* share should have USD 795 (pay USD 120 and not to spend USD 675). It can be added here that to raise the efficiency of the investment, the value of $R(g)$ may be invested in risk-free assets for a period of time equal to the duration of the investment in data risky assets because their value should not be reduced. Prescinding from the considerations in this chapter, it is worth noting that the analysis of Polish and foreign literature shows clearly and unequivocally that there is a need to present the method for determining, by means of the Foster-Hart measurement, the set of risky portfolios that do not expose the investor to the bankruptcy. Such a study cannot be found anywhere, and it may be desirable from the perspective of financial institutions offering investors the portfolio management services, as well as services such as the wealth management. As it is known, the border efficiency drawn up on the basis of the Markowitz's theory, does not contain such a set.

4. Determination of a set of risky portfolios that do not expose the investor to the bankruptcy by using the Foster-Hart measure

The Foster-Hart measure can be used not only for the analysis of individual shares, but for the construction of portfolios as well. By virtue of the properties of this measure, such portfolios in theory should not expose the investor to the bankruptcy. Their construction should begin by determining the amount for the purchase of shares of a company "i" (C_i) and determine the total amount that is allocated to investment in all stocks (C_p), which will be included in the portfolio. Building the portfolio of *n* stocks, we get the relationship:

$$C_P=\sum_{i=1}^{n}C_i,$$

where C_i is equal for each series of the analyzed shares, as the measure of Foster-Hart refers only to individual investments. Knowing the price of the shares *i* ($P_{i,0}$) at the beginning of the investment, there is also the need alongside to determi-

ne a possible sale price of the shares together with certain probabilities to determine the expected rate of return (R_i). Assuming the amount of sales prices of shares at the end of the period of investment, is required for calculating the value $R(g)_i$ (it is the measure of risk). Its value will allow to calculate another value s_i, which is calculated as follows:

$$s_i = P_{i,0} + R(g)_i.$$

The value s_i is therefore the amount that an investor must have in order to purchase shares of a certain company, without being exposed to the bankruptcy. Knowing the predetermined amount C_i, one can calculate the approximate maximum number of shares i (n_i), which can be purchased:

$$n_i = \frac{C_i}{s_i}.$$

Therefore, the total number of shares to be purchased using the Foster-Hart, can be described as n_i' where,

$$n_i' = \text{total amount of } n_i.$$

The value n_i' itself, is the basis needed for the conversion of the amounts that the investor has to spend on the acquisition of shares of individual companies. If we denote these amounts as l_i, then:

$$l_i = P_{i,0} \times n_i',$$

whereas the value one invests in selected shares (V_p) is obtained as follows:

$$V_P = \sum_{i=1}^{n} l_i.$$

It is easy to conclude that the investor's wealth (V_R), which is necessary to avoid the bankruptcy, must be:

$$V_R = C_P - V_P.$$

The value of the investor's wealth can be invested in risk-free assets for the duration of the investment in shares, acting as the depreciation for unfavorable investment results. Return on assets of this type should therefore raise the profitability of the carried out investment in any case. The value *Vp* also allows the calculation of shares of the individual stocks in the portfolio, thus:

$$w_i = \frac{l_i}{V_P},$$

and

$$\sum_{i=1}^{n} w_i = 100\%.$$

The resulting values of shares form a combination that becomes a portfolio which do not expose in the theory the investor to the bankruptcy. Therefore, the measure of the Foster-Hart is a tool that allows an investor to appoint only one portfolio of this quality. The expected return of the portfolio built on the basis of the Foster-Hart measure (R_{F-H}) can be calculated using the formula presented:

$$R_{F\text{-}H} = w_i \times R_{\mathrm{i}}.$$

Knowing the basic parameters of such a portfolio, the risk can also be measured in accordance with the Markowitz [Markowitz 1952] rule, which takes the form:

$$S_{F-H} = \sqrt{\sum_{i=1}^{n} w_i^{\,2} s_i^{\,2} + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j s_i s_j \rho_{i,j}}\,,\ i \neq j,$$

where S_{F-H} is the value of the risk of the risky portfolio, s_i (s_j) is the standard deviation of the *return* rate of the *i*-th (*j*-th) *share* in the portfolio and $\rho_{i,j}$ is a correlation coefficient between the *i*-th share and *j*-th share.

No more is needed to explain besides that, using the Foster-Hart measure, the short sale is not possible, because the shares of the actions in the portfolio will always be positive. The value of the expected rate of return and the risk value (measured in accordance with the rule of Markowitz) of the portfolio, which not exposes to the bankruptcy, allow to specify its location in a coordinate system. Additionally, one can also refer them to the position of other portfolios, which the investor would like to build on the basis of the arbitrary criteria. As a result, the investor would receive a set of portfolios with different characteristics, which can be considered attractive for different types of investors. Also, one can add non-risky portfolio V_R to such a coordinate system if it consisted of risk-free assets with a rate of return equal to R_F. The investor who would have some extra cash, could build a new portfolio, which would consist of risk-free assets (except for the portfolio V_R, which protects the investor). Prescinding the value V_R, this new and non-risky portfolio altogether with a risky portfolio (one can call it the "Foster-Hart portfolio") would create one great collection of investment. The rate of return of this set (R_p) would be dependent on the share of funds invested in the risk-free instruments and the Foster-Hart portfolio. This rate can be calculated according to the formula:

$$R_P = w_F \times R_F + (1 - w_F) \times R_{F-H},$$

where w_F is the share of funds invested in risk-free assets. The risk of a described set of investment (S_P) measured in accordance with the theory of Markowitz, can be calculated by the following equation:

$$S_P = (1 - w_F) \times S_{F-H}.$$

Assuming that there is no short sale, all portfolios consisting of these two types of assets will be on the line, which can be called R_{F-H} – *line*. This line describes the dependence of the rate of return of the investment portfolio on its risk and is given by the following formula:

$$R_{P(F-H)} = R_F + \frac{R_{F\text{-}H} - R_F}{S_{F-H}} \times S_P,$$

where $R_{P(F-H)}$ is the rate of return of the portfolio and has different values from the interval <0; S_{F-H}>. According to the interpretation of the presented line, wallets located on it do not expose the investor to the bankruptcy, while with the increase in the expected rate of return, their risk increases as it is due to the decreasing share of cash invested in risk-free assets. These portfolios make a collection that can offer specific combinations of investment representing an attractive alternative for investors with specific preferences. It should be remembered that the investor must always have a portfolio V_R, because it is the factor levelling the risk of bankruptcy.

5. An example of a theoretical portfolio built using the Foster-Hart measure

In the literature one cannot find a sample of the risky portfolio, which is built on the Foster-Hart measure. Therefore there is a need for such a presentation, however, within the method of presentation of such a portfolio, which in theory does not expose the investor to the bankruptcy, there should be very clearly extended to the fore, the transparency of the conducted argumentation, which will reflect the structure of the portfolio. It should therefore be assumed that it will be consisted of a small number of shares in two scenarios with equal probability of 50%, which will offer a certain rate of return. Building a sample portfolio, one has adopted in the work the following assumptions:

- C_p = USD 10,000,000 and n = 10, hence C_i = USD 1,000,000,
- the investor during the investment does not make the portfolio rebalancing.
- all shares with equal probability, equal to 50%, may be sold at fixed amounts at the end of the period of investment,
- R_F = 1.5%, whereas loans cannot be given and borrowed at the interest rate, moreover, short selling is not possible (this assumption gives our considerations the useful character), so one can only purchase the risk free assets with such a rate of return,

- except for USD 10,000,000, earmarked for the investment, additional funds are available for the purchase of risk-free assets.

For the purposes of the portfolio construction, the theoretical purchase price of 10 shares is proposed, which equally likely offers certain rate of return (Table1). In addition, 45 correlation coefficients of rates of return of these shares were calculated, so that it became possible to use the theory of Markowitz. Ultimately, the measure of Foster-Hart allowed to obtain weights of individual stocks in the portfolio. The values of these weights, along with other data on notional shares, which were obtained on the basis of the conducted calculations, are presented in table 1.

Table 1. Key information on the shares which are included in the portfolio.

Share number	Rate of return		$P_{i,0}$	$P(g)_{i,}$	s_i	R_i	l_i	n_i'	w_i
	Scenario 1	Scenario 2							
1	0.540	−0.45	1,000	2,700	3,700	4.50%	270,000.00	270	5.59%
2	1.125	−0.9375	120	675	795	9.38%	150,960.00	1,258	3.13%
3	0.600	−0.50	18	54	72	5.00%	250,002.00	13,889	5.18%
4	0.756	−0.63	50	189	239	6.30%	209,200.00	4,184	4.33%
5	0.360	−0.30	300	540	840	3.00%	357,000.00	1,190	7.39%
6	0.380	−0.32	270	513	783	3.17%	344,790.00	1,277	7.14%
7	0.030	−0.03	180	27	207	0.25%	869,580.00	4,831	18.01%
8	0.019	−0.02	40	3.78	43.78	0.16%	913,640.00	22,841	18.92%
9	0.086	−0.07	10	4.32	14.32	0.72%	698,320.00	69,832	14.46%
10	0.062	−0.05	700	216	916	0.51%	764,400.00	1,092	15.83%
Probability	0.500	0.50							

Source: own study.

On the basis of the conducted calculations, the following values of certain parameters are obtained:

- V_P = USD 4,827,892 and V_R = USD 5,172,108,
- R_{F-H} =1.79% and S_{F-H} =19.64%,
- $R_{P(F-H)}$ =1.5% + 1.452% × S_P and S_P has different values from the interval of <0; 19.64%>.

Basing on the value of R_{F-H} and of S_{F-H} the position of the built portfolio in a coordinate system (Figure 1) was settled. The advantage of the built portfolio is that the investor learns about the amount which should not be issued, whereby one possibly can invest it in risk-free assets. As it is well known, the model of Markowitz does not produce such information.

The study assumes that there are additional funds for the purchase of risk-free assets at the expected rate of return of 1.5%. Therefore, it became possible to build a new non risky portfolio, which together with the risky "Foster-Hart portfolio" creates a set of investments. Its graphic image is the straight line (Figure 2) of the equation $R_{P(F-H)}$ = 1.5% + 1.452% × S_P.

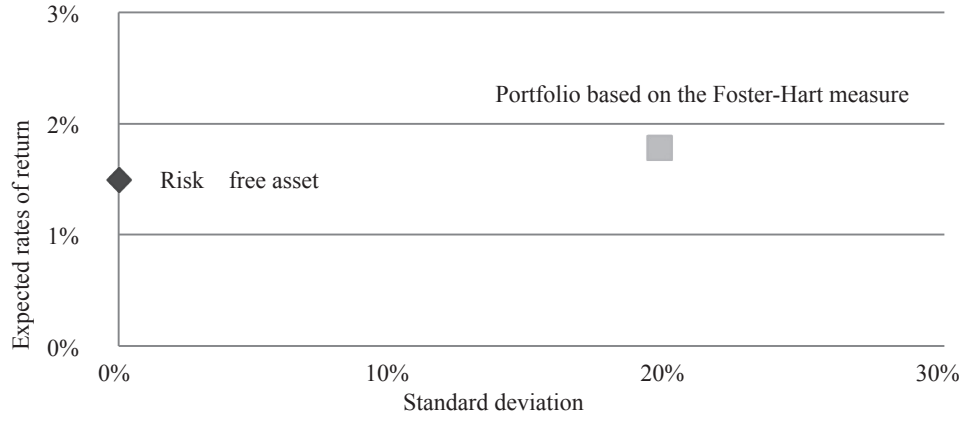

Fig. 1. Location of the portfolio built based on the Foster-Hart measure with regard to the reserve which is invested in risk-free assets

Source: own study.

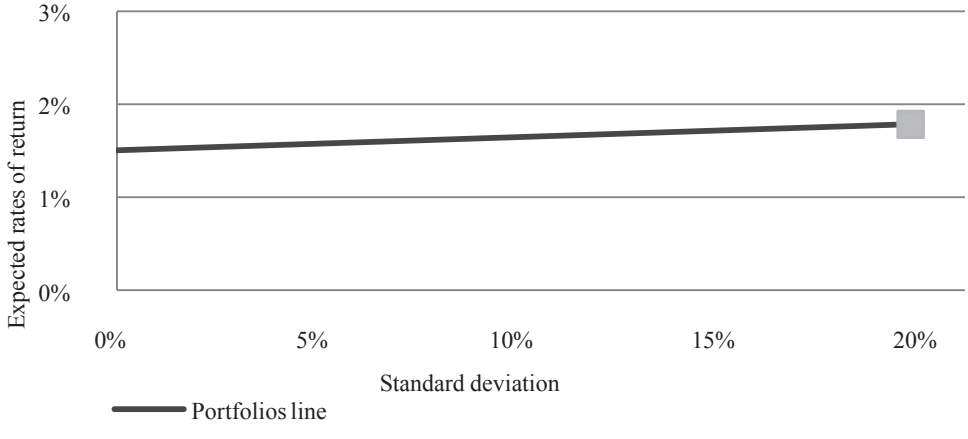

Fig. 2. Investment set

Source: own study.

Empirical analysis of the theoretical portfolio built on the basis of the measure of Foster-Hart, whose selected characteristics are presented in the table leads to the conclusion that it forms the only combination that in theory does not expose the investor to the bankruptcy and which may be a subset of a larger set of the investment, if the opportunity to invest in risk-free assets is taken into account. This larger set also in theory cannot expose investors to the bankruptcy.

6. Conclusions

Standard risk measures do not meet the expectations that are placed on them, but the measure of Foster-Hart seems to be an appropriate tool to support the portfolio management by its qualities, to which we can include: monotonicity, objectivity and universality. In addition, it allows to indicate too risky investments that can lead to the bankruptcy of the investor, and also disregards the objectives and preferences of the investors, not depending on any parameters of an ad hoc [Foster, Hart 2009]. It could be argued that this measure brings important details in the theory of portfolio management for two reasons. Firstly, it takes into account the possibility of the investor's bankruptcy. Secondly, it can be considered as a dynamic risk measurement [Hellmann, Riedel 2015]. As it is known, the recognition of the dynamic view is presently an increasing subject of research, centered around the theme of risk in finance [Detlefsen, Scandolo 2005].

This article is dedicated to the presentation of how to build the portfolio based on the measure of Foster-Hart. The value of shares of stocks in this portfolio make the only combination that in the theory does not expose the investor to the bankruptcy. It was indicated that the use of this measure excludes short sale while allowing at the same time for the construction of a large set of investment opportunities for investing in risk-free assets. Although all the features of the Foster-Hart measurement, which were presented in the article, completely correspond to the requirements imposed to the reckoning-risk, but one cannot eject the thesis that its use will in practice achieve above-average investment performance, while protecting the investor against the risk (bankruptcy). It is worth mentioning that the changes in the size of payments with changing probabilities during the investment may reduce the profitability of the built portfolio. For this reason, the use of the Foster-Hart measure should be combined with the optimization of the portfolio. It should be noted, however, that the issue of optimal portfolios built by using the analyzed measures in the literature, has not been addressed. This issue can therefore be considered as a contribution to a separate study.

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