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THE IDENTIFICATION OF THE BANDWIDTH FOR THE POTENTIAL REDISTRIBUTION INDEX EVALUATION

A decomposition of actual redistributive effect was proposed by Aronson Johnson and Lambert in 1992, in order to evaluate the potential redistributive effect. However this decomposition is not univocally determined, but as it can be calculated after having gathered incomes into groups of "close" equals, de facto it depends on the bandwidth chosen to split the income parade into contiguous income groups; this means that the bandwidth has to be chosen according to proper criteria: Van de Ven, Creedy, Lambert (VCL criterion) propose to choose the bandwidths where potential redistributive indexes are maxima. However the literature proposes more than one index to measure the potential redistribution of a tax system and the maxima associated to each of them do not necessarily coincide and, moreover, they do not generally show a regular sequence of values leading to the global maxima. The main aim of this paper is to contribute to the problem of defining a proper bandwidth which can split the income parade into close equal groups: VCL criterion is considered together with a minimum criterion recently proposed by Vernizzi and Pellegino (VP criterion). Empirical evidence is obtained by a data set of incomes and taxes collected by two Lower-Silesian revenue offices. The analyses were conducted by the author's own programmes written in the "R" language.

Keywords: redistributive effect, close equal group
JEL Classification Numbers: H23; H24

INTRODUCTION

The progressive nature, characterizing most of contemporary income tax systems in the developing countries, suggests that the tax system is an important instrument of income redistribution.

An assessment of redistribution efficiency demands, however, an estimation of tax system characteristics, especially the redistribution capacity of a given tax schedule. The most popular coefficient (*RE*), measuring the extent of redistribution is given as a difference between Gini index before and after taxation(cf. Lambert 1993):

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$$RE = G_y - G_{yt} \tag{1}$$

where G_y denotes Gini index for income before taxation (y) and

G_{yt} – Gini index for income after taxation (yt).

Progressive tax pushes the income of distribution toward an equality uniform distribution. *RE* measures the magnitude of this movement. The greater the tax progressivity and the average tax rate, the larger the redistributive effect. The value of *RE* coefficient could be interpreted as a percentage of income that is transferred from richer to poorer as a result of diversified tax rates. This kind of redistribution does not take the form of direct money transfers. It is a hypothetical value of such transfers that should be made in the case of a hypothetical, proportional tax system to get the tax distribution identical to the analyzed one. However, it is a measure of effective redistribution, comprising both redistribution resulting from progressive tax scale and redistribution being a consequence of unintended tax inequity. A very important problem is the separation of these two effects – and an assessment of a theoretical ("potential") redistribution capacity of the tax schedule and redistribution caused by tax inequity. The second component is useful – even necessary – for an assessment of inequity of taxation understandood as a horizontal equity. This is possible thanks to decomposition of Gini index and following decomposition of redistribution coefficient.

In order to evaluate the "potential" vertical redistribution power of a tax system, together with unfairness which lowers the redistributive potentiality, Aronson and Lambert (1993, 1994), Aronson et al (1994) suggest looking for equal pre-tax incomes sets.

Making use of Gini index decomposition properties, the authors show that the redistribution index can be written as

$$RE = V - H - R, \tag{2}$$

where *V* measures the redistribution that would have occurred if equals had been treated equally: this is called the potential vertical effect. *H* measures horizontal inequity as a loss of redistribution effect due to the unequal treatment of equals (i.e. when individuals with the same pre-tax income or pre-tax living standard do not pay the same tax), and *R* measures the additional effect of reranking caused by the tax system since post tax incomes are often differently ranked than pre-tax income. All these effects are extensively discussed in 0 and 0. The main issue of the decomposition (2) is in response to the question: to what extent the overall redistribution is a consequence of intentional construction of the tax system and to what extent it is restricted by tax inequity.

But in practice we have one problem with using the above described decomposition of *RE*. Calculation of components: *V, H* and *R* in decomposition (2) has to be preceded with a division of the taxpayers' population into groups of individuals with "close" pre-tax income. Describing this problem and its partial solution is a main purpose of the paper.

As we mentioned above, before the assessment of horizontal and vertical equity of tax systems we have to select "close equals groups" – groups with "close" pre-tax income. Close equals groups (CEG) are constituted by taxpayers belonging to the same pre-tax income bracket. The income brackets are created by splitting the pre-tax income range into contiguous income intervals having the same bandwidth. The bandwidth has to be large enough to gather some incomes and small enough to include nearly equal incomes.

Let *Y* be a vector of non-decreasing incomes before taxation for *n* taxpayers:

$$Y = (y_1, y_2, ..., y_n), \quad y_1 \le y_2 \le ... \le y_n,$$

and taxpayers are grouped (with respect to income) into *k* classes, consisting of $n_1, n_2, ..., n_k$ taxpayers respectively. Analogously, *Y-T* would denote incomes after taxation.

Being almost impossible to determine groups of exact pre-tax incomes in real data bases, groups of approximate or close equal pre-tax incomes should be determined: from which it derives the problem to determine "who are the equals" and this necessarily involves the grouping of almost equal incomes. This problem is solved by determining contiguous groups in the pre-tax income parade, by partitioning the whole income range ($y_n - y_1$) into equal income intervals: it follows that in this approach its results are crucial for the decision about such intervals width, as all measures on the right hand side of (2) depend on the income bandwidth. The choice of such a bandwidth should be tackled according to optimality criteria. The main aim of this paper is looking through the problem of defining groups of close equals, by considering in detail some of the suggestions proposed in the literature. In this paper the following, most popular in literature *RE* decompositions will be considered:

1. The Aronson, Johnson, Lambert (1994) decomposition (AJL):

$$RE = V^{AJL} - H^{AJL} - R^{AJL},$$

where:

$V^{AJL} = G_y - G_{yt}^B - G_{yt}^{SW}$,

G_{yt}^B – between groups Gini index is defined as the Gini index for post-tax income when all incomes inside each group are substituted by the group income average,

$G_{yt}^{SW} = \sum_k a_{k,yt} G_{k,y}$, $G_{k,y}$ – is the pre-tax Gini index for the *k*-th group, $a_{k,yt}$ is the product of the *k*-th group population share and post-tax income share, when all incomes in a group are taxed by the same tax rate, by the group average tax rate.

$H^{AJL} = G_{yt}^W - G_{yt}^{SW}$, $G_{yt}^W = \sum_k a_{k,yt} G_{k,y}$, $G_{k,y}$ – is the Gini index for the *k*-th group, $a_{k,yt}$ is the product of the *k*-th group population share and post-tax income share,

$R^{AJL} = G_{yt} - G_{yt}^B - G_{yt}^W$.

2. Van de Ven, Creedy, Lambert (2001) decomposition (VCL):

$RE = V^{VCL} - H^{VCL} - R^{AJL}$,

where

$V^{VCL} = G_y^B - G_{yt}^B$,

G_y^B – between groups Gini index is defined as the Gini index for pre-tax income when all incomes inside each group are substituted by the group income average,

$H^{VCL} = G_{yt}^W - G_y^W$, $G_y^W = \sum_k a_{k,y} G_{k,y}$, $G_{k,yt}$ – is the Gini index for the *k*-th group, $a_{k,y}$ is the product of the *k*-th group population share and post-tax income share.

3. Urban, Lambert (2008) decomposition (UL):

$RE = V^{UL} - H^{UL} - R^{APK}$,

$V^{UL} = G_y - D_{yt}^B - G_{yt}^{SW}$, D_{yt}^B – between groups concentration index for post-tax income. It is defined as the concentration index when all incomes inside each group are substituted by the group income average and additionally groups are ordered according to the pre-tax group average.

The AJL decomposition was the first decomposition of *RE* which extends exactly the methodology of equals groups (groups with exactly the same pre-tax income) to the close equals group. This enhancement was very important because in practice selecting groups with exactly the same pre-tax income is not possible. But AJL decomposition has a drawback, it does not capture within-group and entire-group rerankings, if these occur. The VCL decomposition solves the problem of within-group reranking and this decomposition allows this reranking but the issue of whole-group reranking remains still unenvisaged. The recent UL decomposition enables accounting for all possible rerankings.

There are two ways of choosing the bandwidth for creating close equals groups in literature. Van de Ven, Creedy, Lambert (2001) suggested choosing the bandwidth where V^{VCL} is maximum; their criterion was applied for V^{UL} by Kim and Lambert (2008) and, analogously, it could be applied also for V^{AJL}.

Recently Vernizzi and Pellegrino (2007) (VP) have suggested choosing the bandwidth which minimizes the ratio:

$$\Phi = \frac{\max\left\{|V^{VCL} - V^{UL}|, |V^{VCL} - V^{AJL}|, |V^{AJL} - V^{UL}|\right\}}{\min\left\{V^{VCL}, V^{AJL}, V^{UL}\right\}} \tag{3}$$

The rationale for (3) stays in doubt that each of the three above reported measures considers some important aspects which should be taken into account, without being able to be exhaustive: (3) represents then a conservative compromise. In this paper it is extensively investigated how VCL and VP criteria behave in the framework of income and tax data collected by two Lower-Silesian revenue offices.

1. THE REGULARITY MEASURE FOR CRITERION CHOOSING THE BANDWIDTH FOR CREATING CLOSE EQUALS GROUPS

Obviously some criterion is better than another if it returns a bandwidth for which we receive an estimate of *V, H* and *R* from decompositions of *RE* close to the true vertical effect. The problem is that we do not know the true parameters for specific data and tax system so it is not just to judge this criterion from this point of view. Even if we cannot evaluate the three *V*'s as regards their capabilities in measuring the unknown true potential redistribution index, we may at least request that their identification is obtained by a regular and smoothed function. So we can check how regular is the path which leads to optimal values according either to the VCL or VP criterion.

The criterion is regular if repeated analyses of choosing optimal bandwidth for the same data set receive values of optimal bandwidth close to each other.

The regularity measure here adopted is the average absolute distance between adjacent bandwidths[1]. In the first step, when adopting VCL Criterion, for each of the three *V*'s the bandwidths associated to the ten highest *V* values are registered:

$V_{(1)} = V_{\max}, V_{(2)}, \ldots, V_{(10)}$, where $V_{\max} \geq V_{(2)} \geq \ldots \geq V_{(10)}$

Conversely, when adopting VP Criterion the ten minimum values for the ratio Φ (see column 1 in Table 4) are considered:

$\Phi_{(1)} = \Phi_{\min}, \Phi_{(2)}, \ldots, \Phi_{(10)}$, where $\Phi_{(1)} = \Phi_{\min} \leq \Phi_{(2)} \leq \ldots \leq \Phi_{(10)}$.

Next, we rank these bandwidths in a non-decreasing $\left[\left(V_{\max} - V_i\right) \cdot sign\left(b_i - b_{V_{\max}}\right)\right]$ order for VCL Criterion and in a non-decreasing $\left[\left(\Phi_i - \Phi_{\min}\right) \cdot sign\left(b_i - b_{\Phi_{\min}}\right)\right]$ order for VP Criterion (see column 2 in Table 4). The measure for regularity criterion is defined as the average absolute difference between adjacent bandwidths $\left|b_i - b_{i-1}\right|$, when bandwidths are ranked as explained above. This measure shows how the top ten (lowest ten) bandwidth range is large and about potential reshuffling. The minimum of this measure is equal to the considered step of bandwidth in the analysis and is achieved for full regularity criterion. In our analysis, the step of bandwidth equals 10, so the minimum for the adopted measure is 10 too.

2. EMPIRICAL ANALYSIS

The experiment was conducted on Polish data coming from two Lower-Silesian revenue offices for 2001. This set of data contains information about income tax paid for all taxpayers resident in the Municipality of Wrocław and Wałbrzych. After deleting observations with non-positive gross income, the whole population consists of 130, 494 individuals. The analysis were performed by author's own programmes written in the "*R*" language.

[1] This measure was suggested by Professor Achille Vernizzi from the University of Milan in a personal communication.

In order to conduct the experiment on two different distributions, incomes where aggregated according to families (marriages which account together for income tax) and then considered both as

1. total family income (symbol: *total income*)

and as

2. per-spouse family income (symbol: *per spouse income*).

Table 1 reports the basic indexes for each of the two distributions:

- G_y is the pre-tax Gini index
- G_{yt} is the post tax Gini index
- $RE = G_y - G_{yt}$ is the redistributive effect
- Π^K is the Kakwani index for tax progressivity
- R^{APK} is the Atkinson-Plotnick-Kakwani reranking index.

The above coefficients do not depend on the choice of bandwidth for groups of close equal incomes.

Table 1

Basic indexes for the two income distributions

Coefficient	total income	Per-spouse income
G_{yt}	0,479688666	0,428911806
G_y	0,496753608	0,447737229
RE	0,017064942	0,018825423
Π^K	0,192628192	0,21683269
R^{APK}	0,000313328	0,00032407

Source: own calculations

We can observe that considering total family incomes and income per spouse, in fact we analyse two distributions having different characteristics.

First of all, it is worth to observe how the three potential redistribution indexes behave with respect to the bandwidth. Figure 1 and figure 2 present the relation between bandwidth and vertical effects for the three presented before decompositions of *RE* (V^{VCL}, V^{AJL}, V^{UL}) respectively for total income and per-spouse income. For legibility of the figure, the range of the bandwidth on all figures is divided into three sets:

1. from PLN 20 to PLN 3000. (step PLN 10) – part (a) of the figures

2. from PLN 3000 to PLN 30,000 (step PLN 200) – part (b) of the figures
3. from PLN 30,000 to PLN 300,000 (step PLN 2000) – part (c) of the figures.

This empirical framework extends the analysis on the complete range of possible bandwidth. Additionally, part d) of the figures presents the same relation for the bandwidth from PLN 10 to PLN 800 to provide a magnification of the bandwidths where either V^{AJL} and V^{UL} reaches its maximum or ratio Φ reaches its minimum.

Part (a) and (d) of Figures 1 and 2 show that generally V^{AJL} and V^{UL} are very irregular and present many local maxima when the bandwidth is small. The irregularities are smoothed when the bandwidth and increasing step become larger, as we can see looking at parts (b) and (c) of Figures 1 and 2, that is for increasing steps 3,000 and 30,000 width.

For small bandwidth V^{UL} dominates V^{AJL} and V^{VCL}, additionally V^{UL} decreases meanwhile V^{AJL} and V^{VCL} increases. For the bandwidth from PLN 450 to PLN 600 for total income family (from 350 to 450 for per-spouse income) all the three measures for the potential vertical effects are close together. From a bandwidth larger than PLN 700 V^{AJL} is undistinguishable from V^{UL}. The three indexes decrease to zero when the bandwidth tends to maximum width, however V^{VCL} dominates V^{AJL} and V^{VCL}. It is worth stressing that V^{AJL} and V^{VCL} become lower than *RE* for a bandwidth approximately larger than PLN 6,600 for per-spouse income and larger than PLN 11,600 for total family income V^{VCL} becomes lower than *RE* much later: for a bandwidth approximately larger than PLN 44,000 when dealing with per-spouse income and larger than PLN 104,000 for the total family income. This fact is important as we should look for an optimal bandwidth in range when *V*'s are greater than *RE*. If potential redistribution is lower than actual *RE*, we would have interpretation problems.

Generally, vertical effects behaviours are similar for both kinds of distributions of incomes: total family income and income per spouse. As Table 2 reports, V^{VCL} achieves maximum for a very large bandwidth: PLN 42,000 when we deal with total family income and PLN 20,600 when we deal with per-spouse income distribution: it is quite unreasonable to consider close equals incomes contained in so large bandwidths. Maxima for V^{UL} and V^{AJL} are achieved at bandwidth 350 and 730, respectively, when dealing with total family income, and at bandwidth 130 and 420, respectively, when considering per-spouse incomes. However, as we stressed above, in Figures 1 and 2, part (d) and (a), both V^{UL} and V^{AJL} present many local maxima, which are not so close to one another and then there is the risk that the maxima for these two indexes are irregular.

Table 2

Optimal bandwidth according to VCL and VP Criterion

	VCL Criterion			**VP Criterion**
	max V^{VCL}	max V^{UL}	max V^{AJL}	min Φ
total family income				
bandwidth	42,000	350	730	580
index value	0.0191445	0.0173784	0.0173763	0.0001077
per-spouse income				
bandwidth	20,600	130	420	380
index value	0.0200431	0.0191495	0.0191475	0.0000823

Source: own calculations

To summarize, VCL maximizing vertical effect criterion gives unreasonable bandwidths for V^{VCL} and irregular maxima for V^{UL} and V^{AJL} : further inquiry on regularity V^{VCL} would require looking in a neighbourhood of maxima for a tiny step increase, but in the presence of such a large bandwidth, it is not worth to devote attention to $V^{VCL\,L}$ and it is better to concentrate just on V^{UL} and V^{AJL} .

VP Criterion suggests choosing the bandwidth which minimizes the ratio Φ :

$$\Phi = \frac{\max\left\{|V^{VCL} - V^{UL}|, |V^{VCL} - V^{AJL}|, |V^{AJL} - V^{UL}|\right\}}{\min\left\{V^{VCL}, V^{AJL}, V^{UL}\right\}}.$$

Figure 3 illustrates the behaviour of maximum distance among the three vertical effects (V^{AJL}, V^{VCL}, V^{UL}) towards bandwidth. Maximum distance as a function of bandwidth is very regular and for very small bandwidth decreases and next increases for a bandwidth smaller than PLN 50,000 for income per spouse or than PLN 100,000 for total family income. In the limit maximum distance becomes zero.

However, as Figure 4 shows, the ratio between the maximum distance and the minimum of the three *V*'s, presents the form of a asymmetric *U* shaped form, with a unique minimum [2] Φ achieves its minimum for total family income the minimum is at PLN 580 bandwidth and at PLN 380 for per-spouse income[3]. Table 2 presents the optimal bandwidth according to VCL index maximization criterion: we see that the maximum for V^{VCL} is almost 10% greater than those for V^{AJL} and V^{UL}. However as stressed before, the optimal bandwidth for V^{VCL} cannot be considered as containing close equal incomes, being PLN 42,000 and PLN 20,600 wide. As regards V^{AJL} and V^{UL}, even if their values are rather close, the same does not hold for the bandwidths associated to them. Table 3 reports optimization results according to VP ratio criterion. The index values are quite close to the maxima for V^{AJL} and V^{UL}, and the optimal bandwidth values are now PLN 580, when total family income is considered, and PLN 380 for per spouse income distribution.

Table 3

Optimal bandwidth according to min Φ Criterion

	V^{VCL}	V^{UL}	V^{AJL}
total family income			
bandwidth	580	580	580
index value	0.017378	0.017378	0.017376
income per spouse			
bandwidth	380	380	380
index value	0.019149	0.019149	0.019147

Source: own calculations

Table 4 presents each step for the above described regularity analysis for the identification of the optimal bandwidth according to VP Criterion. Table 5

[2] Vernizzi and Pellegrino in a former version of their paper had suggested to look for the minimum of the maximum distance among the three indexes: in a more recent version they corrected their indication towards the ratio of the minimum of the ratio minimum of the three *V*'s.

[3] Note that for the bandwidth range PLN 10-3000 the curves for the ratio Φ are practically indistinguishable from the curves of the numerator of Φ itself.

gives analogous measures for VCL criterion: only V^{UL} and V^{AJL} are considered due to the non congruity of V^{VCL} maximum with the concept of close equals.

Table 4

Regularity analysis for the identification of the optimal bandwidths according to VP Criterion

Bandwidths (b_i) associated to lowest ten Φ values: b_i's are ranked according to a non decreasing order for Φ				b_i's and Φ's are ranked in a non decreasing order for $\left[(\Phi_i - \Phi_{\min}) \cdot sign(b_i - b_{\min})\right]$		Absolute differences between adjacent bandwidths $\lvert b_i - b_{i-1} \rvert$ b_i's are ranked in a non decreasing order for $\left[(\Phi_i - \Phi_{\min}) \cdot sign(b_i - b_{\min})\right]$	
per-spouse income		total income		per-spouse income	total income	per-spouse income	total income
b_i	Φ	b_i	Φ	b_i		$\lvert b_i - b_{i-1} \rvert$	
380	0.0000823	580	0.0001077	340	520		
390	0.0000859	590	0.0001100	330	500	10	20
400	0.0000902	550	0.0001107	370	570	40	70
360	0.0000918	560	0.0001134	350	560	20	10
350	0.0000930	610	0.0001190	360	550	10	10
370	0.0000939	600	0.0001194	380	580	20	30
410	0.0000947	570	0.0001215	390	590	10	10
330	0.0000957	620	0.0001233	400	610	10	20
340	0.0000958	500	0.0001236	410	600	10	10
420	0.0000995	520	0.0001246	420	620	10	20
					average	**15.56**	**22.22**

Source: own calculations

Table 5

Regularity analysis for the identification of the optimal bandwidths according to VCL Criterion

Bandwidths (b_i) associated to top ten V values: b_i's are ranked according to a non increasing order for V								Absolute differences between adjacent bandwidths $\lvert b_i - b_{i-1}\rvert$; b_i's are ranked in a non decreasing order for $[(V_{\max} - V_i) \cdot sign\{b_i - b_{V_{\max}}\}]$			
per-spouse income		total income		per-spouse income		total income		per-spouse income	total income	per-spouse income	total income
b_i	V^{AJL}	b_i	V^{AJL}	b_i	V^{UL}	b_i	V^{UL}	V^{AJL} $\lvert b_i - b_{i-1}\rvert$		V^{UL} $\lvert b_i - b_{i-1}\rvert$	
420	0.0191475	730	0.0173764	130	0.0191495	350	0.0173784				
350	0.0191475	630	0.0173763	110	0.0191495	210	0.0173784	60	50	10	20
290	0.0191472	590	0.0173761	10	0.0191495	130	0.0173783	40	170	40	40
380	0.0191472	580	0.0173760	30	0.0191495	280	0.0173783	90	80	70	40
730	0.0191471	680	0.0173757	140	0.0191494	140	0.0173783	140	190	10	20
590	0.0191470	610	0.0173757	20	0.0191494	90	0.0173783	70	70	20	60
340	0.0191470	420	0.0173756	90	0.0191494	110	0.0173783	310	100	100	140
540	0.0191470	490	0.0173756	290	0.0191494	70	0.0173783	140	10	20	150
580	0.0191469	660	0.0173756	50	0.0191494	30	0.0173782	50	40	10	80
280	0.0191468	710	0.0173756	60	0.0191494	10	0.0173782	40	100	150	140
							average	**104.44**	**90**	**47.78**	**76.67**

Source: own calculations

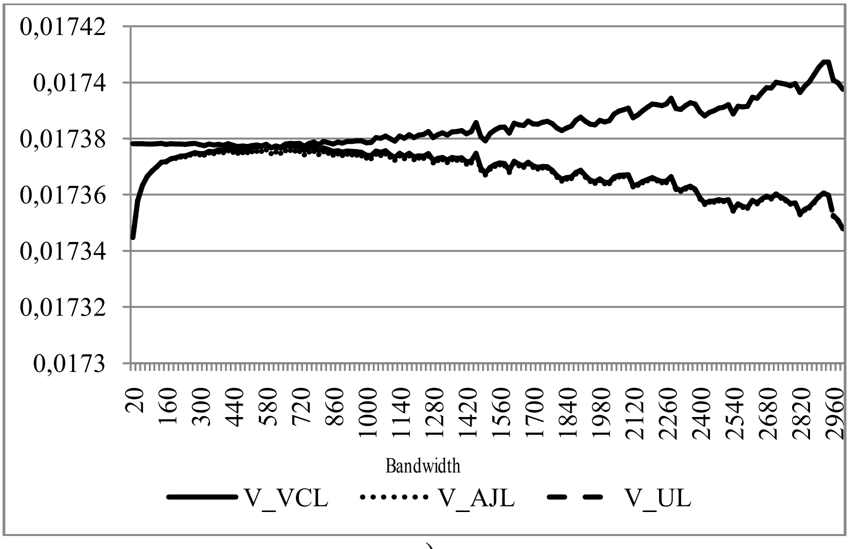
Regularity measures for VP Criterion are equal to 15.56 (minimum is 10) for per-spouse income and 22.22 for total income: they appear to be much lower than the corresponding ones for VCL criterion. For VCL criterion the stability measures are, respectively, 104.44 (per-spouse income) and 90 (total income) for V^{AJL}; when maximizing V^{UL} they are 47.78 and 76.67. In the context of the distributions here considered the criterion which maximizes V^{UL} appears to be more regular than that for maximizing V^{AJL}, however the criterion which minimizes Φ seems to be superior to both of them.

CONCLUSIONS

The behaviour of the redistribution indexes proposed by Urban and Lambert (2008) are analysed using the real income data. The empirical evidence from the data base on individuals and household resident in the Municipality of Wrocław and Wałbrzych, confirms UL's suggestion to find other indexes than V^{VCL} as a measure of the potential redistribution, the bandwidth which maximizes V^{VCL} is too large to be considered as including close equals. When applying VCL maximization criterion to V^{UL} and V^{AJL}, as Kim and Lambert (2008) do for the former, the results of bandwidths can be considered as containing what might actually be looked at as close equals. However, if the step for the grid searches is established, V^{UL} and V^{AJL} curves appear to be very irregular, they show several local maxima so that the absolute maximum appears to be irregular.

In order to override this problem, VP criterion can be applied. This criterion is a compromise of the three indexes and so it preserves the desirable properties that each index owns. Moreover, the minimum of the ratio which identifies VP optimal bandwidth appears to be quite regular.

APPENDIX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1280
1420
1560
1700
1840
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2820
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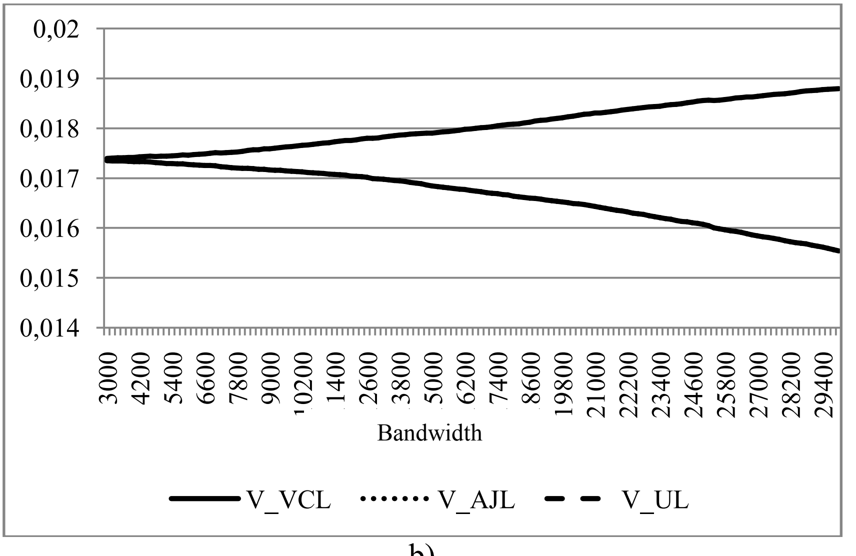
Bandwidth
V_VCL
V_AJL
V_UL

a)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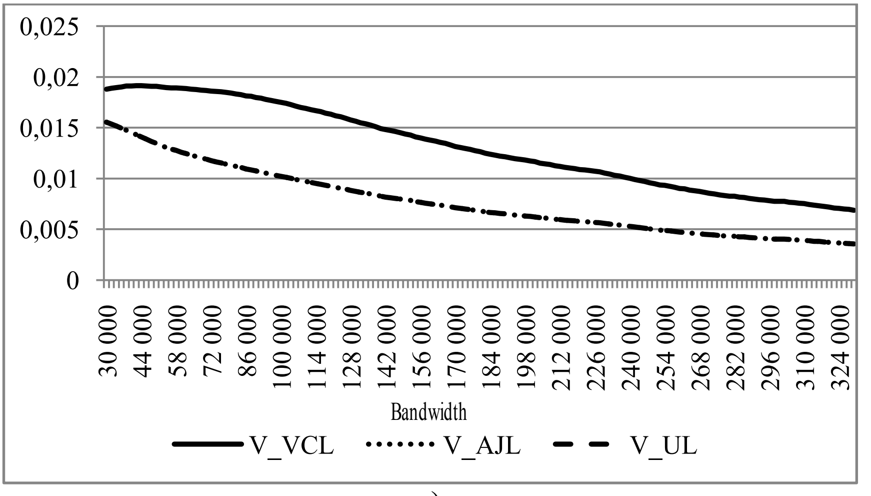
Bandwidth
V_VCL
V_AJL
V_UL

b)



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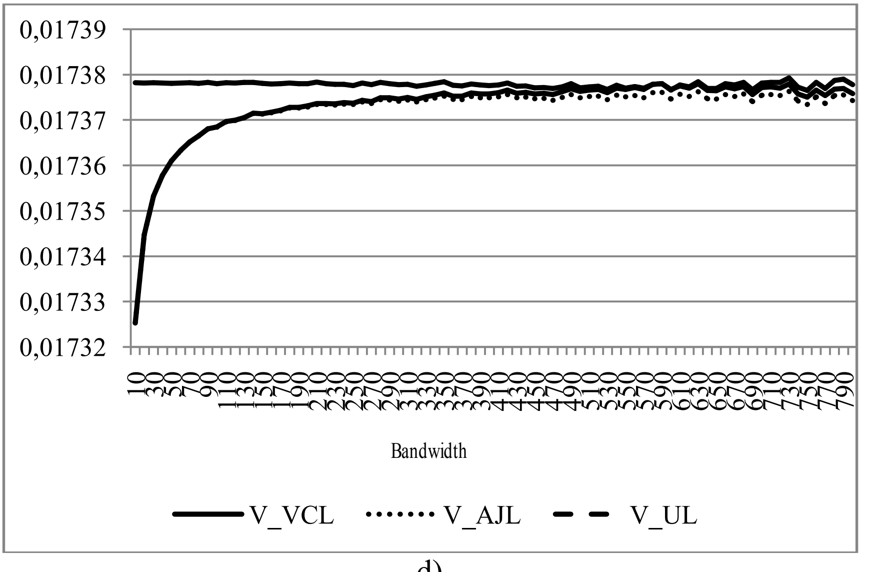
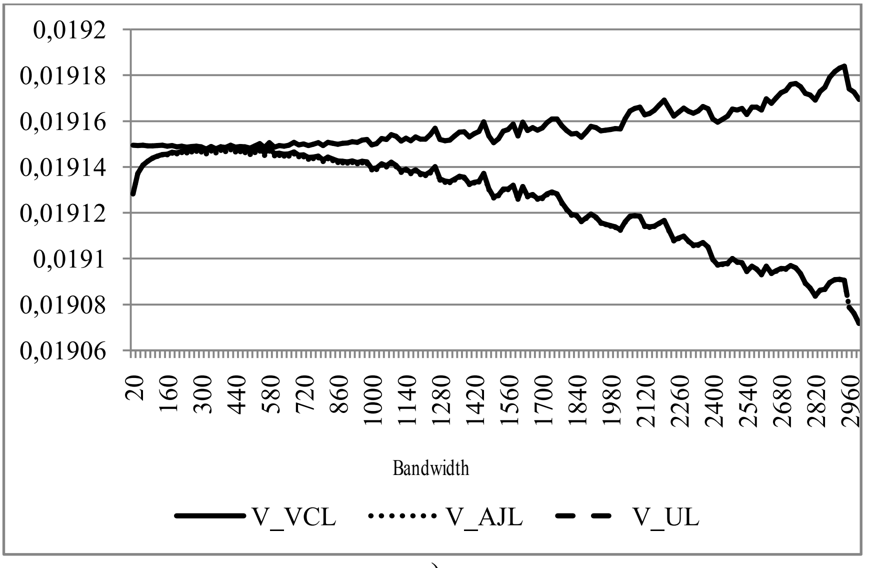


d)

Figure 1:Vertical effect for total family income over different ranges:
a) from PLN 20 to PLN 3,000 PL with step PLN 20;
b) from PLN 3,000 to PLN 30,000 with step PLN 200;
c) from PLN 30,000 to PLN 330,000 with step PLN 2,000;
d) from PLN 10 to PLN 800 with step PLN 10.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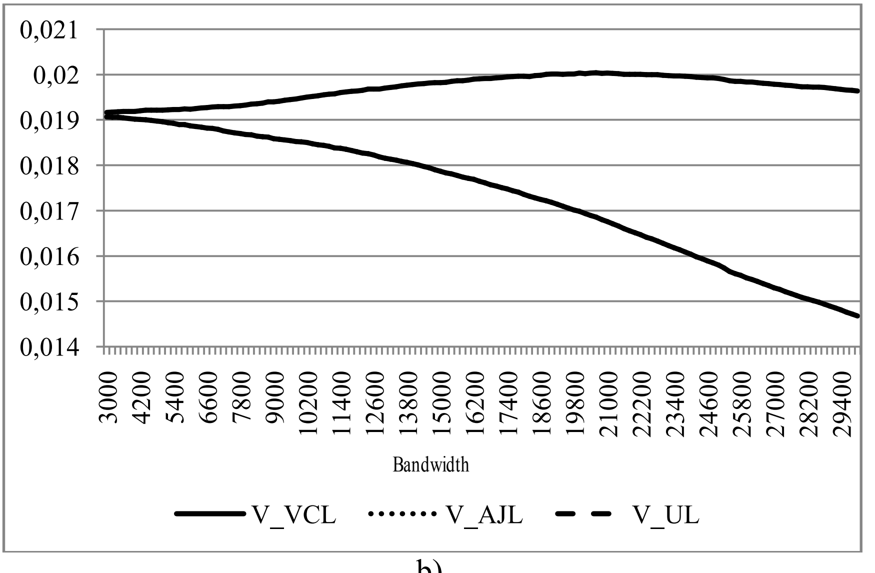
Bandwidth
V_VCL V_AJL V_UL

a)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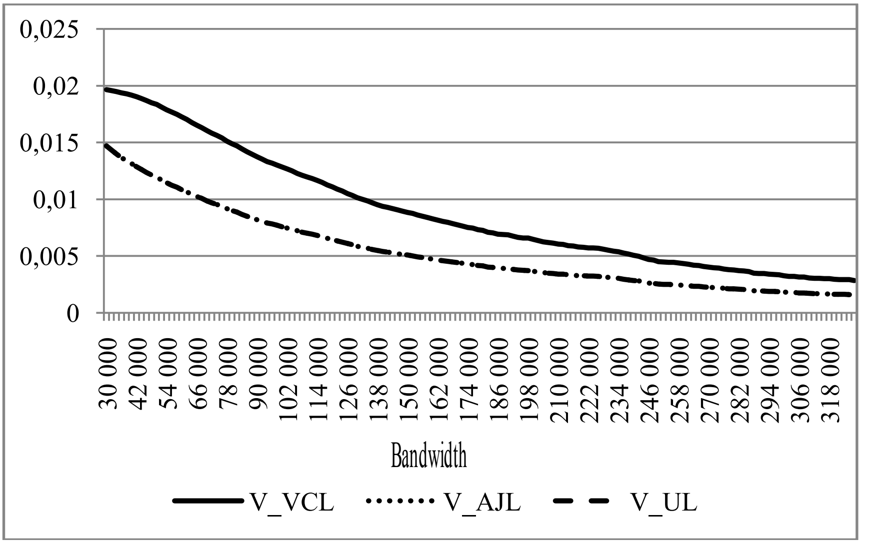
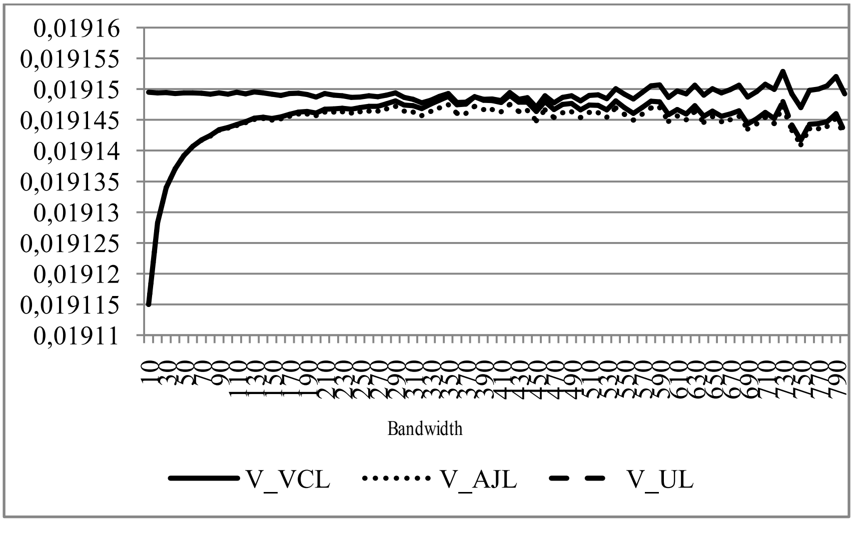
Bandwidth
V_VCL V_AJL V_UL

b)

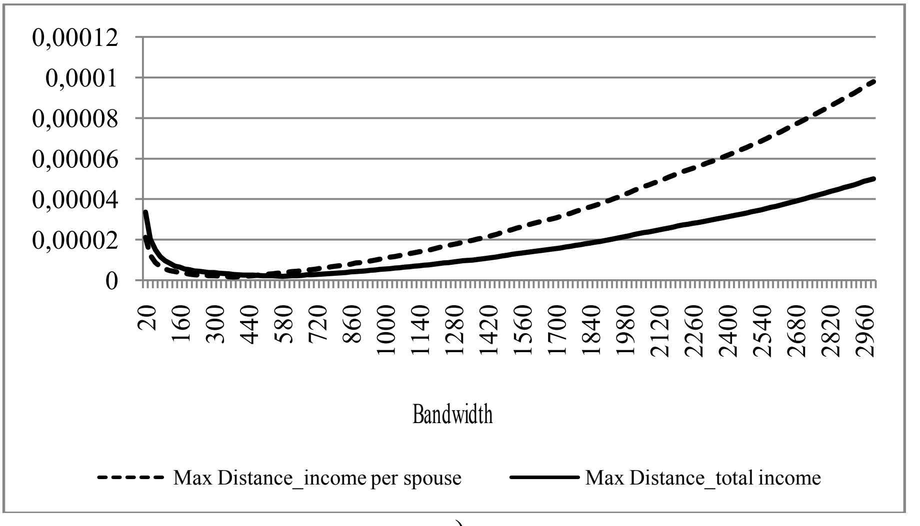


c)

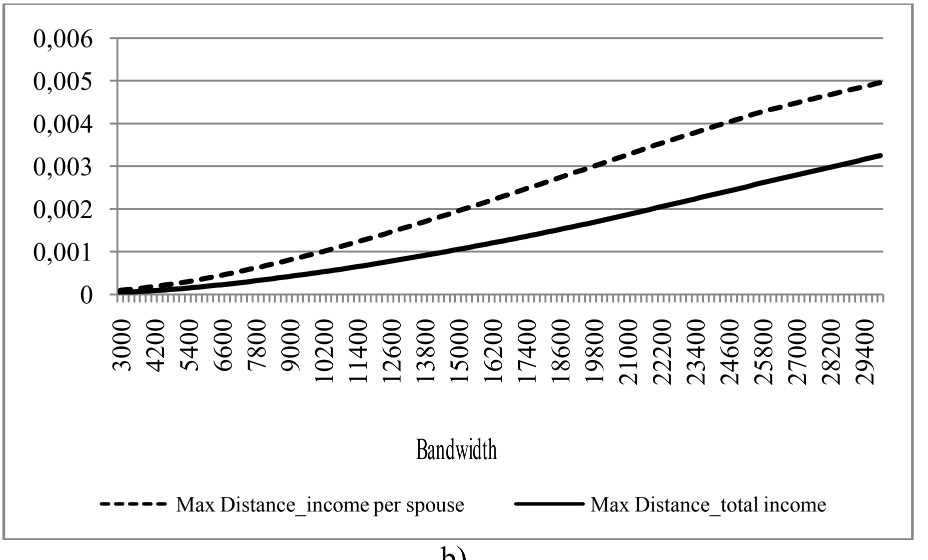


d)

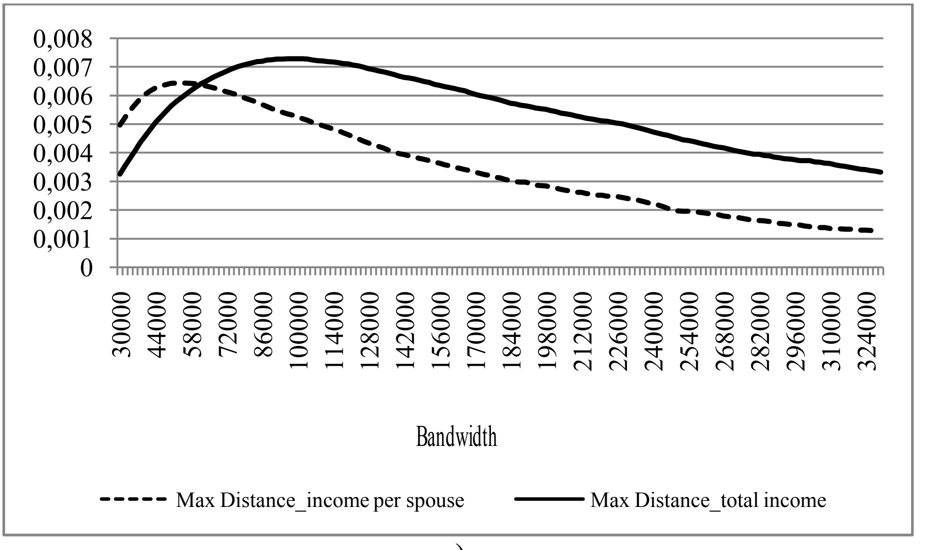
Figure 2:Vertical effect for per-spouse income over different ranges


0,00012
0,0001
0,00008
0,00006
0,00004
0,00002
0
20
160
300
440
580
720
860
1000
1140
1280
1420
1560
1700
1840
1980
2120
2260
2400
2540
2680
2820
2960
Bandwidth
Max Distance_income per spouse
Max Distance_total income

a)

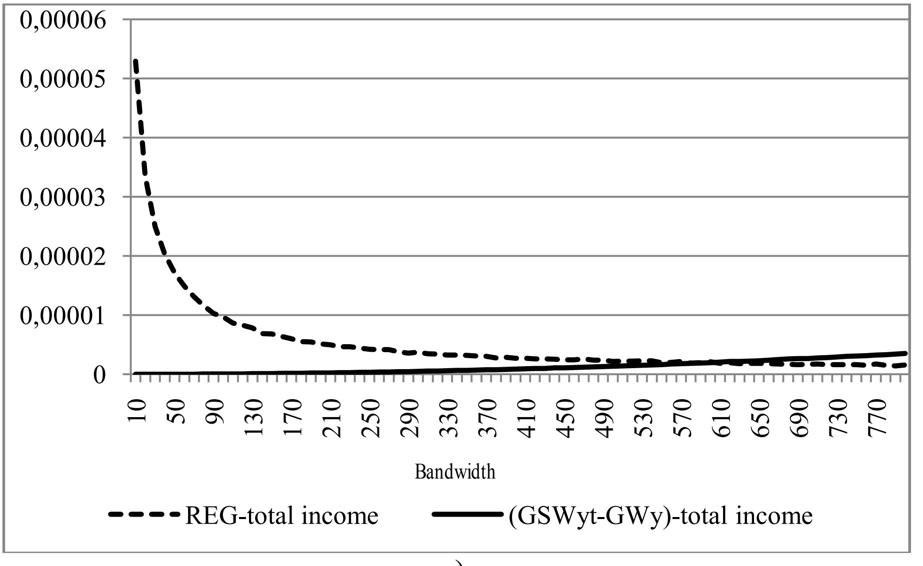

0,006
0,005
0,004
0,003
0,002
0,001
0
3000
4200
5400
6600
7800
9000
10200
11400
12600
13800
15000
16200
17400
18600
19800
21000
22200
23400
24600
25800
27000
28200
29400
Bandwidth
Max Distance_income per spouse
Max Distance_total income

b)

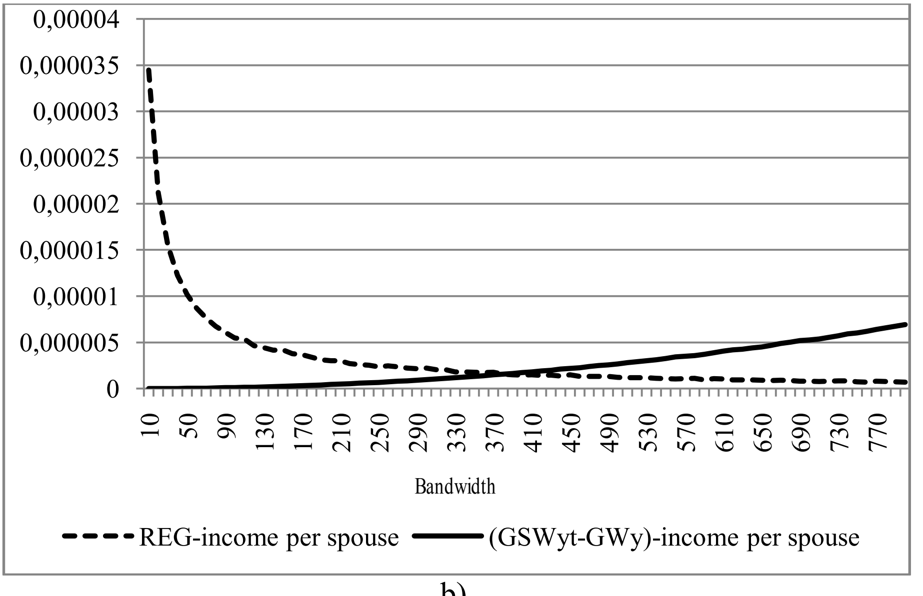


c)

Figure 3: Maximum distance for per-spouse income and total family income over different ranges

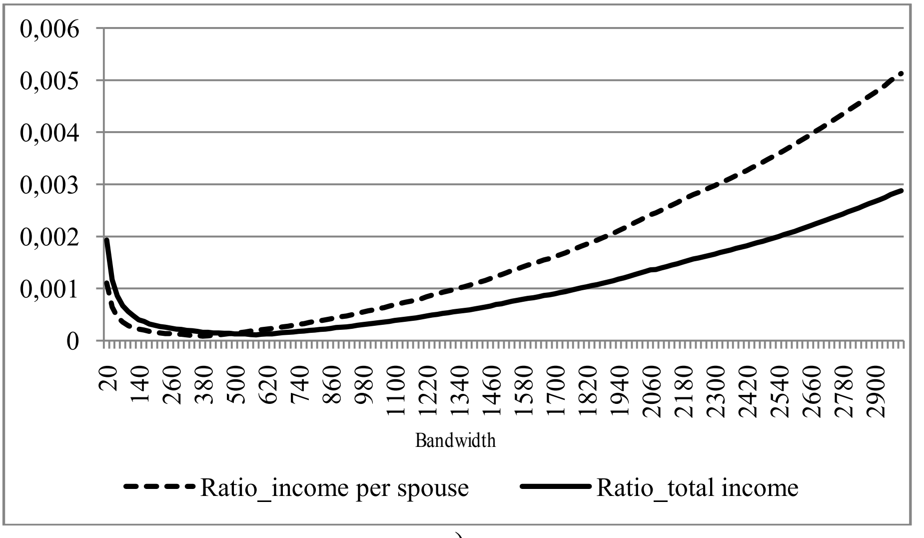


a)

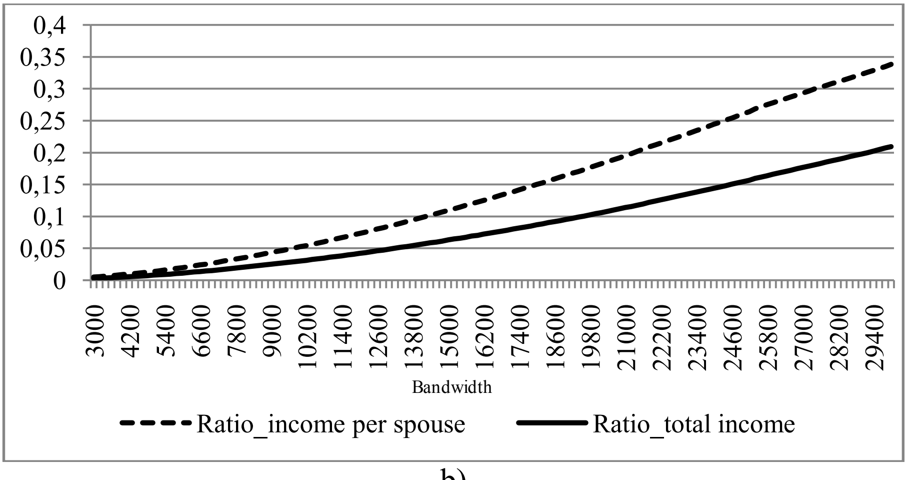


b)

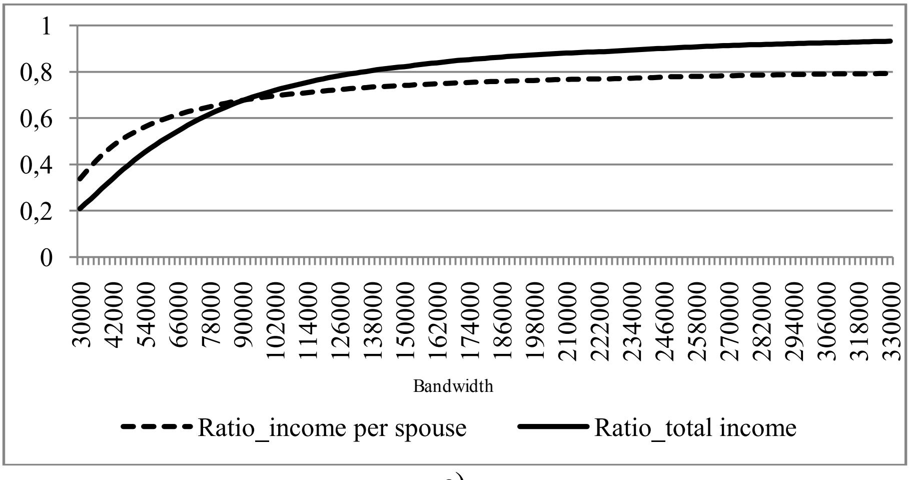
Figure 4: R^{EG} and $\mathrm{G}_{\mathrm{yt}}^{\mathrm{SW}} - \mathrm{G}_{\mathrm{y}}^{\mathrm{W}}$ as function of bandwidth: **a)** for total family income, **b)** for per-spouse income


0,006
0,005
0,004
0,003
0,002
0,001
0
20
140
260
380
500
620
740
860
980
1100
1220
1340
1460
1580
1700
1820
1940
2060
2180
2300
2420
2540
2660
2780
2900
Bandwidth
Ratio_income per spouse
Ratio_total income

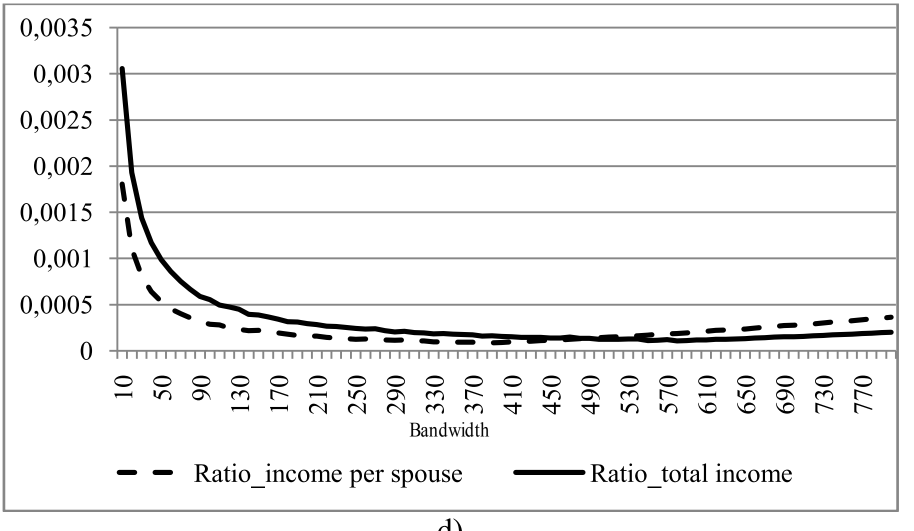
a)


0,4
0,35
0,3
0,25
0,2
0,15
0,1
0,05
0
3000
4200
5400
6600
7800
9000
10200
11400
12600
13800
15000
16200
17400
18600
19800
21000
22200
23400
24600
25800
27000
28200
29400
Bandwidth
Ratio_income per spouse
Ratio_total income

b)



c)



d)

Figure 5: Ratio for per-spouse income and total family income over different ranges

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