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THE ART OF CONJECTURING (ARS CONJECTANDI)[1]
ON THE HISTORICAL ORIGIN OF NORMAL DISTRIBUTION

Ludomir Laudański

Abstract. The paper offers a range of historic investigations regarding the normal distribution, frequently also referred to as the Gaussian distribution. The first one is the Error Analysis, the second one is the Probability Theory with its old exposition called the Theory of Chance. The latter regarded as the essential, despite the fact that the origin of the Error Theory can be associated with Galileo Galilei and his *Dialogo sopra i due massimi sistemi del mondo Tolemaico e Copernico*. However, the normal distribution regarded in this way was not found before 1808-9 as a result of the combined efforts of Robert Adrain and his *Researches Concerning Isotomous Curves* on the one hand and Carl F. Gauss and his *Theoria motus corporum coelestium in sectionibus conicis Solem ambienitum* on the other. While considering the Theory of Chance – it is necessary to acknowledge *The Doctrine of Chances* of Abraham de Moivre – 1733 and the proof contained in this work showing the normal distribution derived as the liming case of the binomial distribution with the number of Bernoulli trials tending to infinity. Therefore the simplest conclusion of the paper is: the normal distribution should be rather attributed to Abraham de Moivre than to Carl Friedrich Gauss.

Keywords: binomial distribution, Errors Theory, normal distribution.

Personal statement. One day I was asked what the origin of Gaussian distribution was and whether I may shortly explain its origin? After one month of efforts dedicated to this question I had to confess that my hasty answer was inaccurate and reflected my underestimation of the problem. But in one respect I was right, i.e. in considering two historical approaches leading towards the right answer.

Error Analysis exposed in the paper (Heller, Paderta 1974) in which the authors present the following axiomatic assertions to derive the normal distribution (quoted literally):

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[1] Reference to a title of an original paper written by Jacob Bernoulli in 1713 means: “forming an opinion or supposition about (something) on the basis of incomplete information”.

“1. the probability of occurrence of small random errors is greater than probability of occurrence of big random errors;

2. random errors of the same absolute value but opposite sign are equally probable”.

Developing this approach they arrived at a functional equation which, as you can easily check, meets the normal function. They also took care of appropriate normalization so that it could meet conditions of the probability density functions. The functional expression they got has only a single constant – the variance, which means that the mean is assumed to be zero. The above described result should be confronted with two important remarks. The first follows a suggestion given in (Juszkiewicz [Ed.] 1977) pointing out that the above stated axioms were first formulated by Galileo Galilei (1564-1642) in his famous *Dialogues* (Galileo Galilei 1962) while analyzing the problem of the astronomic observation of Nova year 1572 – stated by Salviatti (*porte parole* of the Author). Though the reader should be warned about extremely verbose character of the book dialogues (in Polish translation it covers pages 301-341) – therefore a significant effort and inquiring mind are required to fish out these axioms from the slowly flowing discourse in the *Dialogue*. This fact reduces merits of Heller and Paderta (Heller, Paderta 1974). Moreover they were completely unaware of the above described Galileo’s contribution to the field of Error Analysis (quoting instead some third-rate contemporary source book on this matter). But there is also a second important remark – this time pointing out to Carl Friedrich Gauss (1777-1854) – whose book *Theoria motus corporum coelestium in sectionibus conicis Solem ambienitum* published in Hamburg in 1809 is considered to be the first book on mathematical treatment of the experimental errors — therefore it is Gauss with whom they also have to share their final result (Gauss 1857). Thanks to the English translation of the Latin *Theoria motus*… – now accessible via the Internet – it is possible to track – step by step – the entire way in which Gauss obtained his functional equation finally leading to the desirable function of errors (the complete procedure covers pages 249-273, but the resulting functional equation is to be found on p. 258). To the above one has to add yet another finding which, in its part, narrows Gauss contribution to the matter. There was a little known American mathematician – Robert Adrain (1775-1843), who discovered that formula probably earlier (than Gauss) and published a paper related to the evaluation of the quantitative observations of the animal species of the sea (Adrain 1803) – and in this paper he also came to the “bell shaped curve". Now we come to an important question: whether the above described facts really lead to the discovery of the normal curve for the first time? The answer is NO. Therefore there are not so many important reasons to study in detail particular contributors working in this field. Below we shall present instead many more details regarding the other way which leads to the discovery of the normal distribution for the first time. Concluding this paragraph I propose a conclusion that credit for the discovery of the normal curve should not go to Gauss – not questioning his pioneering results in establishing the Error Analysis – he made so many brilliant mathematical discoveries, but in this one he was not the leader but rather a follower.

Theory of Chance, my favorite text on history of mathematics, for a long time has been a single volume book by Carl B. Boyer (Boyer 1985). Unfortunately this time it brought nothing but disappointment. Therefore I switched to the easily accessible in Poland books by Soviet historians (Juszkiewicz 1977) and (Майстров 1967) – despite cautious criticism and lack of confidence – nevertheless the passages devoted to the history of Probability and related disciplines I found in general sufficiently well coinciding with the other sources listed in Bibliography of this paper – among them with the history of mathematics by a German historian H. Wieleitner (Вилейтнер (1956)). It is time to state that the origin of the normal distribution under investigation in fact stems from *binomial distribution*. With respect to this distribution the situation is not so clear and we decided to leave this passage for the future task. The claim of the Russian historians whose books were here in Poland throughout decades easily accessible that we owe it to a Swiss mathematician Jacob Bernoulli (1654-1705) and his book *Ars Conjectandi* [for those who speak Russian I enclose translation of the title which seems to be well done *Исскуство умозаключений ибо Исскуство предположений*] are not true. Regarding the earlier origins the book of A. Edwards *Pascal's Arithmetical Triangle* points out that Pascal was well aware of binomial – more than 50 years earlier. Regarding the position of the book *Ars Conjectandi* I personally share the view that it is a milestone and the really first ever book on Probability. It was published posthumously in 1713 in Basel (later on some other details will be provided). Although this book reflects a dependence on a short paper by Christian Huygens (1629-1695) in Latin entitled *De Ratiociis in Ludo Aleae* (1657), which was republished by Bernoulli as the First Part of *Ars Conjectandi* with an inclusion of his own valuable remarks (which also will be discussed below, even twice). Also, in this place, what has to be acknowledged is the seemingly episodic role of Huygens's teacher – Franciscus van

Schooten (1615-1660), who included *De Ratiociis* as an Appendix to his own book, but first translating it from Flemish into Latin. Therefore perhaps we owe the sophisticated, elaborate Latin title of Huygens's paper to van Schooten. Ten years later Huygens published this paper on his own, but, what is not insignificant – in Flemish. This – one may say detail of secondary importance – in our opinion deserves appropriate attention. This is to underline how misleading sometimes are opinions expressed in a hurry just after a first encounter with the subject not looking at the surrounding facts which may suggest something unseen by them. And the Author of this paper in this respect may declare a guilty conscience of similar kind. Publishing *Ars Conjectandi* affords a prerequisite in a view of the binomial distribution for the second step – towards obtaining the normal distribution. It has been done – may be first for the special case of p = 1/2 by the Frenchman – Abraham (de) Moivre (1667-1754) – in his book *The Doctrine of Chances*. What we would like to expose is his moral strength. As a Huguenot he was expelled after 1686 from native France and settled down for the rest of his life in London. There his mathematical gifts started to flower so fruitfully that he was appreciated by Sir Isaac Newton (1642-1727) as a person who knows mathematics better than Newton himself. Nevertheless as a foreigner, because of the legal barriers, even after 50 years of residence, he could not get a steady position and suffered poverty, earning his living by giving tutorials. In spite of this he became a member of the Royal Society in 1697, and later a member of the Berlin Academy, and also a member of the Paris Academy. Abraham Moivre was a son of a surgeon and added the noble "de" himself. All those colorful facts are not to be found in (Boyer 1985).

My disappointment with the level of inquiries in the book by C.B. Boyer regarding de Moivre grows bigger due to the fact that the following formula – known nowadays as the Poisson's integral (or Euler-Poisson integral):

$$\int_0^\infty \exp(-x^2)dx = \sqrt{\pi}/2 \quad \text{[by the way, according to Wallis} = (\tfrac{1}{2}!)\text{]} \quad (1)$$

according to Boyer was known to de Moivre – and now, for the reader of Boyer's book it is obvious that its author was probably unaware that de Moivre was the discoverer of the two limiting theorems known widely as de Moivre-Laplace's theorems, which are heart of the matter in this context. And here we come to the point where – at least in this geographical region – the honor of historians was saved by Soviet historians (Juszkiewicz 1977; Майстров 1967). The latter monograph, which was published 3 years before the first one – ascribed the mentioned above case p = 1/2 to de Moivre's *The Doctrine of Chances,* but only after he published an Appendix to it in 1733, and the proof of the general case to Pierre Simon, Marquis de Laplace (1749-1827) and his *Treatise on Probability* (which we do not list in References as the author of the paper has never seen this book). However, in (Juszkiewicz 1977), a book of which one chapter (Chapter Four) was written by Maystrov without any additional remarks, the full credit for both limiting theorems (we write about them below) is ascribed to de Moivre. Unfortunately *Historia matematyki...* (Juszkiewicz 1977) despite numerous highly titled contributors holding scientific degrees in Physics and Mathematics – contains also evident mathematical *flaws*. Below I decided to present such a *flaw* in the field here under my attention.

This is the case incorporated into (the mentioned above) Chapter 4 *Theory of probability* – in a short paragraph entitled *Limiting theorems of A. de Moivre's* (it commences on p. 141 vol. 3 of Polish translation) which counts three pages, displays a picture of de Moivre and presents the following formula:

$$\lim_{n\to\infty} P\left\{ a \le \frac{\mu - n\,p}{\sqrt{n\,p\,q}} \le b \right\} = \frac{1}{\sqrt{2\,\pi}} \int_a^b \exp\left(-z^2/2\right) d\,z. \tag{2}$$

It has to be – as the inscription goes "integral theorem of de Moivre-Laplace". But it is not so! The reason is two-fold. First, there is no binomial distribution – hidden under the dumb symbol "P". Therefore, at least the authors of the text have to add what follows:

$$P(\mu) = \binom{n}{\mu} p^{\mu} q^{(n-\mu)} \quad \text{here} \quad \binom{n}{\mu} = \frac{n!}{\mu!(n-\mu)!} \tag{3}$$

But even so – there should be an additional term matching the integral limits („*a*" and „*b*") with the boundaries of the binomial distribution [compare this with (Laudański 2009). By the way our critical remark is also aimed at the publisher of the Polish translation – they overlooked this *flaw* – as well. There is also a quasi-historic remark to be expressed in this place. One may see a paradox in the label of the theorem: the first contributor was not a member of nobility but he uses "de" before his name, and by contrast – the second contributor was a noble but there is no "de" in front of his name. If we simplify the name of this theorem – rightly prescribing it only to de Moivre we may eliminate this paradox.

The above given will be "counterbalanced" by the critical review of the essay by Laplace (Simon 1951), although precisely speaking we shall concentrate our attention entirely on Chapter XVIII entitled *Historical notes concerning the calculus of probabilities* (pp. 185-196). This is the last Chapter of a very famous book and its reading has been done very carefully – which leads to some criticism described briefly below.

I have no doubt about the qualities of the distinguished Author's mind – it is brilliant and witty. But such a mind is hardly open to onerous, systematic and boring efforts of listing successive historic facts one after another. Therefore despite the well known to him significance of a letter which Chevalier de Mere sent to Toulouse to Pierre Fermat (1601-1665) and to Blaise Pascal (1623-1662) (where?), when recounting the story, Laplace begins from Fermat and Pascal. Among Polish language books this episode is described in details and preserves chronology presented in (Laudański 2009). Returning to Laplace I quote from (Simon 1951) how people of 17th century were seen by him (whether the French original is as sophisticated as English translation we may only guess): "illustrious century which has done the greatest honor to the human mind…".

The Enlightenment probably irreversibly impressed Laplace's mentality, therefore he could not help but frequently repeat phrases about the human mind. At the end of the Essay he offers a long passage from Cicero in which he speaks about superstition and religion – which may suggest some effort to disregard the problem of Christianity which he found troubling. This book was written in 1820 – so, there was enough time to completely change the attitude towards French Revolution of 1789 even a several times. It is probably impossible to establish how far the period of revolutionary zeal of Enlightenment reduced or enriched the scientific powers of Laplace. Trying to follow the efforts of a path finder in tracing Laplace's account towards these few scientists who are mainly interesting for us we look how Laplace writes about Huygens, and then Bernoulli. In general there are kind words about them. But our attention focuses when writing about *Ars Conjectandi* – Laplace stated what follows (p. 187 – at the top of the page): "the employment of the formula of the binomial in this kind of question…". Therefore it is rather clear that Laplace is questioning the attribution of the binomial theorem with respect to Bernoulli. And in the light of the mentioned above Treatise of Pascal – it is right questioning! The binomial (distribution) was known well before J. Bernoulli. A further part of the Chapter is for us and for the undertaken problem in this paper less interesting so I leave it without any further comments. A lack of mathematics also in Chapter XVIII – in the opinion of the author of this text – makes the content somewhat shallow or at least imprecise. So from this point of view the question which we struggle here to answer reading (Simon 1951) may cause some disappointment. It also raises a question — whether Laplace really satisfied laymen as he intended to?

The next topic among the literature oriented problems draws our attention again towards the pages of (Майстров 1967). There, on p. 70 the desired formula of the binomial distribution can be seen. Therefore the point has to be reported closer. This is the place where we can get an impression that I have arrived exactly at the desired origin. Looking around p. 70 it is seen that the *Ars Conjectandi* occupies numerous pages commencing from p. 67 to p. 89. Maystrov on his part commences by reporting that it was Nicolaus Bernoulli (1687-1759) who published the book of Jacob Bernoulli in 1713 – surprisingly, eight years after his death. Jacob was the elder brother of Nicolaus Bernoulli (1662-1716) whose son Nicolaus was the only child. The Bernoulli family was unique in history of modern science and "produced" mathematicians for 250 years. But such details are not important for this story, so I leave them aside. The essential story tells us that *Ars Conjectandi* contains four parts and an Appendix (curiously enough only this Appendix can be seen on Internet as a copy was made by the University Library in Upsala as it is documented by the Library stamp). For historians studying the development of the Probability Theory, Part Four is essential as it contains the most significant of Bernoulli's results – his limiting theorem ("weak law of big number"). But for us what becomes most essential is Part One of the book. This part contains a complete reprint of the book by Huygens with valuable comments given by Bernoulli. For the benefit of readers who speak Russian we provide the Russian translation of the title of the First Part of *Ars Conjectandi: Сочинение о возможных расчетах в азартной игре – Христиана Гюйгенса с замечаниями Я. Бернулли.* Translating this title into English we get: *Work on possible calculations in gambling games by Christiaan Huygens with comments of Jacob Bernoulli*, although the title given by the translator of [10] is somewhat different. As a specific feature of Maystrov (Майстров 1967) has to be mentioned the notorious habit of its Author of disregarding the original titles and his embarrassing mistake in calling the original language Dutch and not "Flemish". Maystrov calls the components of this Part „*предложения*" – so, it is difficult to find appropriate an English term – let us try "proposals". And this translation is also justified by (Bernoulli 2006). They can be classified into two groups – most of them are supplied with the comments by Bernoulli, but not all of them. Following the book by Maystrov (Майстров 1967) we finally arrive at the Proposal 12 and especially the valuable comment given by Jacob Bernoulli because here accordingly to (Майстров 1967) we find “Bernoulli’s formula” which is provided there as:

$$P_{m,n} = C_n^m p^m q^n \tag{4}$$

It is expected to be clear for the reader of (Майстров 1967) that the first symbol C_n^m denotes the number of combinations – with n to denote all trials, and m *chosen* trials – which now is frequently denoted by the symbol $\binom{n}{m}$. Interestingly enough L.E. Maystrov left unclear whether (4) is literally copied from *Ars Conjectandi* – or was presented in its contemporary fashion. So, this question cannot be solved without resorting to the pages of (Bernoulli 2006) and we shall clear up this point below. Nevertheless before one may make use of (Bernoulli 2006) the Internet resources offer apparent help. By entering *Ars Conjectandi* and commencing the search it is possible to find the following text: Another key theory developed in this part [Part I of *Ars Conjectandi*] was the probability achieving at least a number of successes from a number of events, today called Bernoulli trials, with multiple outcomes given that the probability of success in each was the same. Bernoulli showed through mathematical induction that given that a was the number of favorable outcomes in each event, b was the number of total outcomes in each event, d was the desired number of successful outcomes, and e was the number of events, the probability could be expressed as

$$P = \sum_{i=0}^{e-d} \binom{e}{d+i} \left(\frac{a}{b}\right)^{a+\upsilon} \left(\frac{b-a}{b}\right)^{e-d-i} . \tag{5}$$

The first part also discussed what is now known as the “Bernoulli distribution”.

For a reader well equipped with mathematical tools a lot of errors in the above given are obvious. Nevertheless, it seemingly mentions something which deserves our attention. A disturbing fact is expressed by the following question: how far is the above passage – on its side – is related to (Bernoulli 2006)? So, this needs some comment. Going this way we return again to the book (Майстров 1967). Studying the book by Bernoulli we cannot find the formula (4) at all – in other words – it was completely invented by Maystrov. Nevertheless, before Maystrov managed this supposition – he took a long ground run – beginning as we already know – with illustrious archaized title which we refresh here *Сочинение о возможных расчетах в азартной игре – Христиана Гюйгенса с замечаниями Я. Бернулли* which has no literal counterpart in (Bernoulli 2006) – but without resorting to the pages of the Latin original we cannot judge who was closer to it – the Russians or Sylla? Moreover, in this place it seems rational to mention the Preface addressed by Christiaan Huygens to Franciscus van Schooten – his teacher and translator of *De ratiociniis in ludo aleae*. We only try to draw the attention of our reader to this specific fact and nothing more – saving the space and volume of this paper. All the foregoing "Propositions" following immediately after the Preface (there are fourteen, not twelve Propositions) lead to the key Proposition XII. And here we quote its initial line describing the problem (Bernoulli 2006, p. 161), the line which we number:

To find with how many dice one may undertake
to throw two sixes on the first try (6)

Even if we are well equipped mathematically, it causes some confusion – what Huygens had exactly in his mind writing (6)? And here we seek the support from Sylla who writes (Bernoulli (2006, p. 345): "When Huygens asks, for instance,

In how many attempts might one undertake to throw a six

[this sentence for a sake of reference we number by (7)] –

he means to ask in how many attempts do the relative chances of achieving what one has undertaken to do become equal to or greater than one-half

[and this part we shall denote by (8)].

Unfortunately, this comment causes more evil than good – and the reader is facing a difficult trial in interpreting all the matter. The most likely – the reason is the translator's profession of Historian. The case prefers a Mathematician not a Historian. Her phrase (7) lost one but very important point – the key matter becomes the dice game not anything else (like for instance tossing a coin). So the phrase (7) does not deserve to be understood as a quotation – the true quotation is denoted by (6). And for her further explanation – denoted as (8) "let us draw the curtain of charity" – as likely said Mark Twain (compare his immortal *The Adventures of Tom Sawyer*). We propose to cut short this passage having in mind the main readers of this paper – i.e. mathematicians.

Instead we propose to turn our attention towards the meaning of "six" used in (6). It is evident that only one of the six facets of a die has six spots. Therefore, throwing a single die the chance to have "six" becomes equal to $\frac{1}{6}$. But "six" also means "six points" – and then – throwing two dice – we expect the following outcomes: (1, 5), (2, 4), and (3, 3). Throwing three dice – the outcomes would be: (1, 1, 4), (2, 2, 2), (3, 1, 2), (4, 1, 1). Throwing four – we expect two pleasurable outcomes: (1, 1, 1, 3), (1, 1, 2, 2). Though throwing five times there will be an empty outcome. The last not trivial outcome corresponds to six throwing – it will be (1, 1, 1, 1, 1, 1). These probabilities cannot be found by using the binomial distribution. And now: what exactly had in his mind Huygens writing the phrase (6)?! Stepping in this way and trying to find the answer – we finally shall arrive at the Comment given for Proposition XII by Bernoulli and here we shall face our main goal: how Bernoulli invented the binomial distribution or if we like to avoid using the term "distribution" – unknown for Bernoulli – how he arrived at formula (4)? First we have to say that the original solution given by Huygens takes less than half-page and the answer is: ten throwing of a single dice or a single throwing of ten dice. But the answer to our question about the binomial formula – given by Bernoulli – shall guide us through pages 163-170. Moreover, the most likely – if in our inquiry we shall drill this hard rock – our way will be much longer. The most likely – due to the balky terminology – we shall rather commence with the Proposition I – then, proceeding step by step and learning all the matter – case-after-case – proceed onwards. Therefore, our true beginning will be paged 133. In a due course of such efforts also our attitude towards the translator of the *Ars Conjectandi* will be softened and we shall finally renounce the previous critical reference towards her. We even may offer some technical explanation to save something which she suggests in (8). But with this we propose to our reader to take into consideration that fact that the proposed below approach possibly does not match this what she really intended to say. Namely, we propose to solve the following problem. Suppose we had a false coin – which secures probability of the Cross equal to $\frac{1}{6}$. We ask about the probability of getting not less than two Crosses in ten throws. This problem falls exactly into the scope of the binomial distribution – and the most likely we will be advised to solve the reverse problem – which is composed entirely out of two compound results – no Crosses, and a single Cross – their sum will lead to the numerical finding 0.484516747 – so, as we see – in this way we shall arrive very close to ½, i.e., to the result suggested by Sylla in (8). Now, it becomes obvious that in this way we annotated as well the book by

Maystrov with respect to the supposition given by (4) and also the quoted from Internet resources false formula (5) – as the formulas impossible to find among the pages of *Ars Conjectandi*.

As a curious fact given by Maystrov we shall consider a remark saying that one of the five unsolved problems left by Huygens for the reader of *De ratiociniis in ludo aleae* was solved and published by the renowned philosopher Baruch Spinoza (1632-1677). Unfortunately, Maystrov did not indicate which exactly problem out of the five unsolved problems was solved by Spinoza. Also the reader of (Bernoulli 2006) has no indicators as how to resolve this dilemma. On its side – Boyer (1985) offers another curious remark with respect to *The Doctrine of Chances* by Abraham de Moivre. Boyer says that de Moivre proceeds from the rules of probability – towards the rules of the combinatorial analysis – what is understood as a procedure which goes against normal procedures in colorful language used at the Polish Eastern borders expressed as „шыворот на выворот”. However, among the scope of the problems undertaken in this paper it is completely out of place.

To close our long search of the historical origins of normal law we have to state that all the details related to the two Theorems of de Moivre-Laplace we are pressed due to the space restrictions to consider as remaining out of the scope of our considerations given here.

In the end of the paper we shall once more turn towards papers (Shafer 1996) and (Edwards 2007) to offer short comments – although these comments have entirely bibliographical interest. Impatient reader may disregard them without any harm with respect to the title matter.

The paper by Glenn Shafer (Rutgers University) to be easily found in the Internet resources (while proceeding a search for *Ars Conjectandi*) is ended with the following mysterious sentences: “This is a prospect that Edith Sylla, Anthony Edwards, and I will soon complete our long-delayed translation, with commentary, of ‘Ars Conjectandi’. I hope that we shall be able soon to speak of the ‘Art of Conjecture’”.

On the other hand a careful reader of the Preface to (Bernoulli 2006) written by the translator E.D. Sylla shall find on p. XIX a comprehensive passage describing the initial stage of the book by Bernoulli especially with respect to the resources of the comments so necessary and vital in such edition. From there one may learn about the true initiator of the project who became mentioned Glenn Shafer – year 1984. He got a grant and he invited a historian Edith Doodley Sylla (North Carolina State University) and a mathematician Anthony W.F. Edwards (Cambridge University) as consultants. Why and when he had chosen these two – no mention. Then a curious thing took place – in 1995 Glenn Shafer withdrew from the project (this fact allows to derive the year of writing the Internet edition of his paper as 1995). An attempt taken by the author of this text to find out what happened regarding to Shafer failed. The same fate befell my efforts to disclose the circumstances of the withdrawal of Professor Anthony W.F. Edwards – who has professional relations with the author of this paper. Whatever could be said at this stage – Edith Doodley Sylla remained alone. She alone translated the Latin book of Jacob Bernoulli into English, she wrote all comments and notes – preparing a majestic book counting 580 pages. And it seems that she sacrificed about 20 years for this mega-project! Edwards on his side wrote a review cited above of which objectives he stated on the front page. Nothing more remains for the author of this paper than what I have already said, that is "to draw the curtain of charity at this stage".

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[In 1809, while analyzing errors in surveying and dead reckoning at sea, Adrain discovered the Gauss Distribution in Probability Theory, demonstrating that errors are distributed according to a bell-shaped curve $f(x) = C$ exp(–*hx*^2). Adrain was unaware that the French mathematician Adrien Legendre had asserted this without proof in 1805, and Karl Friedrich Gauss was to give a more rigorous proof later on, but Adrain's was the first proof – Copied from a text on the Internet: *A History of Mathematics at Rutgers* by Charles Weibel (originally written in 1995)].