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CONSTRUCTION OF A COMPOSITE MEASURE DESCRIBING THE QUALITY OF THE ENVIRONMENT IN POLAND

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Abstract. The concept of sustainable development is a hot and widely discussed issue. In this paper we will discuss only one aspect of sustainable development, namely environmental sustainability. Environmental sustainability is understood as a target state in a positive environment which is claimed by the rational economy of natural resources and outlays for environmental protection. Environmental sustainability assessment involves the construction of suitable metrics. The purpose of this paper is the construction of a composite measure that describes the state of the environment, in particular voivodships in Poland. Composite measure, describing the state of the environment and measures of sub-composite related to the specific domain of environmental sustainability, will be built based on selected indicators presented in "a shortlist of regional indicators" developed by the team of T. Borys (Borys (ed.), 2005). For the construction of composite measures, two different indicators of development will be applied. The arithmetic mean is used as an example of a linear measure. As regards a non-linear measure, we selected a development pattern method. We used these two methods to rank Polish voivodships in terms of the state of their natural environment. Subsequently, these composite measures were compared.

Keywords: composite measure, environmental sustainability, arithmetic mean, development pattern method.

JEL Classification: C39, Q59.

1. Introduction

Economic development and other related developments, including industrial expansion, construction of strip mines, replacement of natural ecosystems with artificial ones, chemicalization of agriculture, construction of roads and motorways, all have an intensely adverse environmental impact. It is vital that people who normally exploit three forms of capital –

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i.e. economic, human and natural capital – should, in the course of satisfying their needs, bear in mind that no form of capital must be either increased or used at the expense of the remaining two. It must be remembered that the natural environment has only so much potential to regenerate itself. Therefore, an analysis of the state of the natural environment seems to be crucial. The study aims to create a composite measure which would facilitate an assessment of the natural environment.

Two different measures of development were employed to create composite measures. An arithmetic mean was selected to represent the linear measure. Non-linear measures were represented by a measure based on the Euclidean distance. We used these two methods to rank Polish voivodships in terms of the state of their natural environment. Subsequently, these composite measures were compared. The comparison of linear and non-linear models involved both a study of the correlation between the values of individual measures (linear correlation coefficient) and an analysis of the concordance of rankings based on measures determined by means of both methods (Kendall's τ coefficient). The paper will test a hypothesis that the values of composite measures yielded by the linear approach will markedly differ from those produced by non-linear methods.

2. Indicator selection

Indicators are a function of one or more features and normally appear as the so-called intensity measures (Borys (ed.), 2005, p. 62.). Their most important quality involves value comparability, enabling researchers to rate an object against other objects. A properly created indicator of sustainable development exhibits a substantive relationship with the studied phenomenon. Furthermore, the indicator should comply with the formal correctness requirement, i.e. it should comply with generally accepted principles underlying the creation of the indicator.

This empirical study was based on "a shortlist of regional indicators" (Borys (ed.), 2005, p. 204), proposed by T. Borys. The shortlist comprises 73 indicators divided into three types of sustainable development: environmental, economic and social. Economic sustainable development is delimited by 18 indicators, and social sustainable development features 26 indicators. The majority of the 29 indicators underlying environmental sustainability are divided into 6 domains reflecting the objectives outlined in a document entitled "County's Ecological Policy 2009-2012 with 2016 Perspective" (*Polityka ekologiczna państwa w latach 2009-2012 z perspektywą do roku 2016*).

Owing to problems with the collection of the 2010 data for all Polish voivodships, we found it impossible to carry out an analysis incorporating all the indicators of environmental sustainability. Considering the availability of statistics on the indicators, we isolated only certain indicators for further analysis. These constitute the so-called final list.

The following criteria were used to select indicators:

– criterion of relevance,
– criterion of data availability,
– statistical criterion.

Ultimately, further analysis comprises of a list of 15 indicators. Data was sourced from Poland's Central Statistical Office[1] publications, including specifically "The Regional Data Bank" (*Bank Danych Regionalnych*) and "The Yearbook of Environmental Protection".

With respect to the indicators which were ultimately disregarded, even if they featured on the original list, it is worth noting that four of the short-listed environmental indicators relating to the share of a given class of water cleanliness in overall water were possible to use. Currently, a new mode of assessment of water cleanliness is in use. In its place, two indicators were employed which strive to assess the cleanliness of surface water. At the time of the drafting of this paper, no data on the share of renewable energy production (in %) in total energy production in 2010 was available. Data on the total use of individual substances depleting the ozone layer and data on the number of endangered species as a proportion of the total number of species (fauna and flora) (in %) is collected for the entire Polish territory with no breakdown by voivodship available.

We tried to purchase from the GIOŚ (Chief Inspectorate for Environmental Protection – *Główny Inspektorat Ochrony Środowiska*) data on four indicators relating to the average share of days when permitted emissions were not exceeded. The request was not granted.

Data indicating the quality of the acoustic climate – the number of inhabitants exposed to excessive noise pollution (in %) based on which maps of acousticsare developed – is collected by GIOŚ (Chief Inspectorate for Environmental Protection) and IOŚ (Institute of Environmental Protection – *Instytut Ochrony Środowiska*) for some cities and towns. However, these towns and cities do not represent all of Poland's voivodships, so the indicator was ignored.

[1] Publication available at the www.stat.gov.pl.website of Poland's Central Statistical Office.

Table 1. Indicators describing the condition of the natural environment

Domain	Symbol	Variable name	Unit	S/D*
D_1 – Water quality and its protection	$X_{1.1}$	Population enjoying access to water treatment plants in the overall population	%	*S*
	$X_{1.2}$	The waters with the status very good or good in the total number of natural water bodies of rivers	%	*S*
	$X_{1.3}$	The waters of the potential good and better than good status in the total number of artificial or heavily modified river water bodies	%	*S*
	$X_{1.4}$	Consumption of water for needs of the industry in total water use	%	*D*
	$X_{1.5}$	Underground water withdrawal as % of available resources	%	*D*
D_2 – Air quality and its protection	$X_{2.6}$	Emissions of particles from plants especially noxious	t/km^2	*D*
	$X_{2.7}$	Emissions of gases from plants especially noxious	t/km^2	*D*
	$X_{2.8}$	Traffic impact	%	*D*
D_3 – Conservation of Earth's surface and raw materials	$X_{3.9}$	Industrial waste utilized economically in the amount of waste produced during the year	%	*S*
	$X_{3.10}$	Municipal waste exported *per capita* per year	t/*per capita*	*D*
	$X_{3.11}$	Sorted waste in the total amount of municipal waste collected	%	*S*
D_4 – Quality and conservation of acoustic climate	$X_{4.12}$	Plants exceeding permissible noise levels as % of establishments inspected	%	*D*
D_5 – Conservation of nature and landscape and spatial management	$X_{5.13}$	Surface of recreational parks and forests per inhabitant	ha/*per capita*	*S*
	$X_{5.14}$	Legally protected areas of outstanding natural qualities in the overall area	%	*S*
D_6 – Protection against radiation and extreme environmental threats	$X_{6.15}$	Number of extraordinary events that might cause environmental hazards in the area of 100 km^2	number/ 100 km^2	*D*

* nature of a variable: *S* – stimulants, *D* – destimulants

Source: own elaboration.

Neither were we able to use an indicator revealing the number of areas particularly exposed to emissions of traffic noise in voivodship's overall area (in %). A replacement indicator offered by GIOŚ called "length of urban streets along which noise pollution exceeds the maximum permitted level as a percentage of analyzed lengths of urban streets" does not lend itself to studies because of the non-variation between voivodships. In nearly all cases, the number of streets with an excessive maximum level is equivalent to 100%. The coefficient of variation for 2010 was below 10%.

Three indicators on the "shortlist of regional indicators of environmental sustainability" representing environmental outlays were also ruled out because of the very nature of this study.[2]

Table 1 shows a synthetic description of indicators that describe the condition of the natural environment selected for further analysis.

The objects of analysis are constituted by voivodships. Their names and abbreviated names are shown in Table 2.

Table 2. Names and abbreviated names of voivodships

Abbreviated names	Names of voivodships
DO	Dolnośląskie
KP	Kujawsko-pomorskie
LL	Lubelskie
LB	Lubuskie
LO	Łódzkie
ML	Małopolskie
MZ	Mazowieckie
OP	Opolskie
PK	Podkarpackie
PD	Podlaskie
PO	Pomorskie
SL	Śląskie
SW	Świętokrzyskie
WM	Warmińsko-mazurskie
WK	Wielkopolskie
ZP	Zachodniopomorskie

Source: own elaboration.

[2] The study aims, in fact, to assess the state of Poland's natural environment.

To ensure the comparability of variable measurement units, the variables need to be normalized. It is crucial to select the right normalization method for further research, and for the purpose of this paper we resolved to apply the zero unitarization method, which has the following formula (Kukuła, 2000, p. 79):

$$x'_{ij.r} = \begin{cases} \dfrac{x_{ij.r} - \min\limits_{i=1,...,16}\{x_{ij.r}\}}{\max\limits_{i=1,...,16}\{x_{ij.r}\} - \min\limits_{i=1,...,16}\{x_{ij.r}\}} & \text{for stimulants} \\ \\ \dfrac{\max\limits_{i=1,...,16}\{x_{ij.r}\} - x_{ij.r}}{\max\limits_{i=1,...,16}\{x_{ij.r}\} - \min\limits_{i=1,...,16}\{x_{ij.r}\}} & \text{for destimulants} \end{cases}, \qquad (1)$$

where: $x'_{i,j.r}$ – value of normalized variable $X'_{j.r}$ for indicator ($r = 1, ..., 15$) pertaining to domain ($j = 1, ..., 6$) and for object O_i ($i = 1, ..., 16$), $x_{i,j.r}$ – value of variable $X_{j.r}$ for object O_i.

The variables subjected to zero unitarization meet normalization's two crucial requirements: they are purged of measurement units in which diagnostic features are expressed and further they bring variables to a comparable level, which means comparable ranges of quality variation, there by allowing their summation.[3] Furthermore, this normalization mode satisfies the requirement of equal length of the variation range in the values of all normalized features and equidistant floor and cap, and especially with regard to zero unitarization we are in effect dealing with a [0,1] variations range. The remaining normalization requirements are likewise met, i.e. it is possible to normalize variables which are positive, negative or equivalent to zero[4] and such normalization yields non-negative values. As indicated by formula (1), there also exist simple formulas which make the nature of a variable uniform.

[3] According to T. Borys, this is one of the main objectives of normalization which can be found in the work (Borys, 1978).

[4] Studies did not feature variables with a negative value, but zero variables were used. Consequently, it was impossible to carry out a quotient transformation with fixed min. and max. parameters.

The fact that the zero unitarization method meets all the requirements of normalization makes the method highly universal and explains why it was employed in the study. It should be borne in mind that variables so normalized have the nature of stimulants and their values measured on a scale fit within [0,1] range.

3. Creating a composite measure

The paper compares two composite measures created by means of different development measures. The first measure concerns a pattern-free linear measure which was determined as an arithmetic mean. The other method is based on Euclidean distance and is a non-linear development pattern method.

A pattern-free sub-composite measure, which is an arithmetic mean of normalized values of characteristics, is computed for different domains in accordance with the following formula:

$$s_{ij} = \frac{1}{m}\sum_{j=1}^{m} x'_{ij.r}, \qquad i = 1,...n,\ \ j = 1,...,d,\ \ r = 1,...,m, \tag{2}$$

where: $x'_{ij.r}$ – normalized value of a *j.r* characteristic of an *i*-object, n – number of analyzed objects (16 voivodships), m – number of characteristics employed (15 indicators) divided into d – domains of environmental sustainability (6 domains).

Likewise, a composite measure describing comprehensively the state of the natural environment is determined as an arithmetic mean of the sub-composite measures relating to individual domains of environmental sustainability and was computed based on the following formula:

$$S_i = \frac{1}{d}\sum_{j=1}^{d} s_{ij}, \qquad i = 1,...n,\ \ j = 1,...,d, \tag{3}$$

where: s_{ij} – sub-composite measure relating to j domains of environmental sustainability for an i-object, n – number of analyzed objects, d – number of domains of environmental sustainability.

The main charge levied against a so created linear measure involves the fact that it overstates the results of composite valuation compared to the requirements of sustainable development; namely it favors the sum of individual indicators while overlooking their diversity.

As regards a non-linear measure, we selected Euclidean distance. A version of the non-linear sub-composite measure used as a development pattern method was computed according to the following formula:

$$m_{ij} = 1 - \frac{d_{i(1)j}}{d_{(0)(1)j}} \tag{4}$$

with: $d_{i(1)j}$ – distance of an *i*-object from the pattern:

$$d_{i(1)j} = \sqrt{\sum_{r=1}^{m} \left(x'_{ij.r} - 1 \right)^2} \tag{5}$$

$d_{(0)(1)j}$ – anti-pattern's distance from the pattern:

$$d_{(0)(1)j} = \sqrt{k}, \tag{6}$$

where: k – number of indicators in a given domain.

An analogous composite measure comprehensively describing the state of the natural environment is computed using the following formula:

$$M_i = 1 - \frac{d_{i(1)}}{d_{(0)(1)}} \tag{7}$$

with: $d_{i(1)}$ – distance of an *i*-object from the pattern:

$$d_{i(1)} = \sqrt{\sum_{r=1}^{m} \left(m_{ij} - 1 \right)^2} \tag{8}$$

$d_{(0)(1)}$ – anti-pattern's distance from the pattern:

$$d_{(0)(1)} = \sqrt{m}, \tag{9}$$

where: m – number of domains.

A non-linear measure offering a comparison to the pattern favors both smoothing and the sum of the levels of individual indicators.[5] Under this method, a high value of the composite measure is assigned only to those objects for which none of the indicator levels is strikingly low. Consequently, it is assumed that from the sustainable development point of view, the method is more rational than the linear model.

Based on the formulas (2-9), we determined the values of sub-composite measures for the individual domains of environmental sustainability and values of the composite measure describing the state of the natural environment in different voivodships. The values of the measures are shown in Table 3.

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[5] For more information see (Borys, 2008, p. 112).

Table 3. The values of the composite measures for individual voivodships 2010

Voivodships	Linear measure							Non-linear measure						
	s_1	s_2	s_3	s_4	s_5	s_6	S	m_1	m_2	m_3	m_4	m_5	m_6	M
Łódzkie	0.52	0.67	0.33	0.29	0.01	1.00	0.47	0.48	0.63	0.29	0.29	0.01	1.00	0.37
Mazowieckie	0.36	0.77	0.47	0.46	0.14	0.66	0.48	0.26	0.74	0.45	0.46	0.14	0.66	0.41
Małopolskie	0.49	0.74	0.84	0.58	0.49	0.00	0.52	0.46	0.72	0.77	0.58	0.44	0.00	0.44
Śląskie	0.54	0.00	0.61	0.18	0.23	0.61	0.36	0.51	0.00	0.51	0.18	0.22	0.61	0.30
Lubelskie	0.40	0.94	0.82	0.59	0.07	0.84	0.61	0.33	0.93	0.77	0.59	0.07	0.84	0.49
Podkarpackie	0.68	0.93	0.87	0.61	0.57	0.82	0.74	0.63	0.92	0.86	0.61	0.57	0.82	0.70
Podlaskie	0.55	0.99	0.54	0.34	0.31	0.84	0.59	0.47	0.99	0.40	0.34	0.31	0.84	0.49
Świętokrzyskie	0.24	0.77	0.79	0.22	0.62	1.00	0.61	0.21	0.76	0.64	0.22	0.46	1.00	0.47
Lubuskie	0.69	0.86	0.45	0.04	0.72	0.77	0.59	0.60	0.79	0.41	0.04	0.61	0.77	0.47
Wielkopolskie	0.27	0.80	0.62	0.00	0.23	0.92	0.47	0.25	0.78	0.58	0.00	0.23	0.92	0.37
Zachodniopomorskie	0.61	0.91	0.32	0.48	0.28	0.65	0.54	0.46	0.91	0.30	0.48	0.25	0.65	0.46
Dolnośląskie	0.37	0.75	0.35	0.13	0.16	0.68	0.41	0.26	0.75	0.30	0.13	0.14	0.68	0.33
Opolskie	0.43	0.71	0.55	0.70	0.20	0.66	0.54	0.38	0.70	0.52	0.70	0.20	0.66	0.49
Kujawsko-pomorskie	0.85	0.83	0.66	1.00	0.18	0.29	0.63	0.80	0.82	0.64	1.00	0.18	0.29	0.52
Pomorskie	0.52	0.87	0.45	0.06	0.42	0.74	0.51	0.41	0.86	0.40	0.06	0.41	0.74	0.42
Warmińsko-mazurskie	0.86	1.00	0.61	0.70	0.47	0.67	0.72	0.83	1.00	0.54	0.70	0.46	0.67	0.65

Source: own calculations.

In domains four (quality and conservation of acoustic climate) and six (protection against radiation and extreme environmental threats), the values of the sub-composite measures determined by means of various methods are equal for both methods. The reason is that both these domains are characterized by a single measure of environmental sustainability. In the remaining cases, the measures determined on the basis of an arithmetic mean have a value higher than the measures based on Euclidean distance.

4. Comparison of the values of composite measures

To illustrate the differences between the values of sub-composite measures for individual domains derived from various methods, the measures values were presented in Fig. 1. Importantly, the figure presents voivodships ordered in accordance with the values of the measure yielded by the non-linear method.

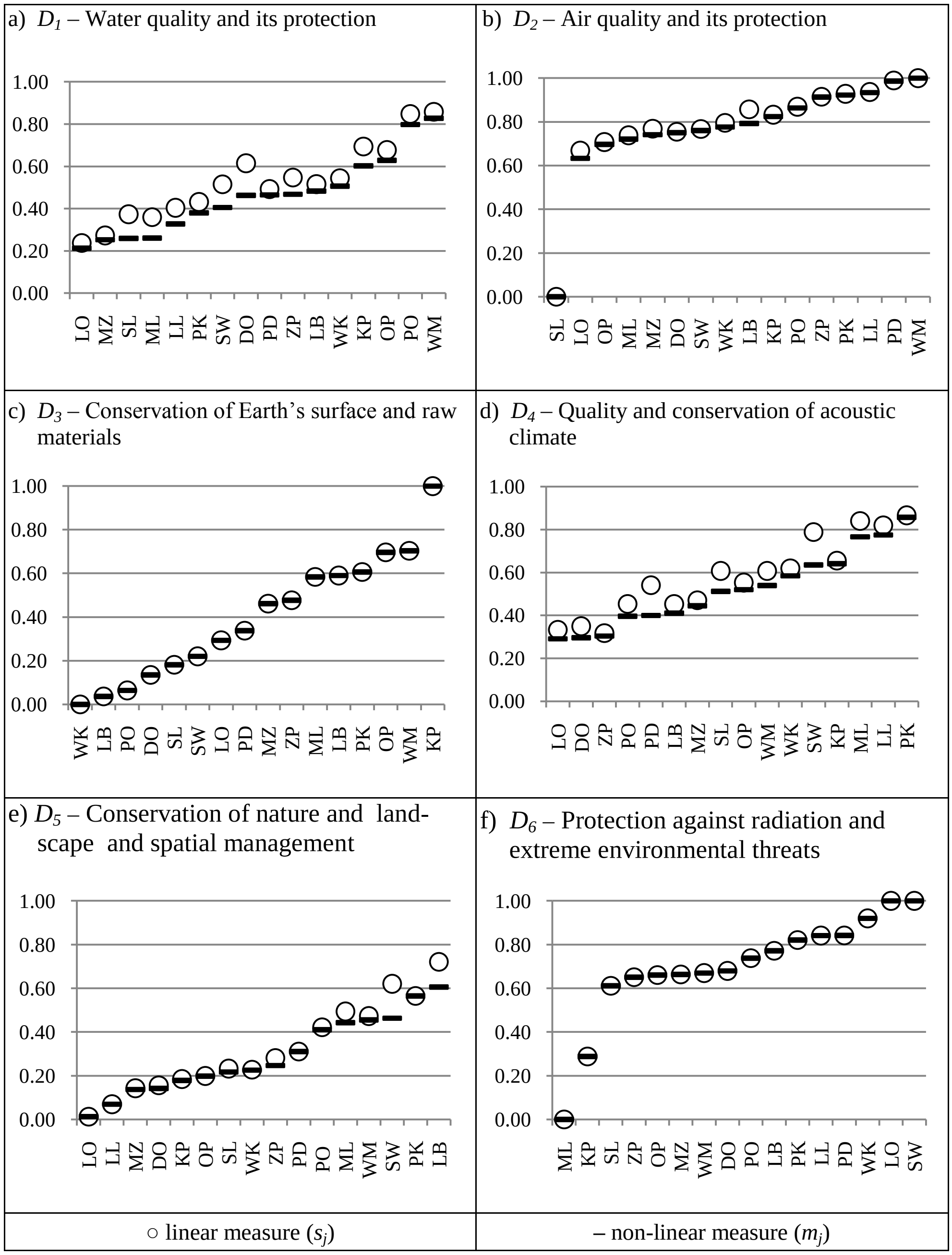

Fig. 1. The values of sub-composite measures for individual domains of environmental sustainability derived from various methods in voivodships in 2010

Source: own elaboration.

Fig. 1 reveals the greatest similarity in the measure values in domain two (air quality and its protection). The *Lubuskie* Voivodship alone reveals the value of the sub-composite measure computed as an arithmetic mean to be markedly higher than the value of the non-linear measure. This voivodship enjoys a very favorable situation in terms of emissions of both particles and gases (the values of the normalized variable were 0.95 and 0.98, respectively), and a slightly less favorable situation in respect of the indicator of traffic impact (the values of normalized variable stood at 0.64). The measure, based on Euclidean distance, incorporated a markedly lower level of one of the indicators and prevented the assignment of a high value of the sub-composite measure to this voivodship.

Somewhat more numerous differences between the results derived from these two methods were ascertained in respect of area five (conservation of nature and landscape and spatial management). In the case of this domain, the values of the measure yielded by the linear method were overstated for the following voivodships: *Zachodniopomorskie*, *Małopolskie*, *Świętokrzyskie* and *Lubuskie*. In the *Zachodniopomorskie* Voivodship, the surface of recreational parks and forests per inhabitant is at an average level (the value of normalized variable stands at 0.51), but the share of legally protected areas of outstanding natural qualities in the overall area of the voivodship is abnormally low (the value of normalized variable is 0.06). The opposite situation was ascertained in the *Małopolskie* Voivodship. The *Świętokrzyskie* and *Lubuskie* Voivodships report the best situation in Poland in terms of one of the two indicators pertaining to this domain (the value of normalized variable called "area of recreational parks and forests per inhabitant" stands at 1 in the *Lubuskie* Voivodship, while in the *Świętokrzyskie* Voivodship the level of 1 is revealed by a normalized variable called "share of legally protected areas of outstanding natural qualities in the voivodship's overall area"), although the value of the latter indicator is markedly lower. The linear measure overlooked variations in values and assigned a higher value of the sub-composite measure to these voivodships.

Notably stronger differences between the values of sub-composite measures computed by means of the different methods can be observed in respect of domains: one (water quality and its protection) and three (conservation of the Earth's surface and raw materials).

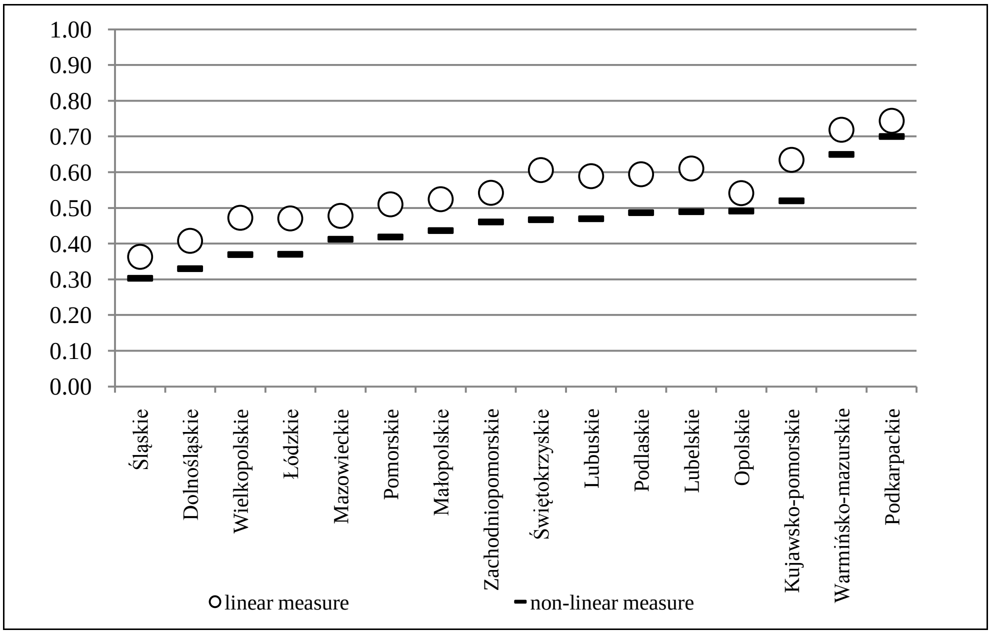

Fig. 2. The value of the composite measure based on all the indicators describing the state of the natural environment in voivodships in 2010

Source: own elaboration.

The strongest differences between the linear and non-linear methods (cf. Fig. 2) are revealed by the value of the composite measure based on all the indicators describing the state of the natural environment. This is particularly obvious in respect of the *Świętokrzyskie* Voivodship, whose arithmetic mean value exceeds that for the *Lubuskie*, *Podlaskie* and *Opolskie* Voivodships. Euclidean distance factored in the fact that in spite of its fairly favorable situation in terms of the majority of indicators, the *Świętokrzyskie* Voivodship fares worst in Poland in terms of the share of population enjoying access to water treatment plants in the overall population (the value of the normalized variable is 0.00).

To compare the values of the measures, we calculated the linear correlation coefficient between the values of the sub-composite measures in individual domains and the values of the composite measure describing the state of the natural environment derived from both methods. The results are shown in Table 4.

The values presented in Table 4 corroborate the initial observations (based on Fig. 1 and 2) of a strong correlation between the composite measures determined by means of different methods. The values of the linear correlation coefficient evidence a very high linear dependence between the values of linear and non-linear measures for all the domains of environmental sustainability. Moreover, the correlation between composite measures comprehensively describing the state of the natural environment is very strong (r = 0.9649).

Table 4. The linear correlation coefficient between values of composite measures

Domain	D_1	D_2	D_3	D_4	D_5	D_6	D
r	0.9775	0.9973	0.9724	1.0000	0.9832	1.0000	0.9649

Source: own calculations.

For the purpose of comparison, we checked if such a strong correlation between the values of the measures yielded by various methods used to create a composite measure would be confirmed if voivodships were ordered on the basis of a createdc omposite measure. By looking at Fig. 1, the reader will notice that in respect of domain one (water quality and its protection) and domain three (conservation of the Earth's surface and raw materials) the order of voivodships derived by means of the arithmetic mean is somewhat different from that based on Euclidean distance.

With a view to that, the Kendall τ coefficient is carried out between rankings compared on account of a composite measure carried out by means of the linear method and orders based on a measure created by means of a non-linear method. The paper uses the following Kendall's τ coefficient (Walesiak, 2006):

$$\Gamma_{lh} = \frac{\sum_{i=2}^{16}\sum_{k=1}^{i-1} a_{ikl} b_{ikh}}{\left[\sum_{i=2}^{16}\sum_{k=1}^{i-1} a_{ikl}^2 \sum_{i=2}^{16}\sum_{k=1}^{i-1} b_{ikh}^2\right]^{\frac{1}{2}}}, \tag{10}$$

where: $i, k = 1, \ldots, n$ – object's number, l, h – variable's number; and substitution:

$$a_{ikl}\left(b_{ikh}\right)=\begin{cases}1 & \text{when} \quad x_{il}>x_{kl} \quad \left(x_{ih}>x_{kh}\right) \\ 0 & \text{when} \quad x_{il}=x_{kl} \quad \left(x_{ih}=x_{kh}\right) \\ -1 & \text{when} \quad x_{il}<x_{kl} \quad \left(x_{ih}<x_{kh}\right)\end{cases}, \tag{11}$$

where: x_{il}, x_{kl} – i observation for an l variable.

Kendall's τ coefficient takes values ranging between [–1,1]. 1 implies utter concordance of ordering, whereas –1 indicates the very opposite.

Table 5. Kendall's τ coefficient between values of composite measures

Domain	D_1	D_2	D_3	D_4	D_5	D_6	D
τ	0.85	0.95	0.85	1.00	0.95	1.00	0.87

Source: own calculations.

The values of Kendall's τ coefficient shown in Table 5 confirm a high concordance of ordering (Γ = 0.95) in domains two (air quality and its protection) and five (conservation of nature and landscape and spatial management), which was also revealed earlier in Fig. 1 (parts: b and e). The differences in ordering ascertained in domains: one (water quality and its protection) and three (conservation of the Earth's surface and raw materials) were confirmed by a lower value of Kendall's τ (Γ = 0.85).

Interestingly enough, despite the fact that the value of the linear correlation coefficient (cf. Table 4) was the lowest in respect of general composite measures, and further, despite the fact that the differences ascertained between the values of individual measures determined by means of these two methods (cf. Fig. 2) were considerable, Kendall's τ coefficient reveals a stronger concordance of ordering (Γ = 0.87) of voivodships based on both models than was ascertained for domains one and three.

5. Conclusion

The research was conducted to create a composite measure which could be used to assess the state of the natural environment in Poland's voivodships. A measure was developed on the basis of indicators of environmental sustainability. It must be emphasized that although the issue of sustainable development is a hot and widely debated issue, yet there are still no adequate or sufficient statistics on individual voivodships to allow the use of all the proposed indicators of environmental sustainability. Consequently, in creating the measure, we were forced to constrain ourselves to a selection of just 15 indicators capturing the state of the natural environment, which were subsequently grouped into 6 domains.

In the first approach, the sub-composite measures for the individual domains and a composite measure comprehensively describing the state of the natural environment were determined by means of the linear method, specifically by calculating the arithmetic mean of normalized variables. The voivodships were ordered on the basis of such measures.

In the second approach, a non-linear one, use was mainly made of a measure based on Euclidean distance, and a square function was used to calculate distance: a generalized measure of Euclidean distance. Pertinent literature underlines the fact that the method better satisfies the requirement of harmonious development, as it does not overstate the values of the measure when any of the indicators assumes abnormally low levels, which the linear method does.

Comparative analysis based on a linear correlation coefficient revealed that the values of measures determined by means of the various methods are strongly correlated. Although observation of individual values of the composite measures revealed that the measures carried out by means of the linear method, as a rule, have a higher value than those computed on the basis of the non-linear method, yet this had no meaningful impact on the ordering of the voivodships, which was shown by the analysis of Kendall's τ coefficient.

The paper aimed to compare two methods of composite measure creation. It was expected that the differences between the variants would be meaningful. Ultimately, it can be assumed that the non-linear method is a better solution, especially in respect of a domain which reveals disparate indicators. Even if somewhat more complex mathematically, the method substitutes the values of individual indicators more rationally than the linear model. A certain limitation of the method arising from the very small size of the comparative group, which somewhat degrades this method of determining a pattern and anti-pattern must, however, be noted.[6]

[6] For more information see (Borys, 2008, p. 114).

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