THE BULLWHIP EFFECT
IN SUPPLY CHAINS
WITH STOCHASTIC LEAD TIMES

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Abstract. In this article we consider a simple two stage supply chain. We quantify the variance amplification of orders – the bullwhip effect in a model with stochastic lead times. Employing the moving average forecasting method for lead time demands we obtain an exact value of the bullwhip effect measure. We analyze the formula using numerical examples.

Keywords: supply chain, bullwhip effect, inventory policy, stochastic lead time, lead time demand forecasting.

1. Introduction

Supply chains are networks of firms (supply chain members) which act in order to deliver a product to the end consumer. Supply chain members are concerned with optimizing their own objectives and this results in the poor performance of the supply chain. In other words, the local optimum policies of members do not result in the global optimum of the chain and they yield the tendency of replenishment orders to increase in variability as one moves up in a supply chain. Such a phenomenon is called the bullwhip effect. This effect was recognized by Forrester (Forrester 1958) in the middle of the twentieth century. The term, bullwhip effect, was coined by Procter & Gamble management. The bullwhip effect is considered harmful because of

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its consequences which are (see e.g. Buchmeister et al. 2008): excessive inventory investment, poor customer service level, lost revenue, reduced productivity, more difficult decision-making, sub-optimal transportation, sub-optimal production etc. This makes it critical to find the root causes of the bullwhip effect and to quantify the increase in demand variability at each stage of the supply chain. In the current state of research typically five main causes of the bullwhip effect are considered (see e.g. Lee et al. 1997a and 1997b): demand forecasting, non-zero lead time, supply shortage, order batching and price fluctuation. To decrease the variance amplification in a supply chain (i.e. to reduce the bullwhip effect) we need to identify all the factors causing the bullwhip effect and to quantify their impact on the effect.

Many researchers assuming a deterministic lead time have studied the influence of different methods of demand forecasting on the bullwhip effect such as simple moving average, exponential smoothing, and minimum mean-squared-error forecasts when demands are independent, identically distributed or constitute an integrated moving-average, autoregressive process or autoregressive-moving average (see Graves 1999, Lee et al. 2000, Chen et al. 2000a and 2000b, Alwan et al. 2003, Zhang 2004 and Duc et al. 2008). Moreover, they quantify the impact of a deterministic lead time on the effect which, as follows from their works, is one of the major factors causing the effect. Stochastic lead times were intensively investigated in inventory systems see Bagchi et al. 1986, Hariharan and Zipkin 1995, Mohebbi and Posner 1998, Sarker and Zangwill 1991, Song 1994a and 1994b, Song and Zipkin 1993 and 1996 and Zipkin 1986. Most of these works consider the so-called exogenous lead times, that is they do not depend on the system e.g. the lead times are independent of the orders and the capacity utilization of a supplier. Moreover, these articles studied how the variable lead times affect the control parameter, the inventory level or the costs. Recently the impact of stochastic lead times on the bullwhip effect has been intensively investigated. One can investigate the so-called endogenous lead times, that is depending on the system. This is analyzed in So and Zheng (2003) showing the impact of endogenous lead times on the amplification of the order variance. In the paper of Kim et al. (2006) the bullwhip effect performance is given under the condition that lead time demands are predicted using the moving average forecasting method. Another work investigating stochastic lead times in the context of the bullwhip effect is the paper of Duc et al. (2008). They assume that lead times are independent and identically distributed and demands constitute a first-order autoregressive AR(1) process or a mixed first-order autoregressive-moving average

ARMA(1,1) process. The disadvantage of this approach is that the lead times are not predicted to make an order. In practice this is not feasible. Michna and Nielsen (2013) present a model where lead times and demands are forecasted separately using the moving average method. They indicate and quantify the impact of lead time forecasting on the bullwhip effect.

The main aim of this article is to quantify the bullwhip effect in a model with stochastic independent identically distributed lead times where orders are made by the lead time demands which are forecasted using the moving average method. We will modify the model of Kim et al. (2006) because their approach and assumptions seem to be infeasible in real supply chains.

2. Supply chains and the bullwhip effect

In recent studies a supply chain is a system of organizations, people, activities, information, and resources involved in moving a product or service from suppliers to customers. More precisely a supply chain consists of customers, retailers, warehouses, distribution centers, manufactures, plants, raw material suppliers etc. They are members or stages (echelons) of a given supply chain. A supply chain has a linear order which means that at the bottom there are customers, above customers there is a retailer, above the retailer there is for example a manufacturer and so on. The linear order is determined by the flow of products which stream down from the supplier through the manufacturer, warehouse, retailer to the customers. Financial and information flows can accompany the flow of products. The simplest supply chain can consist of customers (customers are not regarded as a stage in many articles), a retailer and a manufacturer (a supplier). At every stage (except customers) a member of a supply chain possesses a storehouse and uses a certain stock policy (a replenishment policy) in its inventory control to fulfill its customer's (a member of the supply chain which is immediately below) orders in a timely manner. Commonly used replenishment policies are: the periodic review, the replenishment interval, the order-up-to-level inventory policy (out policy), (s, S) policy, the continuous review, the reorder point (see e.g. Zipkin 2000). A member of a supply chain observes the demands from the stage immediately below and lead times from the stage immediately above. Based on the previous demands and lead times and using a certain stock policy, the member of a chain makes an order to its supplier which is the stage immediately above. Thus at every stage one can observe the demands from the stage below and replenishment orders sent to the stage above. The phenomenon of the variance amplification in replen-

ishment orders if one moves up in a supply chain is called the bullwhip effect (see Disney and Towill 2003 and Geary et al. 2006 for the definition and historical review). Munson et al. (2003) assert: "When each member of a group tries to maximize his or her benefit without regard to the impact on other members of the group, the overall effectiveness may suffer". The bullwhip effect is the key example of a supply chain inefficiency.

Many researchers use the ratio of variances as the bullwhip effect measure that is if q is a random variable describing the demands (orders) of a member of the supply chain to the member above and D is a random variable responsible for the demands of the member below (e.g. q describes the demands of a retailer to a manufacturer (supplier) and D shows the customer's demands to the retailer) then the measure of performance of the bullwhip effect is the following

$$BM = \frac{\mathbf{Var}(\text{orders}) \, / \, \mathbf{E}(\text{orders})}{\mathbf{Var}(\text{demands}) \, / \, \mathbf{E}(\text{demands})} = \frac{\mathbf{Var}q \, / \, \mathbf{E}q}{\mathbf{Var}D \, / \, \mathbf{E}D}.$$

Usually in most models $\mathbf{E}D = \mathbf{E}q$. The value of BM is greater than one in the presence of the bullwhip effect in a supply chain. If BM is equal to one then there is no variance amplification, whereas BM smaller than one indicates dampening which means that the orders are smoothed compared to the demands. Another very important parameter of the supply chain members is the measure of the net stock amplification of a given supply chain member. More precisely let N_s be the level of the net stock of a supply chain member (e.g. a retailer or a supplier) and D be the demands observed from its downstream member (customers or a retailer), then the following measure

$$NSM = \frac{\mathbf{Var}(\text{net stock})}{\mathbf{Var}(\text{demands})} = \frac{\mathbf{Var}(N_s)}{\mathbf{Var}D}$$

is a very important parameter of the supply chain performance. In many models it is assumed that the costs are proportional to $\sqrt{\mathbf{Var}(\text{orders})}$ and to $\sqrt{\mathbf{Var}(N_s)}$.

3. Models with stochastic lead times

The lead time is regarded as the second cause of the bullwhip effect, after demand forecasting. Lead times are made of two components, physical delays and information delays. In the models one does not distinguish between those components. The lead time is the time between when an order

is placed by a member of a supply chain and the period when the product is delivered to the member. The assumption that the lead time is constant is rather unrealistic. Undoubtedly in many supply chains physical and information delays are random which means that a member of a supply chain does not know the values of the future lead times and in the past he/she observed that their values were varied in a stochastic manner. For instance in the paper of So and Zheng (2003), the model of a supply chain with stochastic lead times is motivated by the semiconductor industry where the dramatic boom-and-bust cycles cause delivery lead times to be highly variable, ranging from several weeks during the low demand season to over several months during the high demand season. Moreover, in the models investigating the bullwhip effect one can decide how time is represented. There are two choices, discrete or continuous time. We will analyze the stochastic techniques which use discrete time. We assume that the observations are made at integer moments of time which means that time is represented in units of the review periods and nothing is known about the system in the time between observations.

The main difference in models with stochastic lead times lies in the definition of the lead time demand forecast which is necessary to make an order. Let us recall that the lead time demand at the beginning of a period t (at a certain stage of the supply chain) is defined as follows

$$D_t^L = D_t + D_{t+1} + ... + D_{t+L_t-1} = \sum_{i=0}^{L_t-1} D_{t+i}, \qquad (1)$$

where D_t, D_{t+1}, … denote demands (from a stage below) during t, $t + 1$, ... periods and L_t is the lead time of the order placed at the beginning of the period t (order made to a stage above). This value sets down the demand during a lead time. The demands come from the stage immediately below and the lead times come from the supplier immediately above, that is they are the delivery lead times of the supplier which is immediately above the member. This quantity is necessary to make an order by the member of the supply chain to a supplier which is immediately above. In practice the member of the supply chain does not know its value at the moment t but he/she needs to predict its value to make an order. Thus if we want to analyze the bullwhip effect we need to look closer at the definitions of the lead time demand forecasting $\hat{D}_t^L$. Some of them are less or more realistic or even infeasible in real supply chains with stochastic lead times. Many articles on the bullwhip effect investigate different methods of demand fore-

casting under the assumption that the lead times are constant. The problem of the lead time demand prediction is much more complicated if the lead times are stochastic. Then mere demand forecasting is not sufficient to make an order.

4. A model with lead time demand forecasting

Let us recall the work of Kim et al. (2006). In their approach, lead time demand forecasting is defined as follows

$$\hat{D}_t^L = \tfrac{1}{n}\sum_{j=1}^{n} D_{t-j}^L,$$

where n is the delay parameter of the prediction and D_{t-j}^L is the previous known lead time demand of the order placed at the beginning of the time $t-j$. This method is practically feasible. The problem of the approach of Kim et al. (2006) lies in the impractical definition of the past lead time demands D_{t-j}^L.

Namely they continue

$$\hat{D}_t^L = \tfrac{1}{n}\sum_{j=1}^{n} D_{t-j}^L = \tfrac{1}{n}\sum_{j=1}^{n}\sum_{i=0}^{L-1} D_{t-j+i} = \tfrac{1}{n}\sum_{i=1}^{L-1}\sum_{j=1}^{n} D_{t-j+i},$$

where L is a lead time. Firstly, if we assume that lead times are stochastic then with every lead time demand D_{t-j}^L we associate a different lead time L_{t-j}. Moreover their definition does not work even in the case of a deterministic lead time because at the beginning of the moment t we do not know the values of demands D_{t-j+i} if $j \leq i$ (they explain it is a "mirror image" and "equivalent in terms of a priori statistical analysis").

Let us analyze the bullwhip effect under the above setting but with essential modifications. More precisely let us consider the simplest supply chain, that is one consisting of customers, a retailer and a supplier. We assume that the customers demands constitute an iid sequence $\{D_t\}_{t=-\infty}^{\infty}$. Moreover, lead times are deterministic and equal to L where L is a positive integer, that is $L = 1, 2, ...$ It is assumed that the retailer's replenishment order policy is the order-up-to-level policy and his/her lead time demand forecasting is based on the moving average method. Thus the forecast of the

lead time demand at the beginning of the period t based on the moving average method is as follows

$$\hat{D}_t^L = \tfrac{1}{n} \sum_{i=0}^{n-1} D_{t-L-i}^L. \tag{2}$$

In this approach we have to get back with lead time demands at least to the period $t - L$ because we know the demands till the period $t - 1$. Moreover, let us recall that the demand forecast alone using the moving average is as follows

$$\hat{D}_t = \tfrac{1}{n} \sum_{j=1}^{n} D_{t-j}.$$

Thus substituting into eq. (2) the known values of the previous lead time demands we get

$$\hat{D}_t^L = \tfrac{1}{n} \sum_{i=0}^{n-1} \sum_{j=0}^{L-1} D_{t-L-i+j} = \sum_{j=0}^{L-1} \tfrac{1}{n} \sum_{i=0}^{n-1} D_{t-L-i+j} = \sum_{j=0}^{L-1} \hat{D}_{t-L+j+1} = \sum_{j=0}^{L-1} \hat{D}_{t-j}. \tag{3}$$

Applying the order-up-to-level policy we find that the inventory level of the retailer at time t is

$$S_t = \hat{D}_t^L + z\hat{\sigma}_t, \tag{4}$$

where $\hat{\sigma}_t$ is the error of the lead time demand forecast which is usually defined as follows

$$\hat{\sigma}_t^2 = \mathbf{Var}\left(D_t^L - \hat{D}_t^L\right) \tag{5}$$

and z is a constant called the normal z-score. In some articles, $\hat{\sigma}_t^2$ is defined more practically, that is instead of variance the empirical variance of $D_t^L - \hat{D}_t^L$ is taken. This complicates calculations very much but we must mention that the estimation of $\hat{\sigma}_t^2$ increases the size of the bullwhip effect. These two approaches coincide if $z=0$. It is easy to notice that under the above assumptions $\hat{\sigma}_t$ is independent of t. Thus the order quantity q_t placed by the retailer at the beginning of a period t is

$$q_t = S_t - S_{t-1} + D_{t-1} = \hat{D}_t^L - \hat{D}_{t-1}^L + D_{t-1} =$$

$$\sum_{j=0}^{L-1} \hat{D}_{t-j} - \sum_{j=0}^{L-1} \hat{D}_{t-1-j} + D_{t-1} = \hat{D}_t - \hat{D}_{t-L} + D_{t-1},$$

where in the second last equality we use equation (3).

To calculate the value of q_t we need to consider two cases, that is $L \geq n$ and $L < n$. Thus in the case $L \geq n$ the order q_t placed by the retailer is as follows

$$q_t = \left(\frac{1}{n} + 1\right) D_{t-1} + \frac{1}{n} \sum_{j=2}^{n} D_{t-j} - \frac{1}{n} \sum_{j=1}^{n} D_{t-L-j}$$

and

$$\mathbf{Var}\, q_t = \left[\left(\frac{1}{n} + 1\right)^2 + \frac{2n-1}{n^2} \right] \mathbf{Var}\, D = \left(1 + \frac{4}{n}\right) \mathbf{Var}\, D.$$

In the case of $L < n$ we get

$$q_t = \left(\frac{1}{n} + 1\right) D_{t-1} + \frac{1}{n} \sum_{j=2}^{L} D_{t-j} - \frac{1}{n} \sum_{j=1}^{L} D_{t-n-j}$$

and

$$\mathbf{Var}\, q_t = \left[\left(\frac{1}{n} + 1\right)^2 + \frac{2L-1}{n^2} \right] \mathbf{Var}\, D = \left(1 + \frac{2}{n} + \frac{2L}{n^2}\right) \mathbf{Var}\, D.$$

Proposition 1. *If the lead times are deterministic and positive integer valued that is $L = 1, 2, \ldots$ and lead time demands are forecasted using the moving average method then the bullwhip effect measure is*

$$BM = \frac{\mathbf{Var}\, q_t}{\mathbf{Var}\, D} = \begin{cases} 1 + \frac{2}{n} + \frac{2L}{n^2} & \text{if} \quad L < n \\ 1 + \frac{4}{n} & \text{if} \quad L \geq n \end{cases}.$$

In Figure 1 we plotted the bullwhip effect measure for deterministic lead times when lead time demands are predicted by the moving average method (see Proposition 1). Let us notice that the bullwhip effect function $BM(n)$ as a function n does not have any jump at L that is it smoothly gets across the point $n = L$ (compare Proposition 1 with the similar result of Kim et al. 2006).
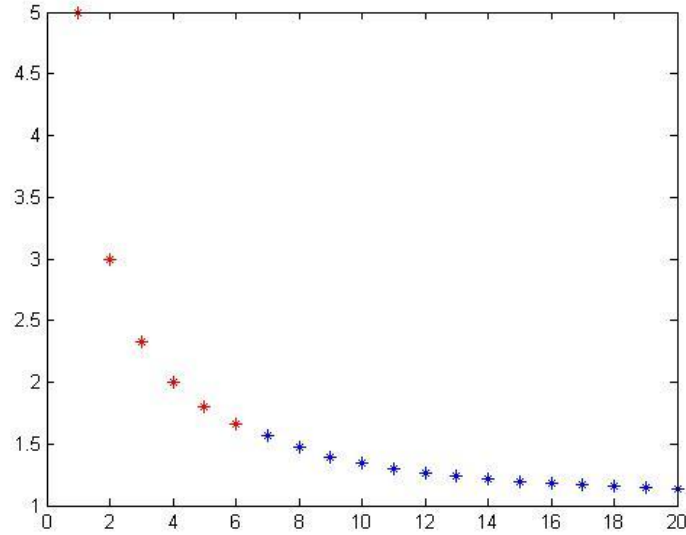
Fig. 1. The plot of the bullwhip effect measure as a function of *n* where $L = 7$

Source: own elaboration.

Now we follow the work of Kim et al. (2006) with certain modifications to find the bullwhip effect measure in the presence of stochastic lead times. We assume that the customers' demands constitute an iid sequence $\{D_t\}_{t=-\infty}^{\infty}$ and the lead times $\{L_t\}_{t=-\infty}^{\infty}$ are also independent and identically distributed and the sequences are mutually independent. Let us put $\mathbf{E}D_t = \mu_D$, $\mathbf{Var}D_t = \sigma_D^2$, $\mathbf{E}L_t = \mu_L$ and $\mathbf{Var}L_t = \sigma_L^2$. Additionally we need to assume that lead times are bounded random variables that is $L_t \leq M$ where M is a positive integer. This assumption is not adopted in Kim et al. (2006) which makes their results slightly impractical because this is necessary to make the prediction of lead time demands. More precisely we get back at least M periods to forecast the lead time demand at time *t,* that is we know the values of lead time demands at times $t - M$, $t - M - 1$, ... which means that we may not know the values of lead times L_{t-M+1}, L_{t-M+2}, ... at time *t* (and further demands) because the orders placed at times $t - M + 1$, t - $M + 2$, ... can be unrealized at moment *t*. This assumption is the main difference between our approach and the model of Kim et al. (2006), and this results in a different bullwhip effect performance measure as we can see

later. Moreover in our approach we need to know the distribution of L_t to calculate the bullwhip effect measure, that is we assume that

$$P(L_t = k) = p_k,$$

where $k = 1, 2, ..., M$ and k is the number of periods (in practice we estimate these probabilities). Later we will see that only the value of p_M is necessary in the bullwhip effect measure. Thus the prediction of the lead time demand at time t using the method of moving average with the length n is as follows

$$\hat{D}_t^L = \frac{1}{n} \sum_{j=0}^{n-1} D_{t-M-j}^L. \tag{6}$$

Let us repeat that the lead time demands $D_{t-M+1}^L, D_{t-M+2}^L, ...$ we may not know at time t, which is why in the lead time forecasting we engage $D_{t-M}^L, D_{t-M-1}^L, ...$ that is the lead time demands up to time $t - M$. Thus by equation (1) we get

$$\hat{D}_t^L = \frac{1}{n} \sum_{j=0}^{n-1} \sum_{i=0}^{L_{t-M-j}-1} D_{t-M-j+i}. \tag{7}$$

We assume that the retailer uses the order-up-to-level policy, thus the level of the inventory at time t is given in equation (4). By the stationarity and independence of the sequences of demands and lead times one can show that $\hat{\sigma}_t^2$ given in (5) does not depend on t. Hence we obtain

$$q_t = \hat{D}_t^L - \hat{D}_{t-1}^L + D_{t-1} = \frac{1}{n} \sum_{j=0}^{n-1} D_{t-M-j}^L - \frac{1}{n} \sum_{j=0}^{n-1} D_{t-1-M-j}^L + D_{t-1}$$

$$= \frac{1}{n} D_{t-M}^L - \frac{1}{n} D_{t-M-n}^L + D_{t-1} = \frac{1}{n} \sum_{i=0}^{L_{t-M}-1} D_{t-M+i} - \frac{1}{n} \sum_{i=0}^{L_{t-M-n}-1} D_{t-M-n+i} + D_{t-1}. \tag{8}$$

Theorem 1. *Under the above assumptions and for $n \geq M$ the bullwhip effect measure is the following*

$$BM = \frac{\mathbf{Var}\, q_t}{\mathbf{Var}\, D_t} = 1 + \frac{2p_M}{n} + \frac{2\mu_L}{n^2} + \frac{2\sigma_L^2 \mu_D^2}{\sigma_D^2 n^2}.$$

Proof. Using the law of total variance we have

$$\mathbf{Var}\, q_t = \mathbf{E}\left(\mathbf{Var}(q_t \mid L_{t-M}, L_{t-M-n})\right) + \mathbf{Var}\, \mathbf{E}\left(q_t \mid L_{t-M}, L_{t-M-n}\right).$$

By equation (8) we get

$$\mathbf{E}\left(q_t \mid L_{t-M}, L_{t-M-n}\right) = \mu_D \left(\frac{L_{t-M} - L_{t-M-n}}{n} + 1\right).$$

Thus

$$\mathbf{Var}\,\mathbf{E}\left(q_t \mid L_{t-M}, L_{t-M-n}\right) = \frac{2\sigma_L^2 \mu_D^2}{n^2}. \tag{9}$$

We need to consider two cases to find $\mathbf{Var}\left(q_t \mid L_{t-M}, L_{t-M-n}\right)$. In the first case $L_{t-M} < M$ we get

$$\mathbf{Var}\left(q_t \mid L_{t-M}, L_{t-M-n}\right) = \sigma_D^2 \left(\frac{L_{t-M} + L_{t-M-n}}{n^2} + 1\right).$$

If $L_{t-M} = M$ we have

$$\mathbf{Var}\left(q_t \mid L_{t-M}, L_{t-M-n}\right) = \sigma_D^2 \left(\frac{L_{t-M} + L_{t-M-n}}{n^2} + 1 + \frac{2}{n}\right).$$

Finally we obtain

$$\mathbf{Var}\left(q_t \mid L_{t-M}, L_{t-M-n}\right) = \sigma_D^2 \left(\frac{L_{t-M} + L_{t-M-n}}{n^2} + 1\right) + \frac{2\sigma_D^2}{n} \mathrm{I}\{L_{t-M} = M\}.$$

where I is the indicator function. Thus we get

$$\mathbf{E}\,\mathbf{Var}\left(q_t \mid L_{t-M}, L_{t-M-n}\right) = \sigma_D^2 \left(\frac{2\mu_L}{n^2} + 1\right) + \frac{2\sigma_D^2 p_M}{n}$$

which together with equation (9) give the assertion.

The formula for the bullwhip effect measure in the case $n < M$ is more complicated and its derivation is rather cumbersome. In practice the case $n \geq M$ is more interesting, because we require to put large n in the forecast to get a more precise prediction. Let us notice that if $L_t = M = L$ (which gives $p_M = 1$ and $\mu_L = L$) is deterministic, the formula of Theorem 1 is consistent with Proposition 1. Compare Theorem 1 with the results of Kim et al. (2006).

We have to mention that in the formula of Theorem 1 the term $\dfrac{2p_M}{n}$ gives the largest contribution in the bullwhip effect for large n because it is $O(1/n)$. This means that in reducing the bullwhip effect the probability of

the largest lead time is very important. It is astonishing that if $p_M = 0$ and we still get back M periods in the prediction of lead time demands, the bullwhip effect measure is reduced by the term $O(1/n)$ and is of the form

$$BM = \frac{\mathbf{Var}\, q_t}{\mathbf{Var}\, D_t} = 1 + \frac{2\mu_L}{n^2} + \frac{2\sigma_L^2 \mu_D^2}{\sigma_D^2 n^2}.$$

Let us compare the values of the bullwhip effect measure under $p_M > 0$ and $p_M = 0$. More precisely, let L_t have the discrete uniform distribution on $\{1, 2, 3\}$ that is $p_k = 1/3$ for $k = 1, 2, 3$ then $M = 3$, $\mu_L = 2$ and $\sigma_L^2 = 2/3$. In the case $p_M = 0$ we assume that L_t has the discrete uniform distribution on $\{1, 2\}$ that is $p_k = 1/2$ for $k = 1, 2$ then $\mu_L = 1.5$ and $\sigma_L^2 = 1/4$ (and we still get back at least $M = 3$ periods to predict the lead time demand). The results are in Table 1.

Table 1. The measure of the bullwhip effect as a function of n
for $M = 3$ and $\sigma_D / \mu_D = 0.5$

n	$p_M > 0$	$p_M = 0$
3	2.259	1.555
4	1.750	1.312
5	1.506	1.200
6	1.370	1.138
7	1.285	1.102
8	1.229	1.078
9	1.189	1.061
10	1.160	1.050
11	1.137	1.041
12	1.120	1.034
13	1.106	1.029
14	1.095	1.025
15	1.085	1.022

Source: own elaboration.

Similarly, we can calculate the bullwhip effect measure for longer lead times. More precisely, let L_t have the discrete uniform distribution on $\{1, 2, \ldots, 7\}$ that is $p_k = 1/7$ for $k = 1, 2, \ldots, 7$ then $M = 7$, $\mu_L = 4$ and $\sigma_L^2 = 4$. In the case $p_M = 0$, we assume that L_t has the discrete uniform distribution on $\{1, 2, \ldots, 6\}$ that is $p_k = 1/6$ for $k = 1, 2, \ldots, 6$ then $\mu_L = 3.5$ and

$\sigma_L^2 = 3.916$ (and we still get back at least $M = 7$ periods to predict the lead time demand). The results are shown in Table 2.

Table 2. The measure of the bullwhip effect as a function of *n*
for $M = 7$ and $\sigma_D / \mu_D = 0.5$

n	$p_M > 0$	$p_M = 0$
7	1.857	1.782
8	1.660	1.598
9	1.525	1.473
10	1.428	1.383
11	1.356	1.316
12	1.301	1.266
13	1.258	1.226
14	1.224	1.195
15	1.196	1.170
16	1.155	1.132
17	1.174	1.149
18	1.139	1.118

Source: own elaboration.

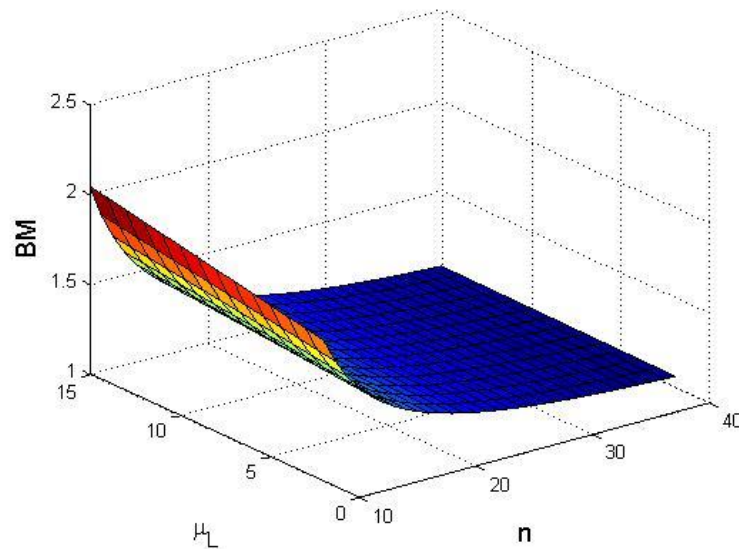

Fig. 2. The plot of the bullwhip effect measure as a function of *n* and μ_L
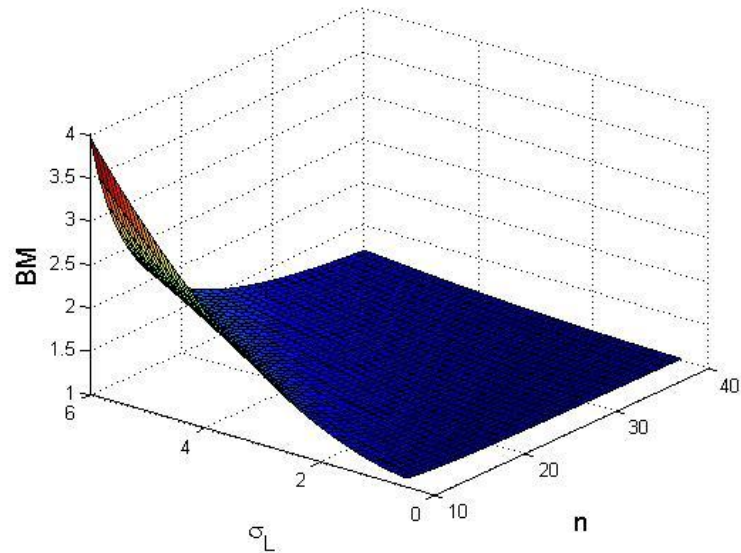where $p_M = 0.1$, $\sigma_D / \mu_D = 0.5$ and $\sigma_L = 3$

Source: own elaboration.

Fig. 3. The plot of the bullwhip effect measure as a function of n and σ_L
where $p_M = 0.1$, $\sigma_D / \mu_D = 0.5$ and $\mu_L = 3$

Source: own elaboration.

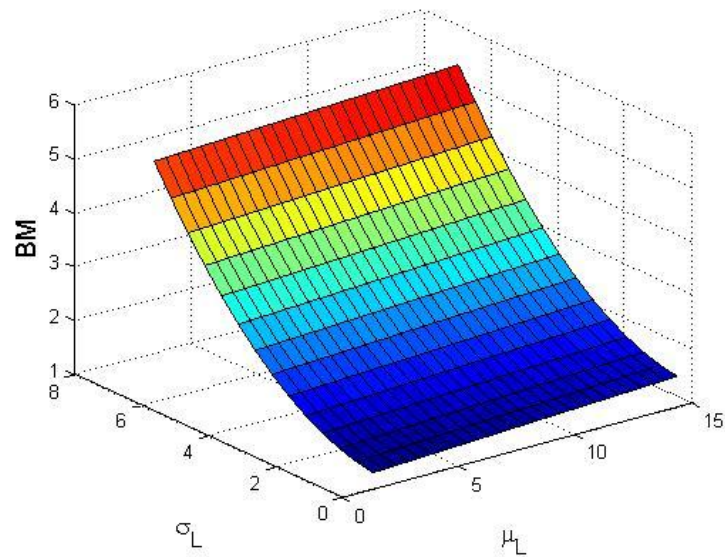

Fig. 4. The plot of the bullwhip effect measure as a function of μ_L and σ_L
where $p_M = 0.1$, $\sigma_D / \mu_D = 0.5$ and $n = 10$

Source: own elaboration.

Moreover, we plotted the bullwhip effect measure as a function of two variables. In Figure 2 we visualized the bullwhip effect measure as a function of the delay parameter n and the mean value of lead times μ_L for $p_M = 0.1$, $\sigma_D / \mu_D = 0.5$ and $\sigma_L = 3$.

Similarly, in Figure 3 we presented *BM* as a function of the delayed parameter n and the standard deviation of lead times σ_L for $p_M = 0.1$, $\sigma_D / \mu_D = 0.5$ and $\mu_L = 3$.

In Figure 4 the plot of bullwhip as a function of the expected value μ_L and the standard deviation of lead times σ_L is shown for $p_M = 0.1$, $\sigma_D / \mu_D = 0.5$ and $n = 10$.

5. Conclusions and future research opportunities

One of the main objectives of the supply chain management is to quantify the bullwhip effect which is a standard example of an inefficiency in supply chains. The quantitative description of the bullwhip effect enables to find all causes of the effect and to reduce its negative results which are a substantial increase in costs and disorders in the continuity of deliveries. Thus the bullwhip effect is the most severe phenomenon observed in supply chains and its analysis is the main challenge of supply chain management. In this article we have quantified the bullwhip effect in the presence of stochastic lead times where the order is made by the prediction of lead time demands using the simple moving average method. We have noticed that the bullwhip effect measure depends essentially on the delay parameter of the forecasting, the probability of the largest lead time, and the expectation and variance of lead times. These factors affect the size of the bullwhip effect, and a proper tuning of them can dampen the effect. Theorem 1 provides a lot of information on the factors of the bullwhip effect in a quantitative manner which is necessary in supply chain management.

Although the bullwhip effect has been investigated since the middle of the twentieth century, there are still a lot of paths which have to be examined and analyzed. A member of a supply chain placing an order has many tools and policies to choose from. So assuming that lead times are random, which is usually observed in supply chains, the member of a supply chain must predict the next lead times to make an order, which is more natural than the lead time demand forecasting itself. More precisely, when making an order one can forecast the next lead time demand by predicting lead times and demands separately. Thus in further approaches to the lead time

forecasting problem one needs to investigate structures other than iid of lead times and demands. Even if one considers a more complicated structure of demands, for example autoregressive-moving average leaving iid structure of lead times, then this will complicate the derivations of the bullwhip effect measure to a significant degree. Other opportunities lie in different forecasting methods for lead times and demands. One can employ simple moving average, exponential smoothing, and minimum-mean-squared-error forecasts to predict lead times and demands. Here one can apply different methods for lead time forecasting and demand forecasting or the same methods but other than the moving average method. In other directions of research one can investigate multi-echelon supply chains in the presence of stochastic lead times being predicted at every stage where the information on demands and lead times is shared or is not shared. The value of the bullwhip effect measure in these situations will be surely valuable for theorists and practitioners in the field of supply chain management.

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