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Contents

Introduction

One of the fastest growing areas in the economic sciences is broadly defined area of finance, with particular emphasis on the financial markets, financial institutions and risk management. Real world challenges stimulate the development of new theories and methods. A large part of the theoretical research concerns the analysis of the risk of not only economic entities, but also households.

The first Wrocław Conference in Finance WROFIN was held in Wrocław between 22nd and 24th of September 2015. The participants of the conference were the leading representatives of academia, practitioners at corporate finance, financial and insurance markets. The conference is a continuation of the two long-standing conferences: INVEST (Financial Investments and Insurance) and ZAFIN (Financial Management – Theory and Practice).

The Conference constitutes a vibrant forum for presenting scientific ideas and results of new research in the areas of investment theory, financial markets, banking, corporate finance, insurance and risk management. Much emphasis is put on practical issues within the fields of finance and insurance. The conference was organized by Finance Management Institute of the Wrocław University of Economics. Scientific Committee of the conference consisted of prof. Diarmuid Bradley, prof. dr hab. Jan Czekaj, prof. dr hab. Andrzej Gospodarowicz, prof. dr hab. Krzysztof Jajuga, prof. dr hab. Adam Kopiński, prof. dr. Hermann Locarek-Junge, prof. dr hab. Monika Marcinkowska, prof. dr hab. Paweł Miłobędzki, prof. dr hab. Jan Monkiewicz, prof. dr Lucjan T. Orłowski, prof. dr hab. Stanisław Owsiak, prof. dr hab. Wanda Ronka-Chmielowiec, prof. dr hab. Jerzy Różański, prof. dr hab. Andrzej Sławiński, dr hab. Tomasz Słoński, prof. Karsten Staehr, prof. dr hab. Jerzy Węcławski, prof. dr hab. Małgorzata Zaleska and prof. dr hab. Dariusz Zarzecki. The Committee on Financial Sciences of Polish Academy of Sciences held the patronage of content and the Rector of the University of Economics in Wroclaw, Prof. Andrzej Gospodarowicz, held the honorary patronage.

The conference was attended by about 120 persons representing the academic, financial and insurance sector, including several people from abroad. During the conference 45 papers on finance and insurance, all in English, were presented. There were also 26 posters.

This publication contains 27 articles. They are listed in alphabetical order. The editors of the book on behalf of the authors and themselves express their deep gratitude to the reviewers of articles – Professors: Jacek Batóg, Joanna Bruzda, Katarzyna Byrka-Kita, Jerzy Dzieża, Teresa Famulska, Piotr Fiszeder, Jerzy Gajdka, Marek Gruszczyński, Magdalena Jerzemowska, Jarosław Kubiak, Tadeusz Kufel, Jacek Li-

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Wanda Ronka-Chmielowiec, Krzysztof Jajuga

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Agnieszka Przybylska-Mazur
Uniwersytet Ekonomiczny w Katowicach
e-mail: agnieszka.przybylska-mazur@ue.katowice.pl

FISCAL RULES AS INSTRUMENT OF ECONOMIC POLICY

REGUŁY FISKALNE JAKO NARZĘDZIE PROWADZENIA POLITYKI GOSPODARCZEJ

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Summary: The basic importance of the implementation of economic policy in the short and medium term, the so-called macroeconomic policy, is at the discretion of monetary policy and fiscal policy. A frequently used method of making decisions are decisions based on rules. According to the most frequently stated definition [Kopits, Symanski 1998] a fiscal policy rule is a permanent constraint on fiscal policy, typically defined in terms of indicators of overall fiscal performance. In this article, to determine the fiscal policy feedback rules, we take into account Quadratic Linear Tracking Problem and the selected dynamic model. The objective of the article is the presentation of fiscal rule, which can be an effective instrument for limiting the generation of excessive public debt. Therefore, it presents selected form of simple fiscal rule that is the solution of Quadratic Linear Tracking Problem.

Keywords: economic policy, feedback fiscal rule, public debt, Quadratic Linear Tracking Problem.

Streszczenie: Podstawowe znaczenia przy realizacji polityki gospodarczej w krótkim i średnim okresie, czyli tak zwanej polityki makroekonomicznej mają polityka pieniężna i polityka fiskalna. Często stosowanym sposobem podejmowania decyzji są decyzje oparte na regułach. Zgodnie z najczęściej podawaną definicją [Kopits, Symanski, 1998] reguła fiskalna jest trwałym ograniczeniem w zakresie polityki fiskalnej odzwierciedlonej we wskaźnikach ogólnej wydajności fiskalnej. W celu wyznaczenia reguły polityki fiskalnej sprzężenia zwrotnego wzięliśmy pod uwagę Kwadratowo-Liniowy Problem Tropiący i wybrany model dynamiczny. Celem artykułu jest przedstawienie reguły fiskalnej, która może być skutecznym narzędziem ograniczającym generowanie nadmiernego długu publicznego. Zatem przedstawiliśmy wybraną postać prostej reguły fiskalnej, która jest rozwiązaniem Kwadratowo-Liniowego Problemu Tropiącego.

Słowa kluczowe: polityka gospodarcza, reguła fiskalna sprzężenia zwrotnego, dług publiczny, Kwadratowo-Liniowy Problem Tropiący.

1. Introduction

The fundamental importance of the implementation of economic policy in the short and medium term, the so-called macroeconomic policies, are at the discretion of the monetary policy and the fiscal policy.

The requirement to reduce public debt to 60% of GDP and the budget deficit to 3% of GDP, introduced the Maastricht Treaty, was signed by EU countries in 1992. The Stability and Growth Pact guarantees that Member States of the European Union will keep public finance in equilibrium and these countries will coordinate the fiscal policy. The pact aims for revision of the excessive public debt and for revision of excessive government deficits.

In the context of correction function of the Stability and Growth Pact, the excessive deficit procedure provides a correction of the excessive deficit or public debt. It is a method of gradual reduction of excessive deficits and the reduction of excessive debt. According to the EU Treaty, as excessive public debt, debt exceeding 60 percent of GDP which does not decrease at the appropriate rate – average of 5% per year over three years, is considered; however, the excessive deficit is a deficit exceeding 3 percent of GDP. Therefore, the annual public debt and budget deficit must be controlled. It is also worth to have regard for the annual changes in fiscal policy that could have a beneficial effect on the performance of stabilization policy, which aims to mitigate the fluctuations in economic activity caused by the change of phases of the business cycle.

Therefore, the objective of this article is the determination of fiscal rule as the solution to the Quadratic Linear Tracking Problem. Therefore, we aim at determination of these fiscal rules which can be an effective instrument for limiting the generation of excessive public debt.

The first part of this article discusses the general fiscal policy based on rules. The second part presents a dynamic model describing the dynamics of public debt and interest rate. It presents also the optimal control problem. In the next part of the article the optimal fiscal policy rules that are the solution of the quadratic linear problem are determined. These rules are the feedback rules The application of these rules allows the economy to develop according to the desired path. The last part of the article presents the results of empirical analysis for Poland. Some remarks about the level of public debt are also presented by Debortoli, Nunes [2012] and Sutherland, Hoeller, Merola [2012].

2. Fiscal policy based on rules

The fiscal policy involves government decisions on the size and structure of public expenditure and the budget deficit. The tools of fiscal policy are:

- various public expenditure,
- budget deficit,
- tax rates.

One of the ways of decision making is the decisions based on rules. Fiscal rules are an effective tool for limiting the generation of excessive public debt and the generation of excessive government deficit. When we conduct the fiscal policy based on rules it is strengthened by the fiscal Policy caution and objectivity in the implementation of this policy. Therefore, of significant practical importance is the knowledge of the fiscal rule, whereby the optimal fiscal decision making in different phases of the business cycle becomes possible. The other authors write also about the importance of fiscal rules in decision making (see e.g. [Marchewka-Bartkowiak 2012]).

There are four types of fiscal rules:

- debt rules,
- budget balance rules,
- expenditure rules,
- revenue rules.

As it is mentioned in the introduction, one of the criteria of the Maastricht Treaty refers to public finance. This criterion says that an EU Member State may not be covered by the excessive deficit procedure. The public debt cannot exceed 60 percent of GDP; however, the budget deficit cannot exceed 3 percent of GDP. Taking into account this convergence criterion, the article focuses on the determination of the debt rule.

In the next part of the article we determine the fiscal policy rules that are feedback rules. The application of these rules allows the economy to develop according to the desired path. In order to determine these rules deterministic control theory is applied. Deterministic control problem is presented below.

3. Dynamic model and the Quadratic Linear Problem

Many of the problems in the economy can be modelled using dynamic models. These models can be the basis to determine the strategy whose effect is the achievement of future desired values of selected variables, such as inflation and output.

In the article, to determine the fiscal policy feedback rules we take into account the dynamic model which can be written in the matrix form as follows [Kendrick, Amman 2011]:

$$X_{t+1} = A \cdot X_t + B \cdot U_t \quad \text{for all } t = 0,1, ..., N-1 \tag{1}$$

with the initial condition

$$X_0 = \widetilde{X}_0 , \tag{2}$$

where: X_t – vector of state variables at time t, U_t – control vector at time t, X_t^* – vector of desired values of the state variables at time t, U_t^* – vector of desired control values at time t, $\widetilde{X}_0$ – given initial value of state vector, the

state vector at time $t = 0$, A – matrix of state vector coefficients at time t, B – matrix of control vector coefficients at time t, that is multiplier matrix of impact of control variables, V_t – symmetric positive definite matrix of penalties of deviations of state variables from the desired values of state variables[1], S_t – symmetric positive definite matrix of penalties of deviations of control variables from the desired values[2].

This article, as state variables takes into account: the inflation rate π_t and GDP growth Y_t, therefore $X_t = \begin{bmatrix} \pi_t \\ Y_t \end{bmatrix}$, while the control variables are the interest rate i_t and public debt D_t, and more specifically the ratio of public debt to GDP, thus $U_t = \begin{bmatrix} i_t \\ D_t \end{bmatrix}$. Moreover, as vectors of desired values of the state variables and control variables, it takes:

$$X_t^* = \begin{bmatrix} \pi_t^* \\ Y_t^* \end{bmatrix} \text{ and } U_t^* = \begin{bmatrix} i_t^* \\ D_t^* \end{bmatrix} \tag{3}$$

where: π_t^* – the inflation target, Y_t^* – the potential output, i_t^* – the natural interest rate, D_t^* – the public debt equal to 55 percent of GDP.

Furthermore, it assumes:

$$V_t = \begin{bmatrix} \lambda_{\pi t} & 0 \\ 0 & \lambda_{Y t} \end{bmatrix}, \; S_t = \begin{bmatrix} \lambda_{i t} & 0 \\ 0 & \lambda_{W t} \end{bmatrix} \tag{4}$$

Now it presents the quadratic linear problem, which it uses to determine the fiscal rule.

This problem is the example of linear deterministic control problem. In the quadratic linear problem, the criterion function is the quadratic function, but as limiting conditions it takes the linear equation system. If in the analysis we allow for the fact that the values of analyzed economic variables are carried out in accordance with the desired trajectory, we should consider the so-called tracking problem. Thus, the article determines the fiscal policy rules as the solution of Quadratic Linear Tracking Problem.

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[1] If V_t is a diagonal matrix, then elements of the main diagonal are the weights assigned to the deviations of state variables vector from the vector of desired state variables.

[2] If S_t is a diagonal matrix, then elements of the main diagonal it treats as the weights assigned to the deviations of control variables vector from the vector of desired control variables.

Quadratic Linear Tracking Problem can be formulated as the following: for each $t = 0,1, ..., N-1$ it determines the control vector U_t for which the function being the cost-to-go [Kendrick 1981], illustrated below (formula (5)), reaches a minimum:

$$\begin{aligned} J = \tfrac{1}{2}\left(X_N - X_N^*\right)^T \cdot V_N \cdot \left(X_N - X_N^*\right) + \\ + \tfrac{1}{2}\sum_{t=0}^{N-1}\left(\left(X_t - X_t^*\right)^T \cdot V_t \cdot \left(X_t - X_t^*\right) + \left(U_t - U_t^*\right)^T \cdot S_t \cdot \left(U_t - U_t^*\right)\right) \end{aligned} \tag{5}$$

4. Fiscal policy feedback rules

The optimal linear feedback rule is the solution of the problem (1) – (5). This rule is given by the following formula [Kendrick, Amman 2011]:

$$U_t = G_t \cdot X_t + g_t \text{ for all } t = 0,1, ..., N-1 \tag{6}$$

where: G_t – the feedback gain matrix at time *t*, g_t – the feedback parameter vector at time *t*,

They are calculated with the following formulas:

$$G_t = -\left(B^T \cdot K_{t+1} \cdot B + S_t^T\right)^{-1} \cdot B^T \cdot K_{t+1} \cdot A \tag{7}$$

$$g_t = -\left(B^T \cdot K_{t+1} \cdot B + S_t^T\right)^{-1} \cdot \left[B^T \cdot p_{t+1} - S_t \cdot U_t^*\right] \tag{8}$$

Matrix K_t and vector p_t fulfil the following Riccati equation for each $t = 1,2, ..., N-1$:

$$\begin{aligned} K_t = V_t + A^T \cdot K_{t+1} \cdot A - A^T \cdot K_{t+1} \cdot B \cdot \left(B^T \cdot K_{t+1} \cdot B + S_t^T\right)^{-1} \cdot \\ B^T \cdot K_{t+1} \cdot A \end{aligned} \tag{9}$$

$$\begin{aligned} p_t = A^T \cdot p_{t+1} - V_t \cdot X_t^* - A^T \cdot K_{t+1} \cdot B \cdot \left(B^T \cdot K_{t+1} \cdot B + S_t^T\right)^{-1} \cdot \\ \left(B^T \cdot p_{t+1} - S_t \cdot U_t^*\right) \end{aligned} \tag{10}$$

Whereas for $t = N$, the following applies:

$$K_N = V_N \tag{11}$$

$$p_N = -V_N \cdot X_N^* \tag{12}$$

Therefore, taking into consideration the matrix representation of feedback rule (6), the fiscal rule may be written down in the following form:

$$D_t = G_{21,t} \cdot \pi_t + G_{22,t} \cdot Y_t + g_2 \tag{13}$$

The fiscal policy feedback rule shows the dependence of the public debt level on the inflation rate and the output.

5. Empirical analysis

To determine the fiscal policy rules we take the annual data on: the inflation rate (corresponding period of the previous year = 100), the GDP growth rate, the interest rate (the annual average of reference rate) and the public debt (percent of GDP).

As desired values of the state variables we take: the inflation target, the potential GDP determined on the basis of the Hodrick – Prescott filter, however the desired values of control vector are: the natural interest rate determined on basis of Hodrick – Prescott filter and the public debt equal to 55 percent of GDP (the lower limit of the second prudential threshold).

For analysis we take into account the data for Poland for the period from 2003 to 2014. Furthermore, we assume the constant weight values for each t, thus $V_t = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.75 \end{bmatrix}$ and we assume $S_t = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ for each t.

On the basis of these data we calculate the matrices G_t and vectors g_t that are used later to determine the fiscal rules.

Below we present the obtained fiscal rules for individual years:

1) in 2003: $D_t = 0.10765 \cdot \pi_t - 0.0427 \cdot Y_t + 54.80738$
2) in 2004: $D_t = 0.107644 \cdot \pi_t - 0.04269 \cdot Y_t + 54.79261$
3) in 2005: $D_t = 0.10762 \cdot \pi_t - 0.04269 \cdot Y_t + 55.04356$
4) in 2006: $D_t = 0.107505 \cdot \pi_t - 0.04262 \cdot Y_t + 54.08015$
5) in 2007: $D_t = 0.107471 \cdot \pi_t - 0.04251 \cdot Y_t + 54.82085$
6) in 2008: $D_t = 0.106994 \cdot \pi_t - 0.04253 \cdot Y_t + 54.75288$
7) in 2009: $D_t = 0.105935 \cdot \pi_t - 0.04152 \cdot Y_t + 54.94816$
8) in 2010: $D_t = 0.105486 \cdot \pi_t - 0.04103 \cdot Y_t + 54.97014$
9) in 2011: $D_t = 0.09817 \cdot \pi_t - 0.03981 \cdot Y_t + 54.77036$
10) in 2012: $D_t = 0.090857 \cdot \pi_t - 0.02837 \cdot Y_t + 54.67866$
11) in 2013: $D_t = 0.078204 \cdot \pi_t - 0.03022 \cdot Y_t + 54.78163$
12) in 2014: $D_t = 0.078204 \cdot \pi_t - 0.03022 \cdot Y_t + 54.78163$

Using these determined fiscal rules for the period 2003-2014 we calculate the optimal value of public debt resulting from the application of these rules. In the following table we contrast them with the real value of public debt.

Table 1. The optimal value and the real value of public debt

Year	The optimal values of public debt (percent of GDP)	The real values of public debt (percent of GDP)
2003	54.74	47.1
2004	54.95	45.7
2005	55.12	47.1
2006	54.92	47.7
2007	54.78	45
2008	55.04	47.1
2009	55.21	50.9
2010	55.09	53.6
2011	55.00	54.8
2012	54.96	54.4
2013	54.80	55.7
2014	54.68	50.1

Source: Author's own study.

The obtained results are also presented by the figure below.

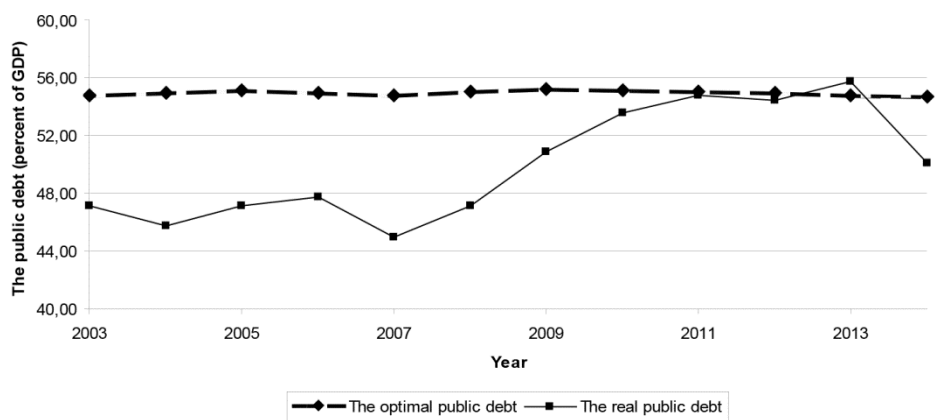

Figure 1. The optimal values and the real values of public debt

Source: Author's own study.

Analyzing the optimal public debt calculated under the proposed fiscal rules being the solution to the quadratic linear tracking problem – the feedback rules and taking the second prudential threshold in Poland which equals to 55 percent as the desired value of public debt that allows to maintain the reserve equal to 5 percent relative to the fiscal Maastricht Treaty criteria (60 percent), it may be concluded that the optimal public debt to GDP ratio, minimizing deviations of the inflation rate from the inflation target, deviations of GDP from potential GDP, deviations of the interest rate from the natural rate, and deviations of public debt to GDP ratio from the second threshold prudential, is close to 55 percent.

In addition, it should be noted that when the effective economic policy is executed, the fiscal policy and the monetary policy must be coordinated. Thus, to achieve a minimum deviation of inflation from inflation target and the production of potential value one has to control not only the public debt, but also the instrument of monetary policy, which in the article is the interest rate. Therefore, in the following table we present the obtained optimal average annual value of the reference rate based on the presented model, comparing them with the real annual average values of reference rate for the period 2003-2014.

Table 2. The optimal and the real annual average values of reference rate

Year	The optimal annual average values of reference rate	The real annual average values of reference rate
2003	2.97	5.56
2004	6.11	5.81
2005	6.07	5.25
2006	3.75	4.06
2007	3.45	4.48
2008	5.68	5.73
2009	5.21	3.67
2010	3.83	3.50
2011	4.06	4.25
2012	3.82	4.60
2013	1.82	2.92
2014	1.08	2.38

Source: own calculation.

The obtained results are also presented by Figure 2.

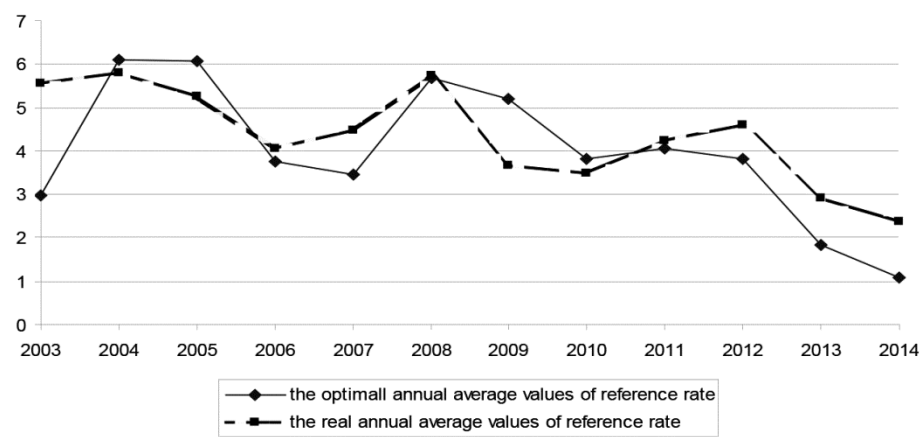

Figure 2. The optimal value and the real value of reference rate

Source: Author's own study.

Therefore, on the basis of the calculated annual average value of reference rate it can be observed that the divergences between optimal and real values exist.

6. Conclusion

In the article we determine the general form of fiscal rules for each analyzed period. These rules are the feedback rules and they are the solution to the quadratic linear tracking problem. The quadratic linear tracking problem is an example of the optimal control problem.

As the economy can be regarded as a dynamic system with control, thus use of the solution of quadratic linear problem in practice, which is calculated above, for fiscal rule will help the economy develop in accordance with the desired path. This study, as the state variables, took into consideration two basic variables, such as inflation and GDP growth, whose values were taken into account when fiscal and monetary policy decisions were made. In the study, as a control variable, which is an instrument of fiscal policy, we used the public debt to GDP ratio.

In this simple proposed optimal fiscal policy rule, the public debt depends on the inflation rate and the GDP growth rate. The obtained fiscal policy rules made it possible to calculate the optimal values of public debt, and more precisely the public debt to GDP ratio in the analyzed period, that is in the years 2003-2014. However, we made the analysis ex-post.

When one knows the predicted values of the state variables: the inflation rate and GDP growth rate and also the forecasts of desirable public debt, of the interest rate, of inflation target and of GDP growth rate, one can calculate the fiscal rules and the optimal values of forecast of public debt on basis of these rules.

An alternative method to the one presented in this article may be applied to the analysis of fiscal policy and the factors affecting its other tools, such as for example the wavelet transform, artificial neural networks, genetic algorithms, etc. [see: Hadaś-Dyduch 2014, 2015a, 2015b].

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