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Contents

Introduction

One of the fastest growing areas in the economic sciences is broadly defined area of finance, with particular emphasis on the financial markets, financial institutions and risk management. Real world challenges stimulate the development of new theories and methods. A large part of the theoretical research concerns the analysis of the risk of not only economic entities, but also households.

The first Wrocław Conference in Finance WROFIN was held in Wrocław between 22nd and 24th of September 2015. The participants of the conference were the leading representatives of academia, practitioners at corporate finance, financial and insurance markets. The conference is a continuation of the two long-standing conferences: INVEST (Financial Investments and Insurance) and ZAFIN (Financial Management – Theory and Practice).

The Conference constitutes a vibrant forum for presenting scientific ideas and results of new research in the areas of investment theory, financial markets, banking, corporate finance, insurance and risk management. Much emphasis is put on practical issues within the fields of finance and insurance. The conference was organized by Finance Management Institute of the Wrocław University of Economics. Scientific Committee of the conference consisted of prof. Diarmuid Bradley, prof. dr hab. Jan Czekaj, prof. dr hab. Andrzej Gospodarowicz, prof. dr hab. Krzysztof Jajuga, prof. dr hab. Adam Kopiński, prof. dr. Hermann Locarek-Junge, prof. dr hab. Monika Marcinkowska, prof. dr hab. Paweł Miłobędzki, prof. dr hab. Jan Monkiewicz, prof. dr Lucjan T. Orłowski, prof. dr hab. Stanisław Owsiak, prof. dr hab. Wanda Ronka-Chmielowiec, prof. dr hab. Jerzy Różański, prof. dr hab. Andrzej Sławiński, dr hab. Tomasz Słoński, prof. Karsten Staehr, prof. dr hab. Jerzy Węcławski, prof. dr hab. Małgorzata Zaleska and prof. dr hab. Dariusz Zarzecki. The Committee on Financial Sciences of Polish Academy of Sciences held the patronage of content and the Rector of the University of Economics in Wroclaw, Prof. Andrzej Gospodarowicz, held the honorary patronage.

The conference was attended by about 120 persons representing the academic, financial and insurance sector, including several people from abroad. During the conference 45 papers on finance and insurance, all in English, were presented. There were also 26 posters.

This publication contains 27 articles. They are listed in alphabetical order. The editors of the book on behalf of the authors and themselves express their deep gratitude to the reviewers of articles – Professors: Jacek Batóg, Joanna Bruzda, Katarzyna Byrka-Kita, Jerzy Dzieża, Teresa Famulska, Piotr Fiszeder, Jerzy Gajdka, Marek Gruszczyński, Magdalena Jerzemowska, Jarosław Kubiak, Tadeusz Kufel, Jacek Li-

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Wanda Ronka-Chmielowiec, Krzysztof Jajuga

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EVIDENCE OF LONG MEMORY AND ASYMMETRY IN THE EUR/PLN EXCHANGE RATE VOLATILITY[1]

EMPIRYCZNA ANALIZA DŁUGIEJ PAMIĘCI PROCESU I ASYMETRII ZMIENNOŚCI KURSU WYMIANY WALUT EUR/PLN

DOI: 10.15611/pn.2016.428.11
JEL Classification: C22, C53, G15, G17, G32

Summary: This paper focuses on capturing the conditional volatility in the foreign exchange Value-at-Risk forecasts. By implementing a variety of GARCH models under different return distributions, we model the volatility of daily returns of EUR/PLN exchange rates. Statistically significant long memory and asymmetry effects in volatility are observed. These characteristics implicate some challenges in volatility forecasting. Therefore, we combine these two effects in the Fractionally Integrated Asymmetric Power ARCH modelling framework which yields the best goodness-of-fit. Furthermore, it outperforms other models in regard to the applied loss functions and is found to provide the best Value-at-Risk estimation results. Our findings contribute to research on volatility of Polish exchange rate and expand the findings related to dynamic volatility in the existing literature and raises awareness of combined volatility effects to practitioners.

Keywords: asymmetry, GARCH, long memory, Value-at-Risk, volatility forecasting, Złoty.

[1] For advice, remarks and hints, we thank the editor, two anonymous referees, Wolfgang Härdle, Hermann Locarek-Junge, Daniel Tillich, Rafał Weron and the participants of the Wrocław Conference in Finance 2015, especially Krzysztof Piontek for his thoughtful discussion. Hien Pham Thu gratefully acknowledges the financial support from the Deutsche Forschungsgemeinschaft via SFB 649 "Ökonomisches Risiko" and International Research Training Group (IRTG) 1792, Humboldt-Universität zu Berlin.

Streszczenie: Artykuł koncentruje się na uchwyceniu warunkowej zmienności obecnej w prognozach wartości zagrożonej dla badanego kursu wymiany walut. Poprzez zastosowanie szerokiej gamy modeli GARCH dla różnych rozkładów, modelowana jest zmienność dziennych stóp zwrotu dla kursu wymiany walut EUR/PLN. Statystycznie istotna długa pamięć procesu oraz efekt asymetrii zmienności są obserwowalne. Te właściwości powodują pewne wyzwania dla prognozowania zmienności. Dlatego, w badaniu efekty te zostają zintegrowane w modelu FIAPARCH (Fractionally Integrated Asymmetric Power ARCH), który wykazuje najlepsze dopasowanie. Ponadto, model ten wykazuje przewagę mierzoną również za pomocą funkcji straty i przynosi najtrafniejszą prognozę wartości zagrożonej dla przeprowadzonych estymacji. Przedstawione badanie stanowi wkład w obszarze modelowania zmienności polskiej waluty, a także poszerza zakres wiedzy dotyczącej dynamiki zmienności i pogłębia wiedze praktyków na temat łączonych efektów zmienności.

Słowa kluczowe: asymetria, GARCH, długa pamięć, wartość zagrożona, prognozowanie zmienności, PLN.

Essentially, all models are wrong, but some are useful.

George E.P. Box (1919-2013)

1. Introduction

Foreign exchange rates are crucial to the functioning of an economy and they also impact the price level within the country as well as export profits. The tendency of a currency to appreciate or depreciate in value is indicated by the exchange rate volatility. A variety of volatility modelling can be found in the literature; such as models with assumption of unconditional volatility when it is constant over time. However, historical time series exhibit a dynamic volatility.

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models [Bollerslev 1996] have been proposed to account for the conditional volatility in the modelling and forecasting process. Nonetheless, these models do not capture the long memory and asymmetry in volatility [Mensi et al. 2014; Kumar 2014]. By implementing the Fractionally Integrated GARCH (FIGARCH) by Baillie et al. [1996], this paper reveals a significant long memory effect in the EUR/PLN time series. Furthermore, asymmetry in volatility of EUR/PLN exchange rates is confirmed through Asymmetric Power ARCH (APARCH) [Ding, et al. 1993]. This effect refers to an asymmetric impact of upward or downward movements on the conditional variance. The model extension allows for a separate integration of movements with different direction. Both effects of long memory and asymmetry are combined in a Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) modelling framework [Tse 1998], which yields the best goodness-of-fit of all aforementioned models.

Since 1999, the Polish Złoty has been classified as free floating exchange rate regime with its characteristic fluctuations due to market mechanisms [Kelm 2015]. The Polish economy is highly integrated with the Euro area as Poland's exports of goods to the euro area was 51.5% and import of goods from the euro area was 54.5% of the total import/export businesses in 2013 [Eurostat 2015]. Hence, understanding and forecasting the dynamics of the volatility of the EUR/PLN exchange rate is of high practical importance.

There are several studies investigating the behaviour of the Złoty, such as volatility clustering and asymmetry [Kočenda, Valachy 2006; Fidrmuc, Horváth 2008] or infinite persistence of shocks [Będowska-Sójka, Kliber 2010]. However, past studies only found separate effects of asymmetry and infinite shock persistence. Our hypothesis is that the conditional volatility has a long memory effect rather than infinite persistence of shocks. This effect can be described as a slowly decaying (hyperbolic) function of the autocorrelation of the squared residuals [Franke et al. 2015]. Popular low parameterized GARCH models can only depict "short memory" (fast, exponential decay of shocks) and non-stationary models, such as Integrated GARCH [Engle, Bollerslev 1986], exhibit an unlimited autocorrelation (infinite persistence). Further, the asymmetry effect in the conditional volatility indicates that negative returns have a different impact on volatility than positive returns. In this paper we show that both effects can be found in the EUR/PLN time series separately, as well as in a combined model.

The presence of long memory and asymmetry imposes some challenges on volatility forecasting and risk management such as Value-at-Risk (VaR) predictions. Disregarding the asymmetry and long memory effect in volatility can lead to significant underestimation of VaR. We test the forecasting performance of GARCH, FIGARCH, APARCH, and FIAPARCH models with different loss functions. The FIAPARCH outperforms all other models in regard to the given loss functions. VaR prediction quality in short and long trading position is compared with the popular tests by Kupiec [1995] and Christoffersen [1998]. The FIAPARCH model is found to provide the best VaR prediction results. Our research contributes to a better understanding of the behaviour of the volatility of the currency pair EUR/PLN. Acknowledging asymmetry and long-memory of volatility is highly beneficial in hedging FX risks.

Our paper is organized as follows. In Section 2, we define the GARCH-type models which we examine in detail. Data and first results are presented in Section 3. The results of the parameter estimation and forecast evaluation are given and discussed in Section 4. Section 5 concludes.

2. Methodology

Throughout this paper, we set for all $t = 0, \dots, T$:

$$\begin{aligned} y_t &= \mu + \varepsilon_t, \\ \varepsilon_t &= z_t \sqrt{h_t} \quad \text{with } z_t \sim \text{Dist}(0,1) \text{ i.i.d.}, \\ h_t &= \text{Var}(y_t | \mathcal{F}_{t-1}), \end{aligned}$$

where: μ = the unconditional mean of the return series $\{y_t\}_{t=0}^{T}$, h_t = the conditional variance at time t, $\mathcal{F}_{t-1}$ = the σ-algebra generated by the past of the time series up to time $t-1$.

The distribution of the random variable z_t is either the normal (N), student-t (t), or the skewed student-t (Sk-t) [Hansen 1994] distribution. We only define the first two moments (mean and variance), while other necessary distribution parameter (kurtosis and skewness) will be estimated along with all other model parameters. In the following, we focus only on the volatility models and neglect the conditional mean.

The GARCH(1,1) can be given by:

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1},$$

with the non-negativity conditions $\omega > 0$ and $\alpha, \beta \geq 0$ and the stationarity condition $\alpha + \beta < 1$. If $\alpha + \beta = 1$, the resulting process is referred to as Integrated GARCH, which is not (weakly) stationary.

The so-called Asymmetric Power ARCH (APARCH) by Ding et al. [1993] incorporates the stylized effect of asymmetry or so-called leverage effect. This feature is accompanied by modelling a variable power of the volatility. The APARCH(1,1) can be written as:

$$h_t^{\delta/2} = \omega + \alpha(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1})^{\delta} + \beta h_{t-1}^{\delta/2},$$

with the restriction of ω (strictly), α, β, and δ being positive. Furthermore, for the leverage parameter, it has to hold that $\gamma \in [-1,1]$. With these two generalizations, APARCH includes seven other models with ARCH and GARCH among them.

In order to depict the property of long memory in volatility, one has to choose a very high order of lags and hence, an excessive amount of parameters if using a GARCH(p,q)-framework. With the purpose of being more parsimonious, the alternative is the Fractionally Integrated GARCH by Baillie et al. [1996]. The FIGARCH(p,d,q) adds the fractional integration (or long memory) parameter d with $0 \leq d \leq 1$. The FIGARCH(1,d,1) is defined as:

$$\begin{aligned} h_t &= \omega + \left(1 - \beta L - (1 - \alpha L)(1 - L)^d\right)\varepsilon_t^2 + \beta h_{t-1} = \\ &\frac{\omega}{1-\beta} + \sum_{i=1}^{\infty} \lambda_i \varepsilon_{t-i}^2, \end{aligned}$$

where L denotes the lag-operator and the restrictions $\omega > 0$, $0 \leq \beta \leq \alpha + d$, and $0 \leq d \leq 1 - 2\alpha$ must hold. The second line in the definition above is the ARCH(∞) representation where λ_i is calculated from the FIGARCH parameters α, d, and β as shown in Bollerslev and Mikkelsen [1996]. Furthermore, $\sum_{i=1}^{\infty} \lambda_i < 1$ is required for stationarity. The FIGARCH nests GARCH ($d = 0$) and IGARCH ($d = 1$).

The Fractionally Integrated Asymmetric Power ARCH [Tse 1998] combines the extensions of APARCH and FIGARCH in a unified model. The FIAPARCH(1,d,1) is a FIGARCH applied on the APARCH innovations given by:

$$h_t^{\frac{\delta}{2}} = \omega + \left(1 - \beta L - (1 - \alpha L)(1 - L)^d\right)(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1})^\delta + \beta h_{t-1}^{\frac{\delta}{2}} = \frac{\omega}{1-\beta} + \sum_{i=1}^{\infty} \lambda_i(|\varepsilon_{t-i}| - \gamma\varepsilon_{t-i})^\delta.$$

All parameter specifications of APARCH and FIGARCH have to hold for FIAPARCH as well.

In order to apply the volatility models for risk management we define the Value-at-Risk as follows. The k day-ahead VaR for each $t \in 1,2, \dots, M$ is:

$$\mathrm{VaR}_{\mathrm{Dist},a,t}^{(k\ \mathrm{day})} = Q_{\mathrm{Dist},a}\sqrt{\hat{h}_{t+k}},$$

where: $Q_{\mathrm{Dist},a}$ = the a-quantile function for a particular distribution (*N*, *t*, or Sk-t).

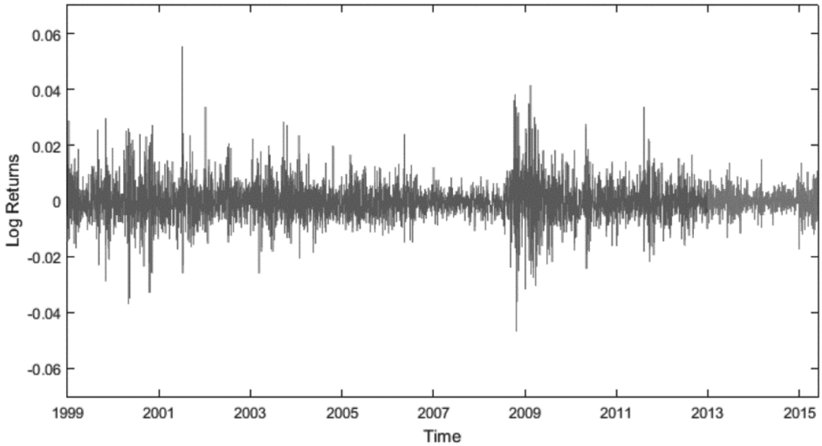
This function is dependent on the estimated parameters of the distributions. Further, $\hat{h}_{t+k}$ is the k day-ahead variance forecast, calculated analytically. The forecast is conducted for 1, 5, and 20 day-ahead to test the performance for daily, weekly, and monthly predictions. We investigate a values of 0.01, 0.05, 0.95, and 0.99 to account for long and short trading positions.

All estimations, forecasts, and evaluations are implemented with MatLab.

3. Data

Our dataset consists of closing prices P_t of the EUR/PLN exchange rate from 01/01/1999 to 05/31/2015 obtained from Bloomberg. We utilize daily logarithmic returns defined as $y_t = \log(P_t) - \log(P_{t-1})$ for $t = 2, \dots, T$. The returns from 2013 to 2015 are used for out-of-sample forecasts and tests thereof. Figure 1 shows the corresponding log returns and the separation between insample and out-of-sample period.

The descriptive statistics and preliminary tests for the EUR/PLN exchange rate return series are given in Table 1. The time series has a zero mean. Furthermore, it shows a deviation from normally distributed samples; the kurtosis, the skewness, as well as the rejected Jarque-Bera test show evidence for this assumption. The non-normality stems from clustering, long memory, and asymmetry in the volatility, which is confirmed by the model estimation results presented in Section 4.



Note: The period 2013 – 2015 is used as out-of-sample.

Figure 1. Log returns of the EUR/PLN exchange rate from 01/01/1999 – 05/31/2015

Source: Authors' own study.

Table 1. Descriptive statistics and preliminary tests for EUR/PLN log returns, 01/01/1999 – 05/31/2015

Descriptive Statistics					
Mean	St. Dev.	Minimum	Maximum	Skewness	Kurtosis
0.0000	0.0068	−0.0466	0.0553	0.3599	8.3450
Preliminary Tests					
Jarque-Bera	Ljung-Box (65)	Peiró-Test	AG-LME y_t^2	ADF	KPSS
5186.0***	3430.9***	0.0463**	0.2512**	−69.48***	0.0267

Note: ADF is the augmented Dickey Fuller statistic, KPSS the Kwiatkowski–Phillips–Schmidt–Shin test statistic, and AG-LME is the Andrews & Guggenberger [2003] long memory estimator. Rejection of the null hypothesis is displayed by *, **, and *** for 10%, 5%, and 1% significance level.

Source: Authors' own study.

By rejecting the assumption of no autocorrelation in the squared returns up to lag 65, the Ljung-Box test suggests heteroskedastic behaviour as well. Testing for asymmetry in the unconditional distribution, we examine the proposed test by Peiró [2004]. It divides the centralized dataset into samples of positive and absolute negative returns and decides whether both samples have the same distribution by a Kolmogorov-Smirnov test. The Peiró-test rejects the null hypothesis of positive and negative EUR/PLN returns being drawn from the same distribution and hints skewness in the distribution.

The long memory property is preliminary tested with the Andrews and Guggenberger [2003] Long Memory Estimator, a bias-reduced version of the popular GPH-estimator [Geweke, Porter-Hudak 1983]. The pseudo-regression estimates the long memory parameter d with $d \in \mathbb{R}$, of autoregressive fractionally integrated moving average (ARFIMA) models[2]. A $d > 0$ show signs of the series having long memory, which is true for the squared returns as a proxy for the variance[3]. Finally, the augmented Dickey-Fuller (ADF) test rejects the hypothesis of the time series being non-stationary and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test for stationarity is not rejected. We assume the series to be stationary.

4. Results & discussion

The estimated parameters and robust standard errors as well as the respective log-likelihood (LL) and Bayesian Information Criterion (BIC) for each model and distribution are reported in Table 2 for the EUR/PLN series.

The FIAPARCH with skewed-t innovations yields the best results regarding log-likelihood and BIC. As anticipated, for each of the four models the goodness-of-fit and BIC increase with higher-parameterized distributions. The skewed-t distribution with its ability to model the unconditional asymmetry (parameter ξ) at a flexible degree of freedom (parameter ν) yields the best results for all models. For the GARCH(1,1) model, we report the so-called IGARCH effect, as $\alpha + \beta \approx 1$, which is well documented throughout the literature for exchange rate time series [Będowska-Sójka, Kliber 2010].

For the APARCH(1,1), we find a statistically significant asymmetry in the conditional variance, reported by γ. The asymmetry parameter is consistently negative over all distributions; the negative sign emphasizes that upward movements have a higher impact on the conditional variance as downward movements. This finding supports the results of Kočenda and Valachy [2006] and Fidrmuc and Horváth [2008], who come to this conclusion with different asymmetric GARCH models for the EUR/PLN exchange rate volatility[4]. They also show that positive conditional skewness is not consistent over all exchange rates, when comparing PLN to other non-EUR exchange rates such as Hungarian Forint, etc. The finding of conditional asymmetry is of particular interest for any forecast, especially for VaR predictions, as different directions and news impact are modelled asymmetrically for the short and long side.

[2] For ARFIMA models, it has to hold that $d \in (-0.5, 0.5)$. A d higher than 0.5 yields a non-stationary ARFIMA series.

[3] We apply the log-periodogram regression for $T^{0.5}$ observations and $r = 3$ additional regressors.

[4] Kočenda and Valachy [2006] use TGARCH-M from 1999 to 2005. Fidrmuc and Horváth [2008] use an augmented TGARCH from 1999 to 2006. The TGARCH (Threshold-GARCH) has a dummy variable for negative returns.

This is beneficial of the forecasting quality since it is an evident improvement over the assumption of symmetric return distributions as presented later. Comparing the studies of Kočenda and Valachy [2006] and Fidrmuc and Horváth [2008] with our results indicates a stability of the direction of news impact. Analysis of the stability of this particular parameter is prone to further research. Additionally, the power parameter δ is close to 2 while not being statistically different from 2 for all three distributions. This leads to the conclusion that the EUR/PLN series features a small, if not virtually zero, correlation between absolute returns (see further: Ding et al. [1993]).

For FIGARCH($1,d,1$), the fractional differencing parameter d is statistically significant which fulfills the stationarity conditions over all distributions. Evidently, this shows a longer persistence of shocks in the variance as the standard GARCH framework is able to depict. This finding is a possible explanation why Będowska-Sójka and Kliber [2010] found IGARCH to be superior over GARCH.

Comparing the goodness-of-fit and BIC, the FIGARCH($1,d,1$) outperforms the GARCH and is approximately at par with the APARCH(1,1). Combining asymmetry and long memory in the FIAPARCH framework with skewed-t innovations yields the best goodness-of-fit and BIC. The power parameter δ is closer to 2 than in the APARCH framework, underlining the correlation of squared returns. The parameters for long memory and asymmetry, d and γ respectively, as well as the skewness shape parameter of the skewed-t distribution, ξ, are statistically significant. Hence, we conclude that combining both effects in a unified model benefits the modelling quality, as the time series features both long memory and asymmetry. The model selection is supported by our findings regarding properties of the series' evaluations and tests in Section 3.

In order to evaluate the models' performance in forecasting volatility, we test how consistently each model predicts future variances for 1, 5, and 20 day-ahead forecasts. The performance is determined by different loss functions and Value-at-Risk tests for different levels and trading sides. These tests are applied on an out-of-sample window from 01/01/2013 to 05/31/2015. From Table 3, we deduce that FIAPARCH with a student-t distribution yields the lowest and hence, the best loss function results for 1-day ahead forecasts. The outperformance of FIAPARCH is confirmed by the findings for the 5 and 20 day-ahead predictions[5].The superiority of FIAPARCH with respect to the tested models emphasizes the importance to introduce factors that account for asymmetry, as well as long memory, when modelling the conditional variance.

Regarding the Value-at-Risk tests, all models show good performance in predicting the Value-at-Risk in a short trading position. This is shown by none rejections for neither the Kupiec nor the Christoffersen test. Among these good results, FIAPARCH-t repeatedly shows the best performance by producing the closest cov-

[5] Due to page limitations we only present the results for 1-day ahead forecast. Results for 5-day and 20-day forecast can be obtained upon request from the authors.

erage to α and the lowest test-statistics. Regarding the long trading position, the picture is somewhat different. Only three and five models out of 12 are not rejected by the Kupiec and the Christoffersen test, respectively. The skewed-t versions of APARCH, FIGARCH, and FIAPARCH pass the Kupiec test. Comparing Figure 2 and the coverage ratios in Table 3, one can see that the coverage is always lower than the level of α for the Value-at-Risk. Hence, all models are too conservative in measuring the risk exposure. A possible explanation for this observation is the relatively short time window for out-of-sample analysis. Further research should compare our results with an analysis of a more flexible time frame. It should be also mentioned that the Kupiec and Christoffersen test are of little power for such small sample sizes and more advanced VaR tests based on loss functions or the empirical distribution of the return series might yield different results [Piontek 2010].

However, we conjecture that accounting for asymmetry in the respective distribution yields better results than neglecting an asymmetry in the time series. The results for the 1 day-ahead Value-at-Risk forecast for $\alpha = 0.05$ are given in Figure 2 and Table 3 as well[6].

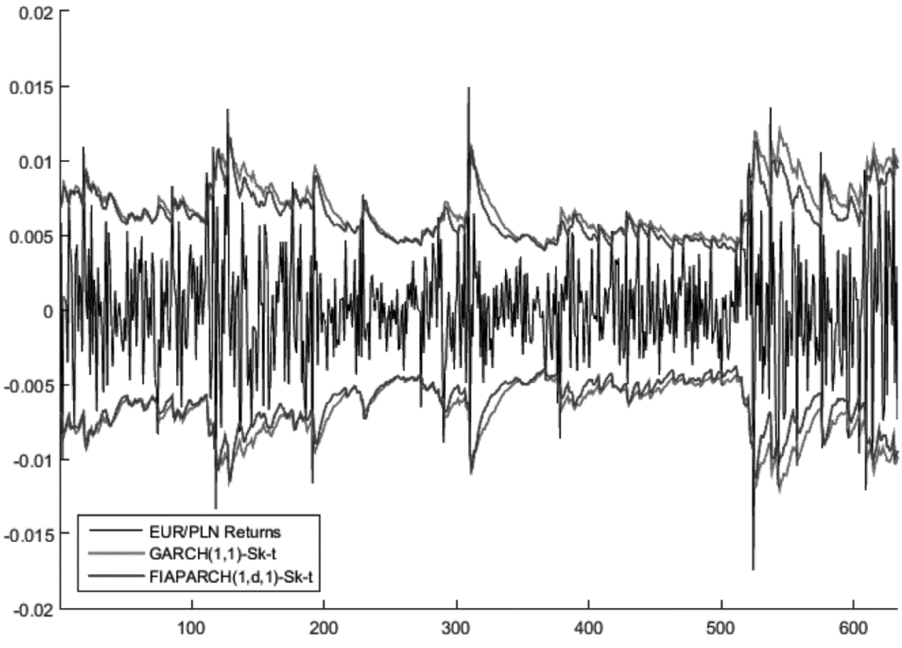


Figure 2. Value-at-Risk (a = 0.05) for 1-day ahead forecast of EUR/PLN 2013-2015

Source: Authors' own study.

[6] Due to page limitations we only present the results for 1-day ahead forecast and $a = 0.05$. Results for 5-day and 20-day forecast for $a = 0.05$ and $a = 0.01$ can be obtained upon request from the authors.

Table 2. Parameter estimates for EUR/PLN log returns, 01/01/1999 – 05/31/2015 (4279 observations)

	GARCH(1,1)			APARCH(1,1)			FIGARCH(1,*d*,1)			FIAPARCH(1,*d*,1)		
	Normal	Student-t	Skewed-t	Normal	Student-t	Skewed-t	Normal	Student-t	Skewed-t	Normal	Student-t	Skewed-t
ω	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
α	0.0899 (0.0146)	0.0863 (0.0134)	0.0864 (0.0112)	0.0865 (0.0147)	0.0867 (0.0118)	0.0869 (0.0136)	0.1878 (0.0550)	0.2204 (0.0678)	0.2222 (0.0733)	0.1586 (0.0711)	0.1865 (0.0659)	0.1888 (0.1524)
β	0.9083 (0.0144)	0.9103 (0.0139)	0.9100 (0.0105)	0.9135 (0.0136)	0.9133 (0.0117)	0.9131 (0.0130)	0.5354 (0.0735)	0.5743 (0.1266)	0.5728 (0.1368)	0.4624 (0.1157)	0.5032 (0.1055)	0.5024 (0.3535)
γ				–0.1846 (0.0636)	–0.1660 (0.0725)	–0.1679 (0.0546)				–0.2130 (0.0597)	–0.1795 (0.0510)	–0.1819 (0.0793)
δ				1.7821 (0.1471)	1.7965 (0.1992)	1.7903 (0.1370)				1.9544 (0.0758)	1.9617 (0.0620)	1.9535 (0.6304)
d							0.4250 (0.0566)	0.4445 (0.1111)	0.4432 (0.1009)	0.3702 (0.0703)	0.4005 (0.0683)	0.4000 (0.2383)
υ		6.6715 (0.6735)	6.7518 (0.6769)		6.6861 (0.6971)	6.7420 (0.6957)		6.9726 (0.8672)	7.0430 (0.7578)		6.9012 (0.7084)	6.9448 (0.9103)
ξ			0.0908 (0.0200)			0.0919 (0.0200)			0.0933 (0.0213)			0.0950 (0.0232)
LL	15941	16058	16068	15948	16062	16072	15964	16069	16079	15979	16077	**16087**
BIC	–31862	–32090	–32104	–31864	–32085	–32098	–31903	–32105	–32119	–31919	–32108	**–32122**

Robust standard errors are in parenthesis. Bold numbers indicate the best model regarding the best goodness-of-fit (LL) and information criterion (BIC). As suggested by Tse [1998], we truncate the ARCH(∞) representation at 1000 lags for FIGARCH and FIAPARCH.
Source: Authors' own study.

Table 3. 1-day ahead forecast loss function and Value-at-Risk ($\alpha = 0.05$) results for EUR/PLN from 01/01/2013 - 05/31/2015 (633 observation)

		GARCH(1,1)			APARCH(1,1)			FIGARCH(1,d,1)			FIAPARCH(1,d,1)		
		Normal	Student-t	Skewed-t	Normal	Student-t	Skewed-t	Normal	Student-t	Skewed-t	Normal	Student-t	Skewed-t
Loss Functions	RMSE (10^{-5})	2.8230**	2.8180	2.8197***	2.7989	2.8006	2.7986	2.8123*	2.8079	2.8102**	2.7831	**2.7825**	2.7835
	MAE (10^{-5})	1.7369***	1.7264***	1.7342***	1.7294***	2.6442***	1.7248***	1.6950***	1.6924***	1.7034***	**1.6341**	1.6410***	1.6509***
	MME(U) (10^{-3})	1.0809	1.0899	1.0828	**1.0730**	1.0799***	1.0775**	1.1221	1.1217	1.1111	1.1662	1.1617	1.1509
	MME(O) (10^{-3})	2.6564***	2.6370***	2.6575***	2.6682***	1.7222***	2.6563***	2.5640**	2.5608**	2.5902***	**2.4249**	2.4405	2.4680
Coverage	short	0.0411	0.0427	0.0411	0.0379	0.0442	0.0379	0.0427	0.0427	0.0411	0.0458	0.0490	0.0427
	long	0.0316	0.0348	0.0348	0.0316	0.0316	0.0379	0.0348	0.0348	0.0379	0.0363	0.0363	0.0395
Kupiec Test	short	11.276	0.7552	11.276	21.161	0.4602	21.161	0.7552	0.7552	11.276	0.2400	0.0141	0.7552
	long	5.1640**	3.4514*	3.4514*	5.1640**	5.1640**	21.161	3.4514*	3.4514*	21.161	2.7386*	2.7386*	15.802
Christoff. Test	short	19.259	14.082	19.259	22.025	0.9868	22.025	14.082	14.082	19.259	0.6585	0.2681	0.8657
	long	7.2973**	5.0426*	5.0426*	7.2973**	7.2973**	32.663	5.0426*	5.0426*	32.663	40.975	40.975	25.439

Loss Functions: The results in bold face show the best result for each loss function. Rejection of the null hypothesis of the Hansen [2005] Super Predictive Ability test (with 10,000 bootstraps) is displayed by *, **, and *** for 10%, 5%, and 1% significance level. The null hypothesis is rejected if the model is inferior to the other models regarding a given loss function. These loss functions are the root mean squared error $\left(\text{RMSE} \coloneqq \sqrt{\frac{1}{m}\sum_{t=1}^{m}\left(\hat{h}_t - h_t\right)^2}\right)$, the mean absolute error $\left(\text{MAE} \coloneqq \frac{1}{m}\sum_{t=1}^{m}|\hat{h}_t - h_t|\right)$, and the mixed mean error for over-predicted values $\left(\text{MME(U)} \coloneqq \frac{1}{m}\left(\sum_{t\in O}|\hat{h}_t - h_t| + \sum_{t\in U}\sqrt{|\hat{h}_t - h_t|}\right)\right)$ and under-predicted values $\left(\text{MME(O)} \coloneqq \frac{1}{m}\left(\sum_{t\in U}|\hat{h}_t - h_t| + \sum_{t\in O}\sqrt{|\hat{h}_t - h_t|}\right)\right)$, where $O \coloneqq \{t \in \{1, \ldots, m\} | \hat{h}_t > h_t\}$ and $U \coloneqq \{t \in \{1, \ldots, m\} | \hat{h}_t < h_t\}$, with m as the number of out-of-sample observations, $\hat{h}_t$ as the estimated variance, and h_t as the real variance (we use the squared residual ε_t^2 as a proxy for h_t). *Value-at-Risk*: The values given represent the test statistics of the Value-at-Risk tests by Kupiec [1995] and Christoffersen [1998] at α for short and long trading positions. Rejection of the null hypothesis is displayed by *, **, and *** for 10%, 5%, and 1% significance level.

However, we conjecture that accounting for asymmetry in the respective distribution yields better results than neglecting an asymmetry in the time series. The results for the 1 day-ahead Value-at-Risk forecast for $\alpha = 0.05$ are given in Fig. 2 and Table 3 as well.[7]

5. Conclusion

Since the Eurozone countries are the major recipient of Polish exports, a reliable modelling of the EUR/PLN exchange rates and fluctuation risks is of great necessity, especially for exporters. We find significant evidence of long memory and asymmetric behaviour in its conditional variance. These long memory and asymmetry effects render simple variance and VaR forecasting methods useless. Neglecting these effects biases any forecast or risk evaluation. This could lead to wrong and unnecessarily costly hedging strategies, as well as underestimated risk exposure.

We present more sophisticated models capturing the time-varying dynamics of volatility and show that an evolvement to the FIAPARCH framework, which unifies long memory and asymmetry, yields a substantial improvement to variance forecasting results. We also find that models which are able to depict long memory and/or asymmetry are clearly superior to a simple GARCH framework. Evidently, a more precise modelling of the conditional variance leads to improved VaR predictions. Implementing the FIAPARCH in risk modelling improves the results obtained for practical application, as the framework is able to depict more effects and reacts more precisely to changes.

Due to the asymmetric modelling, extreme movements like shocks might be detectable earlier giving an advantage over simpler models like the GARCH. We conclude that the aforementioned effects must be included in risk assessment of the EUR/PLN exchange rates in order to obtain more accurate forecasts and prudent risk management.

References

Andrews, D.W.K., Guggenberger, P., 2003, *A Bias-Reduced Log-Periodogram Regression Estimator for the Long-Memory Parameter*, Econometrica, vol. 71, no. 2, pp. 675–712.

Baillie, R.T., 1996, *Long memory processes and fractional integration in econometrics*, Journal of Econometrics, vol. 73, no. 1, pp. 5–59.

Baillie, R.T., Bollerslev, T., Mikkelsen, H.O., 1996, *Fractionally integrated generalized autoregressive conditional heteroskedasticity*, Journal of Econometrics, vol. 74, no. 1, pp. 3–30.

[7] Due to page limitations we only present the results for 1-day ahead forecast and $\alpha = 0.05$. Results for 5-day and 20-day forecast for $\alpha = 0.05$ and $\alpha = 0.01$ can be obtained upon request from the authors.

Będowska-Sójka, B., Kliber, A., 2010, *Realized volatility versus GARCH and stochastic volatility models. The evidence from the WIG20 index and the EUR/PLN foreign exchange market*, Przegląd Statystyczny (Statistical Review), vol. 57, no. 4, pp. 105–127.

Bollerslev, T., Mikkelsen, H.O., 1996, *Modeling and pricing long memory in stock market volatility*, Journal of Econometrics, vol. 73, no. 1, pp. 151–184.

Christoffersen, P.F., 1998, *Evaluating Interval Forecasts*, International Economic Review, vol. 39, no. 4, pp. 841–862.

Ding, Z., Granger, C.W.J., Engle, R.F., 1993, *A long memory property of stock market returns and a new model*, Journal of Empirical Finance, vol. 1, no. 1, pp. 83–106.

Engle, R.F., Bollerslev, T., 1986, *Modelling the persistence of conditional variances*, Econometric Reviews, vol. 5, no. 1, pp. 1–50.

Eurostat, 2015, *GDP per capita, consumption per capita and price level indices*, Statistics Explained, http://ec.europa.eu/eurostat/statistics-explained/index.php/GDP_per_capita_consumption_per_capita_and_price_level_indices.

Fidrmuc, J., Horváth, R., 2008, *Volatility of exchange rates in selected new EU members: Evidence from daily data*, Economic Systems, vol. 32, no. 1, pp. 103–118.

Franke, J., Härdle, W.K., Hafner, C.M., 2015, *Statistics of Financial Markets*, Fourth ed., Springer, Heidelberg.

Geweke, J., Porter-Hudak, S., 1983, *The Estimation and Application of Long Memory Time Series Models*, Journal of Time Series Analysis, vol. 4, no. 4, pp. 221–238.

Hansen, B.E., 1994, *Autoregressive Conditional Density Estimation*, International Economic Review, vol. 35, no. 3, pp. 705–730.

Hansen, P.R., 2005, *A Test for Superior Predictive Ability*, Journal of Business & Economic Statistics, vol. 23, no. 4, pp. 365–380.

Kelm, R., 2015, *Rational Expectations vs . Imperfect Knowledge Economics: What Does Really Drive the Polish Złoty?*

Kočenda, E., Valachy, J., 2006, *Exchange rate volatility and regime change: A Visegrad comparison*, Journal of Comparative Economics, vol. 34, no. 4, pp. 727–753.

Kumar, A., 2014, *Testing for long memory in volatility in the Indian Forex market*, Economic annals, vol. 59, no. 203, pp. 75–90.

Kupiec, P.H., 1995, *Techniques for Verifying the Accuracy of Risk Measurement Models*, The Journal of Derivatives, vol. 3, no. 2, pp. 73–84.

Mensi, W., Hammoudeh, S., Yoon, S.M., 2014, *Structural breaks and long memory in modeling and forecasting volatility of foreign exchange markets of oil exporters: The importance of scheduled and unscheduled news announcements*, International Review of Economics and Finance, vol. 30, no. 1, pp. 101–119.

Peiró, A., 2004, *Asymmetries and tails in stock index returns: are their distributions really asymmetric?*, Quantitative Finance, vol. 4, no. 1, pp. 37–44.

Piontek, K., 2010, *The analysis of power for some chosen VaR backtesting procedures: Simulation approach*, In A. Fink, B. Lausen, W. Seidel, A. Ultsch, eds. *Advances in Data Analysis, Data Handling and Business Intelligence*. Springer-Verlag, Heidelberg, pp. 481–490.

Tse, Y.K., 1998, *The conditional heteroscedasticity of the yen-dollar exchange rate*, Journal of Applied Econometrics, vol. 13, no. 1, pp. 49–55.