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# HOW DO TERM PREMIA CHANGE OVER TIME? EVIDENCE FROM THE US DOLLAR LIBOR DATA USING A FOURIER APPROXIMATION

Using Gallant's (1981) version of the Fourier flexible form we modify the perfect foresight spread equation that allows for a time-varying term premium and estimate it on the weekly sampled US dollar LIBORs ranging from January 1998 to June 2013 to find out that the term premia not only vary over time for the whole spectrum of maturities but are periodic, change with the US business cycle and are broken within the recession periods. We also reveal that – on average – the longer the maturity, the greater the term premium, both in the boom and in the recession. The other significant feature of the term premia for all maturities is that the boom premia are many times smaller than their recession counterparts. For the maturities of 35 weeks and over, the yield spread is a good predictor of future changes in the short interest rate. For other maturities it turns out to be a downwards biased predictor. Except for the two shortest maturities, all restricted perfect foresight spread regressions, i.e. those ignoring the change of premia with the business cycle and their breaks, are misspecified.

**Keywords:** term structure of interest rates, expectations hypothesis, perfect foresight spread, time-varying term premium, Fourier approximation, US dollar LIBORs

**JEL Classification:** E43

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## 1. INTRODUCTION

Although a time-varying term premium long ago became a standard assumption in modelling the term structure of interest rates, and since then it has been empirically well documented (see Fama (1976a), (1976b), (1984a), (1984b), Startz (1982), Fama, Bliss (1987), McCulloch (1987), Froot (1989), Lee, Jo (1996), Tzavalis, Wickens (1997), (1998), Cuthbertson, Bredin (2001), Cuthbertson, Nitzsche (2003), Gravelle, Morley (2005)), the possibility that term premia can not only be volatile but also change with a business cycle, the decisions of monetary authorities, and due to a market turmoil they may be subject to breaks of an unknown form and number, is rather ignored. Instead of that the dynamics of term premia is usually conditioned on these few but exclusively influential factors. For instance, in

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the asset pricing literature term premia are either related to the volatility of interest rates (Cox et al. (1985)), their conditional variances (Fama (1976), Shiller et al. (1983), Engle et al. (1987), Klemkosky, Pilotte (1992)) or conditional covariances with the equity premium (Merton (1973), Hess, Kamara (2005)). Fama (1986) and Fama and Bliss (1987), suggest that the ordering of risks and rewards as well as risk premia change with the business cycle. The structural change in term premia is usually considered within Markov switching models (Bekaert et al. (2001), Psaradakis et al. (2006), Yoo (2010)). Thus leaving out any from the above list of potential factors underlying the dynamics of term premia may seriously bias estimators employed to estimate the term structure of interest rates models and invalidate the results of testing for the expectations hypothesis (EH).

This paper shows how to circumvent this problem in the regression framework. In doing so we build on a simple modification of the asset pricing theory in line with Engle et al. (1987), and follow the methodology of Pagan and Ullah (1988) that allows for modelling risk terms. Similarly to Gerlach (2003) and Liau and Yang (2009), who analyzed the term structure of interbank interest rates in Hong Kong and commercial papers in Taiwan, we augment a conventional perfect foresight spread (PFS) equation by a natural logarithm of the conditional variance of innovations in the short rate, a proxy of the time-varying term premium. To control for the premium's unknown nature (possible change with a business cycle, decisions of monetary authorities, and/or breaks) we modify it using a variant of Gallant's (1981) flexible Fourier form. Since the premium's proxy may differ from the true premium causing an 'error-in-variables' problem, we instrument it accordingly and consistently estimate the PFS equation using the general method of moments (GMM). The relevant computations are performed using Stata and R.

The empirical illustration that follows is based on a weekly sampled data which consisted of the US dollar LIBORs[1] with maturities ranging from 1 to 52 weeks, beginning in January 1998 and ending in June 2013.[2] We also use information on the US business cycle and GDP growth to interpret the dynamics of term premia more accurately.[3] Since the LIBORs serve as the

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[1] LIBOR stands for the London Interbank Offered Rate.

[2] The data were provided by Thomson Reuters.

[3] We fix recession periods according to The National Bureau of Economic Research Business Cycle Committee announcements (see the dates of US Business Cycle Expansions and Contractions at http://www.nber.org/ cycles.html). The data on the US quarterly GDP growth comes from The Bureau of Economic Analysis (see Table 1.1.3. Real Gross Domestic Product, Quantity Indexes downloaded from http://www.bea.gov/national/nipaweb/ Down SS2.asp on August 19, 2014).

primary benchmark for short term interest rates globally and are determined by the equilibrium between demand and supply in the funds market, their dollar rates almost ideally exhibit the cost of borrowing in US dollars for a very wide spectrum of market agents in the United States and worldwide.[4]

The general conclusion stemming from the analysis is that except for the two shortest maturities, all the restricted regressions that ignore the 'true' nature of the term premium (its change with a business cycle, breaks) are misspecified. Having inferred this from the unrestricted regressions, we cannot reject the hypothesis stating that the slope of the term structure equals one, as predicted by the EH, but only for the longer maturities. For the shorter maturities, the yield spread is a downwards biased predictor of the change in the short interest rate. For all maturities the term premia change with the US business cycle, are broken and significantly rise in the recession periods.

The rest of the paper proceeds as follows. Section 2 sketches the PFS model of the term structure with the time-varying term premium allowing for its change with a business cycle and breaks of an unknown form and number, and shows the way it is consistently estimated and validated. Section 3 discusses the empirical findings. The last section briefly concludes.

## 2. MODIFIED PERFECT FORESIGHT SPREAD MODEL

We start with the PFS equation modified by Gerlach (2003) and Liau and Yang (2009),

$$\sum_{i=1}^{n-1}\left(1-i/n\right)\Delta R_{t+i}^{(1)}=\alpha^{(n)}+\gamma^{(n)}\ln\left(\sigma_t^2\right)+\beta^{(n)}S_t^{(n)}+\zeta_{t+n-1}^{(n)} \tag{1}$$

in which the dependent variable is a spread that would be predicted by the model if agents had a perfect foresight about the future interest rates, $S_t^{(n)}=R_t^{(n)}-R_t^{(1)}$ is a yield spread between the long and the short rate, $\ln\left(\sigma_t^2\right)$ is the natural logarithm of the conditional variance of innovations in the short rate exhibiting a time-varying term premium, $\zeta_{t+n-1}^{(n)}$ is an error term

[4] According to The ICE Benchmark Administration which became the administrator of LIBOR on February 1, 2014, LIBOR is referenced to by an estimated US$350 trillion of outstanding business in maturities ranging from overnight to more than 30 years (see *Position Paper on the Evolution of ICE LIBOR* downloaded from https://www.theice.com/iba/libor#governance on August 19, 2014).

that follows a moving average (MA) process of order $(n-2)$, $t=1,2,\ldots,T$ and $n=2,4,\ldots,52$ for the weekly sampled data.

Estimation of Equation (1) like an estimation of any other type ARMA-GARCH (in-mean) model is not straightforward in case it may be misspecified (Francq, Zakoïan, (2006), (2010)). The most likely misspecification is that $\ln\left(\sigma_t^2\right)$ alone may not accurately exhibit periodicity and breaks in the time-varying term premium leading to inconsistency of the conventional quasi-maximum likelihood estimator (QMLE).[5] The other very likely problem is referred to as the density misspecification, in which case the parametric likelihood family does not properly exhibit the heavily-tailed and asymmetric innovations which requires the use of 3-step QLME modification (Fan et al. (2014)). That is why we respecify Equation (1) and implement a consistent estimation procedure as proposed by Pagan and Ullah (1988).

First, as Gallant (1984), Davies (1987), Gallant and Souza (1991) and Becker et al. (2004) show that a Fourier approximation can capture the behaviour of an unknown function even if the function itself is not periodic, and Becker et al. (2006) and Enders and Lee (2012) – that a series with various types of breaks can often be captured using a selected frequency components of the Fourier approximation, we append

$$\sum_{k=1}^{r}\varphi_k^{(n)}\sin\left(2\pi kt/T\right)+\sum_{k=1}^{r}\omega_k^{(n)}\cos\left(2\pi kt/T\right)+ \\ +\left[\sum_{k=1}^{r}\delta_k^{(n)}\sin\left(2\pi kt/T\right)+\sum_{k=1}^{r}\eta_k^{(n)}\cos\left(2\pi kt/T\right)\right]\ln\left(\sigma_t^2\right) \tag{2}$$

to the right hand side of Equation (1). That enables us to test further for the following hypotheses of interest: $\bigwedge_k \varphi_k^{(n)}=\omega_k^{(n)}=\delta_k^{(n)}=\eta_k^{(n)}=\gamma^n=0$ (the term premium is not time-varying), $\bigwedge_k \varphi_k^{(n)}=\omega_k^{(n)}=\delta_k^{(n)}=\eta_k^{(n)}=0$ (the term premium is not changing with a business cycle and (or) has no breaks), and $\beta^{(n)}=1$ (the yield spread is an unbiased predictor of future changes in the short rate). Next, taking into account that for a given $n$ the PFS is a linear combination of future changes in the short rate, we assume that the change of the latter follows a low order ARMA($l$,$q$) process with possible GARCH($p$,$s$)-in-mean innovations:

[5] Even though were the use of QMLE valid, it would be difficult to avoid convergence problems of the stepping algorithm caused by a high order MA process for $\zeta_{t+n-1}^{(n)}$.

$$\Delta R_t^{(1)} = \psi_0 + \psi_1 g\left(\sigma_t^2\right) + \sum\nolimits_{i=1}^{l} \varphi_i \Delta R_{t-i}^{(1)} + \sum\nolimits_{j=1}^{q} \kappa_{t-j} \xi_{t-j} + \xi_t \,, \qquad (3)$$

$$\sigma_t^2 = \phi + \sum\nolimits_{j=1}^{p} \gamma_j \xi_{t-j}^2 + \sum\nolimits_{k=1}^{s} \delta_k \sigma_{t-k}^2 \,, \qquad (4)$$

where $g(\cdot)$ is a function of $\sigma_t^2$, $\xi_t = \sigma_t \varepsilon_t$ and $\varepsilon_t$ follows Student's *t* to exhibit heavy tails of the innovations. Since the true values of $\sigma_t^2$ are unknown and replacing them by their sample estimates, $\hat{\sigma}_t^2$, leads to an 'errors-in-variables' problem, we consistently estimate Equation (1)+(2) using a GMM in the way proposed by Pagan and Ullah (1988), in which the unknown conditional variance $\sigma_t^2$ is replaced with the square of fitted error from model (3)-(4), $\hat{\xi}_t^2$, and the latter – a noisy measure of the short rate volatility – is instrumented by the estimated conditional variance and its lags up to an appropriate order. The choice of instruments is later ascertained performing a bundle of tests including the Kleibergen-Paap underidentification test (KP), the endogeneity test for endogenous regressors (END), and the Hansen test (HAN) (see Kleibergen, Paap (2006), Baum et al. (2007)).

The functional form, whose structure is chosen upon the Akaike information criterion (AIC), is validated throughout the Ramsey type error specification test (RES) robust to heteroskedasticity and autocorrelation, adapted for the IV (GMM) estimation by Pagan and Hall (1983) and Pesaran and Taylor (1999). Decisions regarding the GARCH part of the model are made upon the AIC and the Schwarz Bayesian information criterion (BIC) as well as the Ljung-Box portmanteau test applied to standardized residuals from Equation (3) and their squares.

Since the individual *k*'s from Equation (1)+(2) are not known but estimated, under the null of no change with a business cycle or breaks of whatever the form and number, they become unidentified nuisance parameters. In such circumstances we obtain the critical values of the relevant test statistics distributions employed in testing for the significance of the Fourier components performing a Monte Carlo experiment with the DGP in Equation (1)+(2).[6] The required characteristics of $\zeta_t^{(n)}$ we recover from the residuals of previously estimated models.

[6] The other reason for conducting a Monte Carlo experiment is that Bekaert et al. (1997) and Garganas and Hall (2011) show that the expectation hypothesis tests of the term structure of interest rates are extremely biased in finite samples in the case interest rates are persistent.

## 3. EMPIRICAL RESULTS

Since interest rates are usually found to be $I(1)$, variables estimation of Equation (1)+(2) is preceded by testing for the presence of a unit root in the US dollar LIBORs and their spreads as well as their stationarity. To capture the influence of a business cycle or a market turmoil on interest rates causing their possible periodicity and breaks, we employ the modified versions of ADF and KPSS tests invented by Becker et al. (2006) and Enders and Lee (2012), in which their deterministic part of an auxiliary regression is based on the Fourier approximation. The results of both tests indicate that the change in US dollar LIBORs, including one week (short) rate and all their spreads, are stationary processes.[7]

The search for a proxy of the time-varying term premium with the use of model (3)-(4) is performed as in Gerlach (2003) on the data driven basis. In doing so we follow Bollerslev et al. (1992), who advised to implement a parsimonious GARCH(*p,s*) not to overfit the data.[8] We find that out of all the possible specifications in which $p,s \leq 2$, it is GARCH(2,2)-in-mean that is preferred by the AIC and BIC (see Table 1). More interestingly, the estimates of the *IG* test statistics show that for every specification we cannot reject the null of IGARCH-in-mean. Besides that, even though we additionally run their ARMA versions in which $l,q \leq 3$, we are unable to solve for autocorrelation in their standardized residuals except for IGARCH(1,1)-in-mean. That is why we choose the following:

$$\Delta R_t^{(1)} = -0.068\sigma_t + \xi_t\,,\ \sigma_t^2 = 3.12\times 10^{-6} + 0.248\xi_{t-1}^2 + 0.752\sigma_{t-1}^2\,,\ df = 2.413\,,^{9}$$
$$LL = 1660.50\,,\ AIC = -3313.00\,,\ BIC = -3294.22\,,$$

for further analysis.

The Ljung-Box portmanteau test for white noise in its standardized residuals gives $Q(1) = 0.07$ [0.80], $Q(13) = 15.30$ [0.29], $Q(26) = 33.15$ [0.16] and $Q(52) = 62.46$ [0.15], where $Q(\cdot)$ are the sample estimates of the relevant test statistics, under the null of no autocorrelation up to the $(\cdot)$-th order distributed as $\chi^2(\cdot)$, and the numbers in brackets are their

[7] The results of both tests are available for inspection upon request.

[8] They report that most empirical implementations of GARCH( $p,s$ ) adopt low orders of $p$ and $s$ lags, both not exceeding two.

[9] The standard errors of the parameters are 0.017, $1.06\times 10^{-6}$, 0.027, 0.027 and 0.078, respectively.

Table 1

Summary characteristics of GARCH($p$,$s$)-in-mean specifications

| GARCH | | | | | IGARCH | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Standardized residuals | | | | Squares of standardized residuals | | | |
| ($p$, $s$) | *LL* | *AIC* | *BIC* | *IG*[a] | $Q(1)$ | $Q(13)$ | $Q(26)$ | $Q(52)$ | $Q(1)$ | $Q(13)$ | $Q(26)$ | $Q(52)$ |
| (2, 2) | 1713.65 | –3413.29 | –3380.44 | 1.41 [0.23] | 0.11 [0.74] | 17.66 [0.17] | 42.25 [0.02] | 80.35 [0.01] | 0.03 [0.87] | 0.78 [1.0] | 1.22 [1.0] | 3.65 [1.0] |
| (2, 1) | 1711.77 | –3411.55 | –3383.39 | 2.07 [0.15] | 0.11 [0.74] | 18.46 [0.14] | 42.70 [0.02] | 78.77 [0.01] | 0.03 [0.87] | 0.77 [1.0] | 1.20 [1.0] | 3.02 [1.0] |
| (1, 2) | 1704.05 | –3396.10 | –3367.94 | 1.05 [0.31] | 0.07 [0.80] | 16.06 [0.25] | 37.66 [0.07] | 70.15 [0.05] | 0.01 [0.93] | 0.58 [1.0] | 1.15 [1.0] | 3.17 [1.0] |
| (1, 1) | 1688.22 | –3366.43 | –3342.97 | 0.76 [0.39] | 0.07 [0.80] | 15.30 [0.29] | 33.15 [0.16] | 62.46 [0.15] | 0.00 [0.99] | 0.51 [1.0] | 1.05 [1.0] | 2.56 [1.0] |

*LL* – sample log likelihood estimate; [a] *IG* – Wald test statistics to test for the IGARCH effect, under $H_0$ $\left(\sum_{j=1}^{p}\gamma_j + \sum_{k=1}^{s}\delta_k = 1\right)$ distributed as $\chi^2(1)$; $Q(s)$ – Ljung-Box test statistics, under the null of no autocorrelation up to the $s$-th order distributed as $\chi^2(s)$, $p$-values in brackets.

Source: author’s own

*p*-values. The same test in the squares of standardized residuals shows $Q(1) = 0.00$ [0.99], $Q(13) = 0.51$ [1.0], $Q(26) = 1.05$ [1.0] and $Q(52) = 2.56$ [1.0]. Thus the model seems to be adequate in describing conditional heteroskedasticity of the data at the 5 per cent significance level. If that is correct, the change in the short rate is strictly stationary, but not weakly stationary, because it does not have two first moments. As such it contradicts the results obtained from the Fourier type ADF and KPSS tests.

On the other hand it is well known that IGARCH or nearly IGARCH parameter estimates may be produced while fitting a misspecified GARCH to a true GARCH process exhibiting either structural breaks or occasional level shifts in the volatility (see Diebold (1986), Lamoureux, Lastrapes (1990), Franses (1995), Caporale et al. (2003), Mikosch, Starica (2004) and Tsay (2005), p. 122, among many others). This might be the case as an inspection of Figure 1 reveals an increased volatility of the short rate change within both recession periods (March–November 2001, obs. no 166–200,

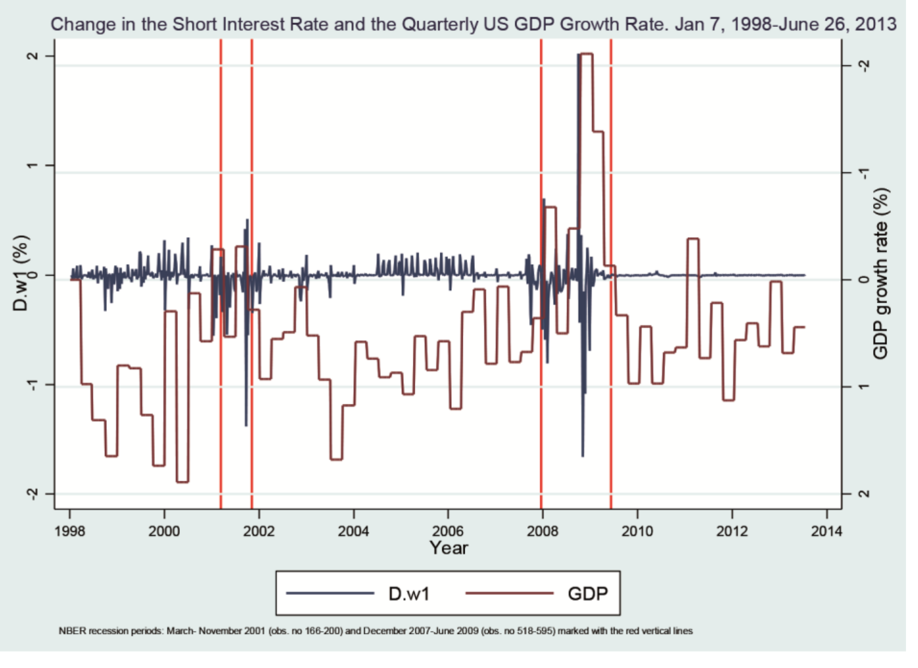


Figure 1. Change in the Short Interest Rate and the Quarterly US GDP Growth Rate. January 7, 1998-June 26, 2013

Source: author's own

rec. #1, and December 2007–June 2009, obs. no 518–595, rec. #2) and several weeks around them, as well as some differences in the volatility among particular booms (January 1998–February 2001, obs. no 1–165, boom #1, December 2001–November 2007, obs. no 201–517, boom #2, and July 2009–June 2013, obs. no 596–804, boom #3). The second boom is also not homogenous as far as the volatility is concerned.

To decide upon a possible misspecification of the term premium proxy with the use of the IGARCH(1,1)-in-mean process, we estimate a couple of the Markov switching GARCH(1,1)-in-mean models that allow for an asymmetric response of the short rate change volatility to its positive and negative innovations, as well as differences in volatility levels in boom and in recession. Since the sample estimates of their AICs and BICs are smaller than that of our reference process, but we cannot solve for autocorrelation in their standardized residuals and the squares of their standardized residuals (see Table 2), we find them all inferior to our IGARCH(1,1)-in-mean. Nevertheless, we still take the natural logarithm of the squared residuals from the latter as a raw and somewhat imprecise proxy of the time-varying term premium that may overstate its true volatility. Fortunately enough, unlike a true nonstationary process, the conditional variance of the IGARCH process is a geometrically decaying function of the current and past realizations of the $\left\{\xi_t^2\right\}$ sequence, so its natural logarithm can still serve as a regressor in model (1)+(2) being estimated by the GMM (see Enders (2004), p. 140–141).

The series of the time-varying term premium proxy and its instrument, the natural logarithm of the estimated conditional variance, are both periodic with two breaks occurring within the recession periods and seem to be level stationary (see Figure 2). The results of the Fourier flexible form type ADF and KPSS tests support this observation.[10]

Next, assuming that the time-varying term premia do not change with the business cycle and exhibit any breaks, we estimate the restricted version of Equation (1)+(2) using the GMM with the Newey-West standard errors accounting for autocorrelation of order (*n-2*) and a possible heteroscedasticity

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[10] The estimates of the Fourier type ADF and KPSS test statistics for the proxy equal to –3.54 (for optimal $k = 5$) and 0.0579 (for cumulated frequencies up to $k = 3$), respectively. The lag 13 is set to remove autocorrelation up to the 4 order (F-ADF) and the lag 20 – according to the Schwert criterion (F-KPSS). The relevant estimates for the instrument show: –4.61 (for optimal $k = 3$) and 0.0785 (for cumulated frequencies up to $k = 3$). The same lag removes autocorrelation.

Table 2

Summary characteristics of MS GARCH(1,1)-in-mean and MS IGARCH(1,1)-in-mean specifications

| Model | C.v. equation restrictions | $LL^a$ | *AIC* | *BIC* | Standardized residuals | | | | Squares of standardized residuals | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | $Q(1)$ | $Q(13)$ | $Q(26)$ | $Q(52)$ | $Q(1)$ | $Q(13)$ | $Q(26)$ | $Q(52)$ |
| GARCH | None | 1708.45 | –3392.89 | –3336.57 | 0.03 [0.86] | 32.22 [0.00] | 84.02 [0.00] | 135.17 [0.00] | 0.14 [0.70] | 5.89 [0.95] | 30.55 [0.25] | 69.19 [0.06] |
| IGARCH | Both eq | 1709.27 | –3398.55 | –3351.61 | 0.06 [0.81] | 33.59 [0.00] | 90.49 [0.00] | 145.33 [0.00] | 0.15 [0.70] | 7.09 [0.90] | 39.19 [0.05] | 84.13 [0.00] |
| IGARCH | First eq | 1709.49 | –3396.99 | –3345.36 | 0.05 [0.83] | 36.60 [0.00] | 88.47 [0.00] | 141.13 [0.00] | 0.14 [0.70] | 5.05 [0.97] | 24.26 [0.56] | 57.31 [0.28] |
| IGARCH | Second eq | 1711.88 | –3401.76 | –3350.13 | 0.07 [0.80] | 32.01 [0.00] | 87.07 [0.00] | 140.94 [0.00] | 0.15 [0.70] | 6.49 [0.93] | 35.28 [0.11] | 77.52 [0.01] |

Conditional variance equation: $\sigma_{it}^2 = \phi_i + \left(\gamma_i^+ 1_{\{\xi_{t-1}\geq 0\}} + \gamma_i^- 1_{\{\xi_{t-1}<0\}}\right)\xi_{t-1}^2 + \delta_i \sigma_{it}^2$, $i = 1, 2$; *LL* – sample log likelihood estimate (optimization procedure in DOptim used for MS GARCH estimation is that the number of constraints set on parameters to estimate an IGARCH version of the model is less than the number of constraints in its GARCH counterpart); $Q(s)$ – Ljung-Box test statistics, under the null of no autocorrelation up to the $s$-th order distributed as $\chi^2(s)$, $p$-values in brackets.

Source: author's own

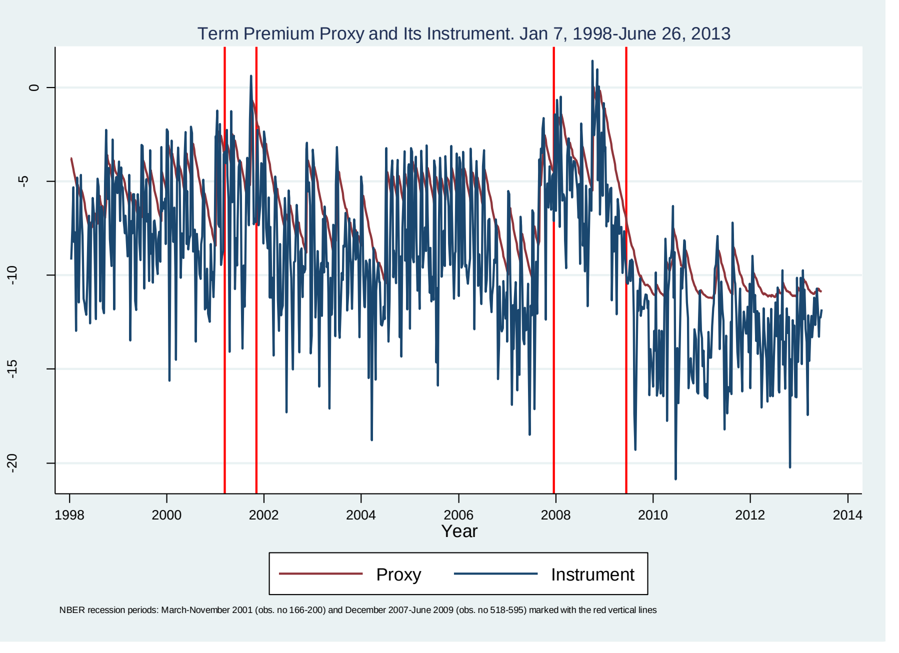


Figure 2. Term Premium Proxy and Its Instrument. January 7, 1998-June 26, 2013

Source: author's own

in $\zeta_{t+n-1}^{(n)}$.[11] Even though we have possibly utilized rather a raw and imprecise term premium proxy, the results reported in Table 3 to some extent, as in Gerlach (2003), stand for the EH. The KP, END and HAN tests support the choice of instruments at least at the 10 per cent significance level for all *n*. The estimates of the slope parameter have correct signs and range from 0.1494 (*n*=4) to 0.8739 (*n* = 52). For the maturities of 42 weeks and over, the null stating that $\beta^{(n)} = 1$ is not rejected at the 5 per cent significance level. For other maturities the yield spread occurs to be a downwards biased predictor of the change in the short interest rate so that the failure of the EH to predict the relationship between long and short LIBORs can be explained according to the overreaction hypothesis (see Mankiw, Summers (1984), Campbell, Shiller (1991), Tzavalis, Wickens (1998), Cuthbertson, Nitzsche (2003)). The null of $\gamma^{(n)} = 0$ is rejected in all regressions indicating that the term premia are time-varying. Only the results of the Ramsey type error

[11] See Newey, West (1987).

Table 3

Results of GMM estimation for $PFS_t^{(n)} = \alpha^{(n)} + \beta^{(n)} S_t^{(n)} + \gamma^{(n)} \ln\left(\sigma_t^2\right) + \omega_{t+n-1}^{(n)}$,

Jan 7, 1998–June 26, 2013

| Maturity $n$ | Obs | Parameter | | | Hypothesis [a] | | | Test statistics | | | | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\alpha$ | $\beta$ | $\gamma$ | $\alpha=0$ | $\beta=1$ | $\gamma=0$ | KP | HAN | END | RES | |
| 2 | 648 | –0.0510 | 0.4048 | –0.0043 | 8.94 | 6.55 | 6.65 | 54.95 | 0.88 | 4.22 | 0.91 | 0.08 |
| | | (0.0170) | (0.2325) | (0.0017) | [0.00] | [0.01] | [0.01] | [0.00] | [0.35] | [0.04] | [0.34] | |
| 4 | 807 | –0.1029 | 0.1494 | –0.0095 | 9.71 | 41.92 | 8.45 | 64.88 | 0.37 | 4.04 | 1.63 | 0.05 |
| | | (0.0330) | (0.1314) | (0.0033) | [0.00] | [0.00] | [0.00] | [0.00] | [0.54] | [0.04] | [0.20] | |
| 9 | 804 | –0.3086 | 0.3063 | –0.0270 | 10.11 | 18.35 | 8.11 | 63.05 | 0.15 | 4.24 | 48.76 | 0.12 |
| | | (0.0970) | (0.1619) | (0.0095) | [0.00] | [0.00] | [0.00] | [0.00] | [0.70] | [0.04] | [0.00] | |
| 13 | 800 | –0.4100 | 0.3755 | –0.0339 | 10.54 | 18.42 | 7.47 | 63.42 | 0.01 | 4.06 | 89.66 | 0.15 |
| | | (0.1263) | (0.1455) | (0.0124) | [0.00] | [0.00] | [0.01] | [0.00] | [0.93] | [0.04] | [0.00] | |
| 17 | 796 | –0.5040 | 0.4627 | –0.0391 | 11.38 | 16.29 | 7.27 | 63.42 | 0.07 | 4.42 | 109.93 | 0.17 |
| | | (0.1494) | (0.1331) | (0.0145) | [0.00] | [0.00] | [0.01] | [0.00] | [0.80] | [0.04] | [0.00] | |
| 22 | 791 | –0.5991 | 0.5313 | –0.0438 | 12.60 | 13.31 | 7.25 | 62.79 | 0.14 | 4.53 | 91.64 | 0.20 |
| | | (0.1688) | (0.1284) | (0.0163) | [0.00] | [0.00] | [0.01] | [0.00] | [0.71] | [0.03] | [0.00] | |
| 26 | 787 | –0.6615 | 0.5572 | –0.0461 | 13.24 | 12.56 | 6.96 | 61.95 | 0.12 | 4.31 | 81.81 | 0.21 |
| | | (0.1818) | (0.1249) | (0.0175) | [0.00] | [0.00] | [0.01] | [0.00] | [0.73] | [0.04] | [0.00] | |
| 30 | 783 | –0.9062 | 0.6497 | –0.0569 | 25.83 | 8.85 | 11.00 | 61.12 | 0.76 | 6.80 | 62.81 | 0.27 |
| | | (0.1783) | (0.1178) | (0.0171) | [0.00] | [0.00] | [0.00] | [0.00] | [0.38] | [0.01] | [0.00] | |
| 35 | 778 | –0.9353 | 0.7101 | –0.0552 | 23.89 | 5.30 | 8.87 | 60.41 | 0.73 | 5.35 | 63.77 | 0.28 |
| | | (0.1913) | (0.1259) | (0.0185) | [0.00] | [0.02] | [0.00] | [0.00] | [0.39] | [0.02] | [0.00] | |
| 39 | 774 | –0.9610 | 0.7537 | –0.0534 | 22.97 | 3.69 | 7.40 | 59.78 | 0.77 | 4.48 | 58.25 | 0.29 |
| | | (0.2005) | (0.1282) | (0.0196) | [0.00] | [0.05] | [0.01] | [0.00] | [0.38] | [0.03] | [0.00] | |
| 43 | 770 | –0.9982 | 0.7973 | –0.0521 | 22.95 | 2.41 | 6.41 | 59.30 | 0.88 | 3.97 | 55.66 | 0.30 |
| | | (0.2084) | (0.1305) | (0.0206) | [0.00] | [0.12] | [0.01] | [0.00] | [0.35] | [0.05] | [0.00] | |
| 48 | 765 | –1.0548 | 0.8471 | –0.0521 | 23.21 | 1.31 | 5.69 | 58.53 | 0.92 | 3.66 | 53.10 | 0.30 |
| | | (0.2190) | (0.1336) | (0.0218) | [0.00] | [0.25] | [0.02] | [0.00] | [0.34] | [0.06] | [0.00] | |
| 52 | 761 | –1.0939 | 0.8739 | –0.0510 | 23.18 | 0.89 | 5.00 | 57.91 | 0.93 | 3.29 | 48.68 | 0.31 |
| | | (0.2272) | (0.1340) | (0.0228) | [0.00] | [0.35] | [0.03] | [0.00] | [0.33] | [0.07] | [0.00] | |

GMM weight matrix – HAC Bartlett with 6 lags; Instrumented: $\ln\left(\hat{\xi}_t^2\right)$; Included instruments: $S_t^{(n)}$; Excluded instruments: $\ln\left(\hat{\sigma}_t^2\right)$, $\ln\left(\hat{\sigma}_{t-1}^2\right)$; [a] Wald test statistics to test for the significance of structural parameters, under $H_0$ distributed as $\chi^2(s)$, $s$ – number of constraints set on parameters; KP – Kleibergen-Paap underidentification test statistics, under $H_0$ (excluded instruments are not correlated with the endogenous regressor) distributed as $\chi^2(2)$; HAN – Hansen's J statistics, under $H_0$ (instruments are correlated with the error term and the excluded instruments are correctly excluded from the estimated equation) distributed as $\chi^2(1)$; END – endogeneity test for endogenous regressor $\ln\left(\hat{\xi}_t^2\right)$, under $H_0$ (endogenous regressor can be treated as exogenous) test statistics distributed as $\chi^2(1)$; RES – Ramsey-Pesaran-Taylor reset test heteroskedastic and autocorrelation robust, under $H_0$ (linearity of functional form) test statistics distributed as $\chi^2(1)$; Standard errors in parentheses, *p*-values in brackets.

Source: autor's own

specification test (RES) prove that for all $n \geq 9$ the modified conventional PFS regression poorly exhibits the relationship between the optimal forecast of future changes in the short rate and the slope of the term structure. This conclusion is supported by the regressions' poor fit as measured by $R^2$ coefficient of fit ranging from 0.05 ($n$ = 4) to 0.31 ($n$ = 52).

Finally, we run the unrestricted regressions with the number of components in the Fourier approximation $k$ equal up to 5. Now the results are more promising for the validity of the EH compared to the restricted regressions (see the results gathered in Table 4). The KP, HAN and END tests support the choice of instruments for all $n$. The estimates of the slope parameter have correct signs and range from 0.0864 ($n$ = 4) to 0.9664 ($n$ = 52). The term spread occurs to be a good predictor of future changes in the short rate for the maturities of 35 weeks and over as $H_0 : \beta^{(n)} = 1$ is now not rejected at the 5 per cent significance level. For other maturities the yield spread – as in the restricted case – is a downwards biased predictor of the change in the short interest rate. More interestingly, the hypothesis stating that the term premium is not changing with the business cycle and (or) has no breaks of whatever the form and number is rejected for all maturities but $n$ = 2.[12] The Ramsey-Pesaran-Taylor version of the reset test soundly stands

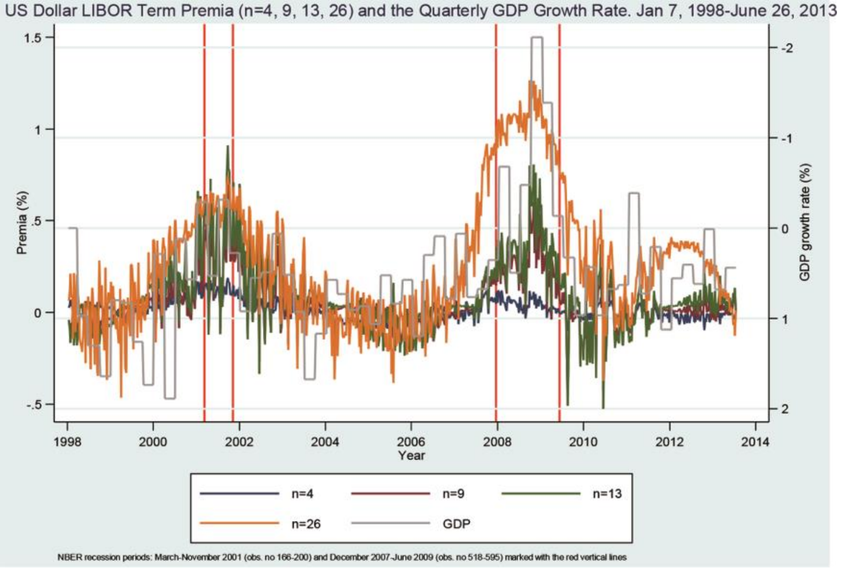


Figure 3. US Dollar LIBOR Term Premia (n = 4, 9, 13, 26) and the Quarterly GDP Growth Rate. January 7, 1998-June 26, 2013

Source: author's own

[12] However, the term premium for $n$ = 2is time-varying as $H_0 : \gamma^{(n)} = 0$ is rejected at the 5 per cent significance level.

Table 4. Results of GMM estimation for

$$PFS_t^{(n)} = \alpha^{(n)} + \sum_{k=1}^{l} \varphi_k^{(n)} \sin(2\pi kt/T) + \sum_{k=1}^{l} \omega_k^{(n)} \cos(2\pi kt/T) + \left[\gamma^{(n)} + \sum_{k=1}^{l} \delta_k^{(n)} \sin(2\pi kt/T) + \sum_{k=1}^{l} \eta_k^{(n)} \cos(2\pi kt/T)\right] \ln(\sigma_t^2) + \beta^{(n)} S_t^{(n)} + \varepsilon_{t+n-1}^{(n)},$$

Jan 7, 1998–June 26, 2013

| Maturity | Obs | Parameter | | | Fourier components | | Hypothesis [a] | | | | | Test statistics | HAN | END | RES | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | | $\alpha$ | $\beta$ | $\gamma$ | A | B | $\alpha = 1$ | $\beta = 0$ | $\gamma = 0$ | $\alpha = \gamma = 0$ | $\varphi_k = \omega_k =$ | KP | | | | |
| | | | | | C | D | | | | $\beta = 1$ | $\delta_k = \eta_k = 0$ [b] | | | | | |
| 2 | 648 | –0.0510 | 0.4048 | –0.0043 | - | - | 8.94 | 6.55 | 6.65 | 41.84 | - | 54.95 | 0.88 | 4.22 | 0.91 | 0.08 |
| | | (0.0170) | (0.2325) | (0.0017) | - | - | [0.00] | [0.01] | [0.01] | [0.00] | - | [0.00] | [0.35] | [0.04] | [0.34] | |
| 4 | 807 | –0.1189 | 0.0864 | –0.0116 | 3 | 1,2,5 | 18.37 | 52.75 | 16.53 | 86.39 | 38.81 | 61.54 | 0.77 | 6.72 | 0.81 | 0.10 |
| | | (0.0277) | (0.1258) | (0.0028) | - | - | [0.00] | [0.00] | [0.00] | [0.00] | *31.43* | [0.00] | [0.38] | [0.01] | [0.37] | |
| 9 | 804 | –0.1916 | 0.2795 | –0.0155 | 5 | 2 | 7.93 | 33.57 | 5.53 | 114.04 | 26.12 | 44.87 | 1.43 | 14.08 | 2.21 | 0.23 |
| | | (0.0681) | (0.1244) | (0.0066) | - | 2 | [0.00] | [0.00] | [0.02] | [0.00] | *23.22* | [0.00] | [0.23] | [0.00] | [0.14] | |
| 13 | 800 | –0.2696 | 0.3650 | –0.0202 | - | 2 | 9.38 | 37.35 | 5.73 | 194.58 | 31.54 | 41.24 | 0.66 | 16.51 | 1.20 | 0.29 |
| | | (0.0880) | (0.1039) | (0.0084) | - | 2 | [0.00] | [0.00] | [0.02] | [0.00] | *22.37* | [0.00] | [0.42] | [0.00] | [0.27] | |
| 17 | 796 | –0.5181 | 0.6085 | –0.0403 | 1 | 2 | 17.47 | 23.02 | 11.80 | 229.10 | 71.42 | 49.18 | 2.10 | 12.31 | 2.80 | 0.40 |
| | | (0.1240) | (0.0816) | (0.0117) | 1 | 2 | [0.00] | [0.00] | [0.00] | [0.00] | *28.19* | [0.00] | [0.15] | [0.01] | [0.09] | |
| 22 | 791 | –0.5625 | 0.8160 | –0.0289 | 1 | 2 | 29.66 | 3.93 | 12.48 | 261.68 | 128.50 | 18.50 | 0.51 | 20.88 | 2.19 | 0.51 |
| | | (0.1033) | (0.0928) | (0.0082) | 1 | 1,2 | [0.00] | [0.05] | [0.00] | [0.00] | *25.48* | [0.00] | [0.48] | [0.00] | [0.14] | |
| 26 | 787 | –0.6372 | 0.7324 | –0.0386 | 1,2 | 2,3 | 12.36 | 5.57 | 4.79 | 374.95 | 146.84 | 52.83 | 0.18 | 9.30 | 2.25 | 0.61 |
| | | (0.1813) | (0.1134) | (0.0176) | - | 3,4 | [0.00] | [0.02] | [0.03] | [0.00] | 21.95 | [0.00] | [0.67] | [0.03] | [0.13] | |
| 30 | 783 | –0.7556 | 0.7482 | –0.0373 | 1,2,3,4 | 1,2,3 | 19.56 | 5.28 | 5.04 | 783.36 | 288.17 | 46.61 | 0.30 | 3.31 | 1.38 | 0.68 |
| | | (0.1704) | (0.1095) | (0.0166) | - | - | [0.00] | [0.02] | [0.02] | [0.00] | *24.04* | [0.00] | [0.59] | [0.07] | [0.24] | |
| 35 | 778 | –0.4639 | 0.7912 | - | 1,2,3,4 | 1,2,3 | 75.22 | 3.07 | - | 761.63 | 373.47 | 61.96 | 2.78 | 11.04 | 0.51 | 0.74 |
| | | (0.0535) | (0.1192) | - | 5 | 4 | [0.00] | [0.08] | - | [0.00] | *58.08* | [0.00] | [0.25] | [0.00] | [0.47] | |
| 39 | 774 | –0.7943 | 0.8704 | –0.0304 | 1,2,4 | 1,2,3 | 25.20 | 1.84 | 3.62 | 812.17 | 448.36 | 46.22 | 0.00 | 9.78 | 0.28 | 0.77 |
| | | (0.1583) | (0.0954) | (0.0160) | 3 | - | [0.00] | [0.17] | [0.06] | [0.00] | *23.37* | [0.00] | [0.98] | [0.01] | [0.60] | |

Table 4, cont.

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 43 | 770 | –0.5837 | 0.8932 | - | 1,2,3,4 | 1,2,3 | 107.96 | 1.03 | - | 971.42 | 588.60 | 62.14 | 2.09 | 13.11 | 0.03 | 0.81 |
| | | (0.0562) | (0.1051) | - | 5 | 4 | [0.00] | [0.31] | - | [0.00] | *50.59* | [0.00] | [0.35] | [0.00] | [0.86] | |
| 48 | 765 | –0.6511 | 0.9421 | - | 1,2,3,4 | 1,2,3 | 129.88 | 0.35 | - | 999.75 | 695.41 | 62.55 | 2.29 | 12.45 | 0.20 | 0.84 |
| | | (0.0571) | (0.0979) | - | 5 | 4 | [0.00] | [0.55] | - | [0.00] | *70.04* | [0.00] | [0.32] | [0.00] | [0.66] | |
| 52 | 761 | –0.7082 | 0.9664 | - | 1,2,3,4 | 1,2,3 | 148.04 | 0.13 | - | 1031.21 | 769.37 | 63.13 | 2.62 | 11.87 | 0.36 | 0.85 |
| | | (0.0582) | (0.0918) | - | 5 | 4 | [0.00] | [0.71] | | [0.00] | *72.53* | [0.00] | [0.27] | [0.00] | [0.55] | |

GMM weight matrix – HAC Bartlett with 6 lags; Instrumented variables: $\ln\left(\hat{\xi}_t^2\right)$, $\sin(2\pi kt/T)\times\ln\left(\hat{\xi}_t^2\right)$, $\cos(2\pi kt/T)\times\ln\left(\hat{\xi}_t^2\right)$, $k = 1, 2, \ldots, l$; Included instruments: $S_t^{(n)}$; Excluded instruments: $\ln\left(\hat{\sigma}_t^2\right)$, $\sin(2\pi kt/T)\times\ln\left(\hat{\sigma}_t^2\right)$, $\cos(2\pi kt/T)\times\ln\left(\hat{\sigma}_t^2\right)$, $\ln\left(\hat{\sigma}_{t-1}^2\right)$, $\sin(2\pi kt/T)\times\ln\left(\hat{\sigma}_{t-1}^2\right)$, $\cos(2\pi kt/T)\times\ln\left(\hat{\sigma}_{t-1}^2\right)$, $k = 1, 2, \ldots, l$; Fourier components: A – $\sin(\cdot)$, B – $\cos(\cdot)$, C – $\sin(\cdot)\times\ln\left(\sigma_t^2\right)$, D – $\cos(\cdot)\times\ln\left(\sigma_t^2\right)$;

[a] Wald test statistics to test for significance of structural parameters, under $H_0$ distributed as $\chi^2(s)$, $s$ – number of constraints set on parameters; [b] Wald test statistics to test for significance of structural parameters, critical values obtained throughout the Monte Carlo experiment with 50000 replications; KP – Kleibergen-Paap underidentification test statistics, under $H_0$ (excluded instruments are not correlated with the endogenous regressor) distributed as $\chi^2(2)$; HAN – Hansen's J statistics, under $H_0$ (instruments are correlated with the error term and the excluded instruments are correctly excluded from the estimated equation) distributed as $\chi^2(\mathbf{1})$; END – endogeneity test for endogenous regressors, under $H_0$ (endogenous regressors can be treated as exogenous) test statistics distributed as $\chi^2(e)$, $e$ – number of endogenous regressors; RES – Ramsey-Pesaran-Taylor reset test heteroskedastic and autocorrelation robust, under $H_0$ (linearity of functional form) test statistics distributed as $\chi^2(\mathbf{1})$; Standard errors ($p$-values, 5 per cent critical values obtained from the Monte Carlo experiment) under the test statistics estimates in parentheses (brackets, italics).

Source: author's own

Table 5

Descriptive statistics of the term premia for $n = 4, 9, 13$ and 26 weeks

| Statistics | Premium | | | | | | | Statistics | Premium | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $n = 4$ | | | | | | | | $n = 9$ | | | | | | |
| | Period | | | | | | | | Period | | | | | | |
| | Boom | | | | Recession | | | | Boom | | | | Recession | | |
| | #1 | #2 | #3 | Overall | #1 | #2 | Overall | | #1 | #2 | #3 | Overall | #1 | #2 | Overall |
| | Obs. no | | | | Obs. no | | | | Obs. no | | | | Obs. no | | |
| | 1–165 | 201–517 | 596–808 | | 166–200 | 518–595 | | | 1–165 | 201–517 | 596–808 | | 166–200 | 518–595 | |
| Max | 0.1682 | 0.1480 | 0.0810 | 0.1682 | 0.1876 | 0.1191 | 0.1876 | Max | 0.4835 | 0.5539 | 0.2612 | 0.5539 | 0.7082 | 0.6024 | 0.7082 |
| Min | –0.0769 | –0.1392 | –0.1024 | –0.1392 | 0.0252 | –0.0392 | –0.0353 | Min | –0.1362 | –0.2678 | –0.4479 | –0.4479 | –0.1047 | –0.0393 | –0.1047 |
| Mean | 0.0300 | –0.0018 | –0.0105 | 0.0030 | 0.1141 | 0.0449 | 0.0664 | Mean | 0.0444 | 0.0316 | –0.0092 | 0.0221 | 0.3203 | 0.2667 | 0.2833 |
| Std dev | 0.0446 | 0.0505 | 0.0305 | 0.0465 | 0.0413 | 0.3581 | 0.0493 | Std dev | 0.1065 | 0.1283 | 0.0978 | 0.1164 | 0.1929 | 0.1283 | 0.1524 |
| Coef of var | 1.4867 | –28.0556 | –2.9048 | 15.5500 | 0.3620 | 7.9755 | 0.7425 | Coef of var | 2.3986 | 4.0601 | –10.6304 | 5.2670 | 0.3203 | 0.4811 | 0.5379 |
| Skewness | 0.4443 | 0.3219 | 0.0685 | 0.4185 | –0.3390 | 0.1128 | 0.3433 | Skewness | 1.6266 | 1.2308 | –1.4307 | 0.8531 | –0.2717 | 0.4037 | 0.1991 |
| Kurtosis | 3.6513 | 2.9778 | 3.4362 | 3.5253 | 2.4315 | 2.6524 | 2.5009 | Kurtosis | 6.9485 | 5.4519 | 6.8942 | 6.7493 | 2.7093 | 3.5085 | 3.2503 |
| Statistics | Premium | | | | | | | Statistics | Premium | | | | | | |
| | $n = 13$ | | | | | | | | $n = 26$ | | | | | | |
| | Period | | | | | | | | Period | | | | | | |
| | Boom | | | | Recession | | | | Boom | | | | Recession | | |
| | #1 | #2 | #3 | Overall | #1 | #2 | Overall | | #1 | #2 | #3 | Overall | #1 | #2 | Overall |
| | Obs. no | | | | Obs. no | | | | Obs. no | | | | Obs. no | | |
| | 1–165 | 201–517 | 596–808 | | 166–200 | 518–595 | | | 1–165 | 201–517 | 596–808 | | 166–200 | 518–595 | |
| Max | 0.6619 | 0.7085 | 0.3703 | 0.7085 | 0.9107 | 0.8051 | 0.9107 | Max | 0.6206 | 0.9221 | 0.7633 | 0.9221 | 0.7437 | 1.2659 | 1.2659 |
| Min | –0.1362 | –0.3341 | –0.5247 | –0.5247 | –0.1209 | 0.0077 | –0.1209 | Min | –0.4643 | –0.3849 | –0.3746 | –0.4643 | 0.4596 | 0.7677 | 0.4596 |
| Mean | 0.0444 | 0.0599 | 0.0154 | 0.0499 | 0.4179 | 0.3643 | 0.3809 | Mean | 0.1487 | 0.1461 | 0.2497 | 0.1785 | 0.5951 | 1.0262 | 0.8927 |
| Std dev | 0.1065 | 0.1651 | 0.1264 | 0.1543 | 0.2495 | 0.1662 | 0.1963 | Std dev | 0.2420 | 0.2765 | 0.1906 | 0.2490 | 0.0648 | 0.1097 | 0.2228 |
| Coef of var | 2.3986 | 2.7563 | 8.2078 | 3.0922 | 0.5970 | 0.4562 | 0.5153 | Coef of var | 1.6274 | 1.8925 | 0.7633 | 1.3950 | 0.1089 | 0.1069 | 0.2496 |
| Skewness | 1.1923 | 1.1397 | –1.1701 | 0.8289 | –0.2890 | 0.5661 | 0.2122 | Skewness | –0.0698 | 0.8929 | 0.0648 | 0.4108 | 0.0899 | –0.1992 | –0.4153 |
| Kurtosis | 4.9568 | 5.0758 | 5.9055 | 5.7976 | 2.6407 | 3.5188 | 3.2054 | Kurtosis | 2.3142 | 3.2887 | 3.3750 | 3.0068 | 2.8229 | 3.0028 | 1.8177 |

Source: author's own

for the linear specification of the modified conventional PFS regression for all $n$. The estimates of $R^2$ coefficient of fit also rise significantly, especially for the longer maturities. Thus we can summarize that the use of the Fourier approximation in all regressions for which $n \geq 4$ enables us to properly exhibit the nature of term premiums despite the fact that we may have imprecisely estimated its initial proxy.

We plot the exemplary estimated premia for $n$=4, 9, 13 and 26 weeks in Figure 3. When compared to the US quarterly GDP growth rate they seem to evolve alike over time, achieving local maxima within the recession periods. The estimates of their descriptive statistics gathered in Table 5 show that – on average – the longer the maturity the greater the term premium, both in the boom and in the recession. They also indicate that the boom premium for every maturity is many times smaller than its recession counterpart. The variability of term premia, as measured by the coefficient of variation, displays a reverse property: the shorter the maturity the greater the variability, and the boom variability for every maturity is many times greater than the recession variability. The tails of the boom premia distributions are thicker than the tails of their recession counterparts indicating that both high and low premia in the boom are more frequently occurring than in the recession.

## CONCLUSION

The present study of the interbank rates term structure based on the Fourier approximation of term premia that uses the weekly sampled series of US dollar LIBORs from January 1998 to June 2013 reveals that the term premia not only vary over time for the whole spectrum of maturities, as earlier proved by Hurn et al. (1995), Cuthbertson (1996) and Gerlach (2003), but are periodic, change with the US business cycle and are broken within the recession periods. It also shows that – on average – the longer the maturity the greater the premium, both in boom and in recession. The other significant feature of the term premium for every maturity is that the boom premium is many times smaller than its recession counterpart. For the maturities of 35 weeks and over, the yield spread is a good predictor of future changes in the short interest rate. For other maturities it turns out to be a downwards biased predictor, so that the failure of the EH to predict the relationship between long and short LIBORs can account for the overreaction of long rates to the expected changes in the short rate. Finally we stress that except for the two shortest maturities, all restricted regressions that ignore the 'true' nature of the term premium, i.e. its change with the business cycle and breaks, are misspecified.

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