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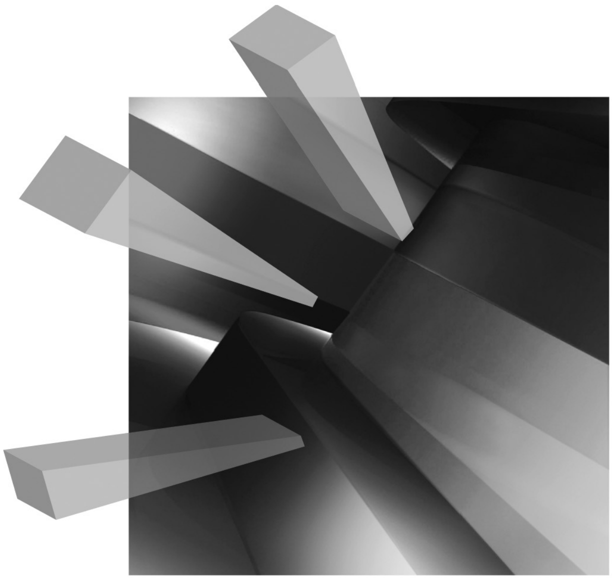

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Summaries

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STATISTICAL MODELS FOR INCOME DISTRIBUTION IN THE SOCIO-ECONOMIC SYSTEMS

Summary: The given article is devoted to agent-based approach used to the construction of income distribution models, at which the economic systems are designed as an aggregate of autonomous interactive agents. The considered models open a new way for the income distribution analysis, which manifests itself in the interaction between a large number of economic agents, which allows to set between the desired stationary distributions.

Keywords: income distribution, agent-based approach, Laplace distribution, asymmetric Subbotin distribution, statistical model, socio-economic system.

1. Introduction

Differentiation in the distribution of income is part of the economic reality of any country, and therefore it is constantly in the view of researchers and periodically becomes the subject to acute socio-political debate. In order to respond effectively to the growing stratification of society by income, economic and social policy should be based on full and adequate understanding of how this inequality is formed, which groups of population by income have the greatest contribution to the change in inequality of income distribution.

The effect of different population groups by income on the formation of inequality is important, because it depends on the selection of socio-economic policy. The increased differentiation due to the income growth of highly skilled population by increasing the return on investment into the human capital can be considered as a positive trend. The simultaneous increase of the poor and secured members of society in the contribution of inequality indicates the deepening of social stratification and leads to the growth of social tensions, hindering the economic growth.

The change in income inequality is determined by the increase or decrease in its performance. It is known that indicators of inequality are functions of the characteristics of income distribution. Therefore, changes in indicators of inequality are a consequence of changes in the characteristics of distribution, which causes the relevance of the construction and the analysis of models of income distribution in the socio-economic systems.

2. Analysis of recent research and publications

In recent decades, the world has seen the increasing interest in the problems of income distribution in the economy caused first of all by the rapid development of new information technologies, which greatly facilitated empirical research in this area and initiated a new approach to the formation of the income distribution in an agent-based economic model.

In the earlier studies [Champernowne 1953; Gibrat 1931], the dynamics of income are described as a stochastic process and they determine their own probabilistic distributions. This early type of modeling is called «one-agent» approach, since the deviation of income is considered independent for each economic agent. In contrast to earlier studies, modeling in the economy has been recently implemented in the agent-based approach in which systems are modeled as a collection of autonomous interacting agents [Bouchaud, Mezard 2000; Chatterjee et al., 2005; Dragulescu, Yakovenko 2000; Scafetta et al., 2004]. This opens up another way for the formation of the income distribution, which manifests itself in the interaction among a large number of agents. Thus, even if it is difficult or impossible to write «microeconomic behavior equation» for each economic agent, complex economic systems can be studied at different levels of the hierarchy. After developing the model of cash exchanges between economic agents even cash has developed random exchange models.

The purpose of the article is the construction and analysis of statistical models of income distribution in the socio-economic systems based on the use of agent-based approach.

3. The main material

In a class of statistical models of income distribution there are distinguished homogeneous and heterogeneous kinetic models of income distribution [Dragulescu, Yakovenko 2000]. Socio-economic system is perceived as a system of closed type. In the natural sciences those systems belong to closed ones that neither interact with the environment nor exchange matter or energy with it. In this case, the system is closed in the sense that it does not exchange variables which are similar to matter and energy with the environment. In the matter of the economic system it means the number of economic agents without their functional and other division, in energy – the amount of income in the economic system. The result is considered to be an economic system with a constant number of economic agents, who together receive a constant income for a specified period of time. Statistical models of income distribution in the socio-economic system can show how this amount of income is distributed among its economic agents.

Thus, let us consider a homogeneous kinetic model of income distribution, in which the final income distribution occurs depending on the conditions under which exchanges take place, that is to share parts of income between economic agents in a closed socio-economic system. It is assumed that the exchange does not depend on previous transactions, and the kinetic model is presented in the following form:

$$\begin{vmatrix} m_i(t+1) \\ m_j(t+1) \end{vmatrix} = \mathrm{M} \begin{vmatrix} m_i(t) \\ m_j(t) \end{vmatrix}, \tag{1}$$

where: $m_i(t)$ – the income of economic agent *i* in time *t*,

M – matrix describing the mechanism of revenues exchange.

In the model of global economic interaction the system consists of *N* economic agents who share income through virtual bank. This bank plays the role of a mechanism for the effective redistribution of income that allows economic agents to make a deposit and credit transactions (in other words, to receive income in the form of debt obligations). It is assumed that each agent initially has a certain amount of income m_0 only in the form of deposit. In each time he interacts with other economic agent randomly, and therefore is determined by *N* pairs. In addition, all economic agents operating at the same time in each of these pairs of one economic agent act either as a payer – or as a buyer while one unit of income is transferred at a time. Exchanges end at the time when the bank refuses to provide loans to payers due to certain restrictions on credit portfolio on the total value of loans *D*. To simplify the situation, it is assumed that the bank does not charge interest on credit.

Since the economic agent can make a deposit as well as credit transactions, then it creates a virtual deposit and loan accounts. Therefore, the value of income of an agent *i* is determined as the difference between its savings and liabilities:

$$m_i = s_i - l_i, \tag{2}$$

where: m_i – amount of income of an agent *i*,

s_i – value of savings of an agent *i*,

l_i – value of liabilities of an agent *i*.

According to the condition (2) all agents can be divided into three groups:

$\{N_+\}$ – agents who have accumulated savings,

$\{N_0\}$ – agents whose balances on bank accounts are zero,

$\{N_-\}$ – agents with debt.

Exchange in which the income of each unit varies for each transaction takes place between economic agents. It is assumed that the system reaches a certain equilibrium condition stipulated only by the interaction of economic agents (for example, different types of competition), subject to the termination of crediting agents. In other words, even if in the future there are changes in income for each agent, income distribution reaches an equilibrium state due to the restriction of bank lending. Obviously, there are two mechanisms that underlie the formation of income distribution before and after border restrictions. Therefore, we consider the evolution of income distribution in a dynamic process, and the formation of the final distribution at a steady state.

Income density function at the initial time has the following form:

$$p(m,0)=\delta(m-m_0)=\begin{cases}1 & \text{for } m=m_0,\\ 0 & \text{for } m\neq m_0.\end{cases} \tag{3}$$

Exchange of income between economic agents is arbitrary and carries out an exchange between them. As a result income density function $p(m,t)$ can be described by linear diffusion equation:

$$\frac{\partial p}{\partial t}-\frac{\partial^2 p}{\partial m^2}=0. \tag{4}$$

The solution of equation (4) based on condition (3) has the form of:

$$p(m,t)=\frac{1}{\sqrt{4\pi t}}e^{-\frac{(m-m_0)^2}{4t}}. \tag{5}$$

Therefore, income distribution is determined by the Poisson. As it can be seen from solution (5), once asked the original value of the income distribution, it is determined for any point in time t. However, when it reaches the limit of credit resource distribution, it is no longer determined by equality (5). The time interval from the initial moment to t_D we refer to as free time diffusion, so exchange until t_D is a free diffusion process and the evolution of the density distribution of income is given by Poisson.

The probability of income distribution during free time diffusion, resulting from the use of empirical (conditional) data and the theoretical function defined by the formula of Poisson is shown in Figure 1.

We consider two cases:

a) It is assumed that the initial value of income of each agent given in the form of deposit is 10 and 20 conv. units. respectively at the moment t = 50 (days). Income of economic agent m (marked by points in Figure 1) during the determined time transaction (taking the form of deposit or credit) takes positive or negative values. It is worth noting that changing the initial value of income an economic agent does not affect the shape of the empirical and theoretical distribution function, but only shifts it by 10 conv. units right at m_0 = 20 in comparison with m_0 = 10. Thus change m_0 affects the change in income, but does not change the amount of inequalities in their distribution.

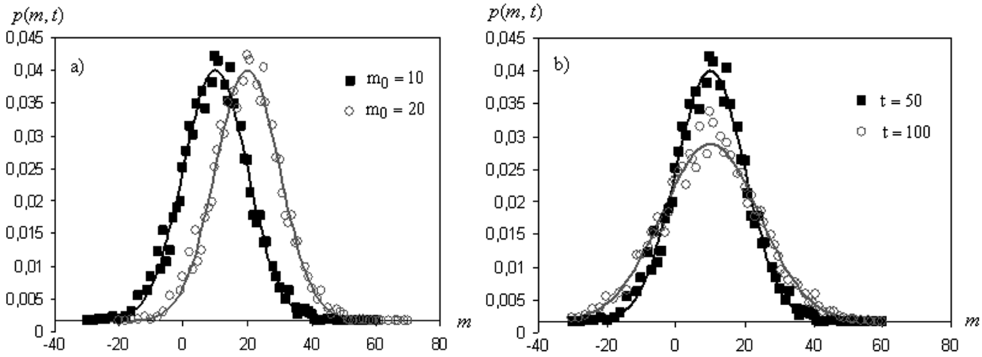

Figure 1. Empirical (points) and theoretical (solid line) curves of income distribution during free time diffusion: a) different initial values $m_0 = 10, 20$ at time $t = 50$; b) different times $t = 50,100$ with $m_0 = 10$

Source: http://www.ukr.stat.gov.ua.

b) It is assumed that the value of initial income is constant and equal to m_0 = 10, but different intervals of time allocated for exchanges between economic agents t = 50 and t = 100 (days) are considered, respectively. As it is seen from the figure, the change of time parameter affects the shape of the distribution function: increasing the duration exchanges «attracts» the empirical and theoretical distribution curves to the horizontal axis, i.e. reducing inequality in the distribution of income between economic agents. Therefore a change in parameter t allows to determine time to create the desired distribution of income.

According to the rules of exchanges, even if the exchange process stopped after exceeding value t_D, distribution is still changing until it reaches a steady state. This is the so-called relaxation period which covers the interval from t_D until such time as the transition occurs a statistical equilibrium.

Another case is when N agents redistribute the excess of proceeds which will appear to some distribution $\{N_i\}$, consisting of n_0 agents without income, n_{m+} agents with accumulated income $m_+ \left(m_+ > 0\right)$ and n_{m-} agents with borrowings (debt) $m_- \left(m_- < 0\right)$. Distribution N_i agents on m_i income groups are determined in the class of combinatorial statistics. The number of microeconomic conditions W given distribution $\{N_i\}$ can be defined as follows:

$$W = \frac{N!}{n_0! \prod_{m+} n_{m+}! \prod_{m-} n_{m-}!} \tag{6}$$

and presented as

$$\delta N = \delta\left(n_0 + \sum_{m_+} n_{m_+} + \sum_{m_-} n_{m_-}\right) = \delta n_0 + \delta \sum_{m_+} n_{m_+} + \delta \sum_{m_-} n_{m_-} = 0, \tag{7}$$

$$\delta D = \delta\left(\sum_{m_-} m_- n_{m_-}\right) = \sum_{m_-} m_- \delta n_{m_-} = 0, \tag{8}$$

$$\delta M = \delta\left(\sum_{m_+} m_+ n_{m_+}\right) = \sum_{m_+} m_+ \delta n_{m_+} = 0, \tag{9}$$

$$\delta M_0 = \delta\left(m_0 N\right) = 0, \tag{10}$$

where the conditions (7) − (10) express the steady state. Condition (7) shows that the number of agents equals N. Due to the limited boundaries of debt, the amount of bank loan eventually reaches its maximum, i.e. total debt agents $m\_$ (condition (8)). Given the initial values (condition (10)), the amount of accumulated revenue of agents reaches its upper value (condition (9)) as the total income remains at the upper limit when the system is in a stable condition.

Obviously, it is necessary to determine the most probable distribution, which corresponds to the largest number of microeconomic conditions. Thus, the stationary distribution can be obtained by maximizing ln W subject to the limitations (7) − (10). Using the method of Lagrange multipliers, we obtain the most probable distribution:

$$\delta \ln W - \alpha\delta N - \beta\delta D - \gamma\delta M_+ - \lambda\delta M_0 = 0. \tag{11}$$

Substituting conditions (6) − (10) for equation (11) and making simple transformations, we obtain expressions for n_0, n_{m_+} and n_m:

$$\begin{aligned} n_0 &= e^{-\alpha-\lambda m_0}, \\ n_{m_+} &= e^{-\alpha-\lambda m_0-\gamma m_+} = n_0 e^{-\gamma m_+}, \\ n_{m_-} &= e^{-\alpha-\lambda m_0-\beta m_-} = n_0 e^{-\beta m_-}. \end{aligned} \tag{12}$$

Giving the initial values of income of each economic agent determines the number of agents without income which is also relevant to the maximum bank loan amount D, and the number of agents N who participated in transactions. Furthermore, n_{m+} and $n_{m\_}$ depend on n_0, and are presented in the form of the exponential function. Thus, the stationary distribution of density can be expressed as:

$$p_0 = \frac{n_0}{N},\; p_+ = \frac{n_0}{N} e^{-\gamma m_+},\;\; p_- = \frac{n_0}{N} e^{-\beta m_-}. \tag{13}$$

Fixed income distribution, which is subject to the limitations on the maximum amount of bank loans, is represented as asymmetric Laplace distribution, displayed on equalities (13). In other words, the distribution at steady state is determined by the initial value of income, limit of debt and the total number of agents participating in the exchange. Then the system reaches data initial conditions determined by some stationary distribution function.

Figure 2 presents the graphs of density of probability distribution of incomes of economic agents in the steady state for different occasions:

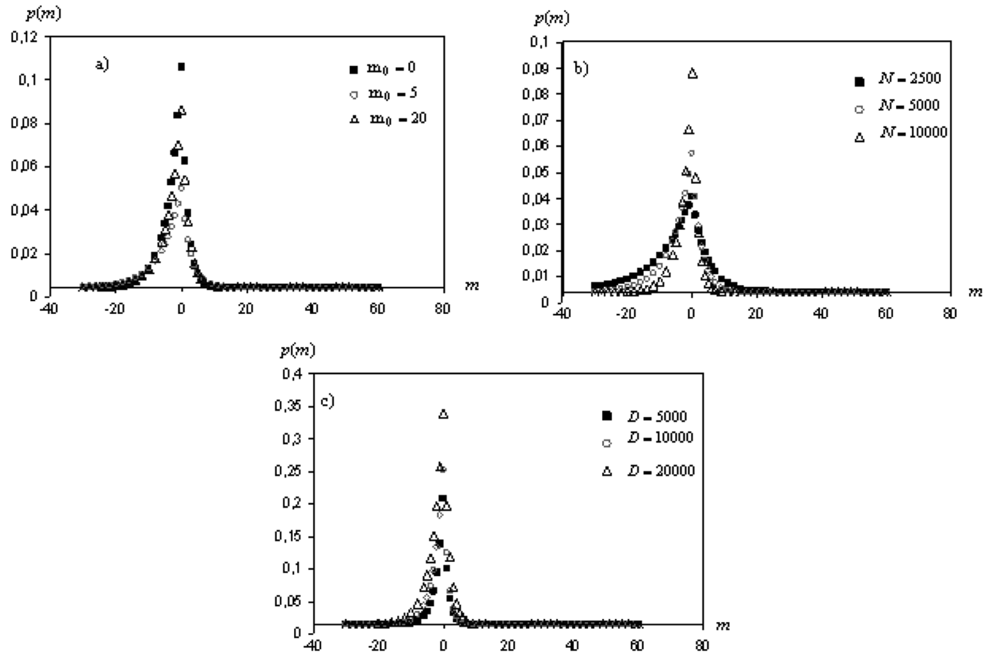

Figure 2. Probability density of income distribution in the steady state: a) $m_0 = 0,5,20$; N = 5000; D = 10 000; b) N = 2500, 5000, 10 000; $m_0 = 5$; D = 10 000; c) D = 5000, 10 000, 20 000; $m_0 = 10$; N = 5000

Source: http://www.ukr.stat.gov.ua.

a) changes of the value of the initial income ($m_0 = 0,5,20$ conv. units. respectively). In this case there is the constant number of agents participating in exchanges (N = 5000 people) and the maximum value of bank loan (D = 10 000 conv. units.). As a result of the growth of m_0 from 0 to 20 conv. units we have a right hand shift of distribution form (which is logical because the initial amount of income increases) and a decline in inequalities in the distribution of income between economic agents. This circumstance can be explained by the fact that most of the income is formed as deposit resources, i.e. the change in the deposit exceeds the change in the credit of economic agents. If credit resources are interpreted as the amount of social grants to economic agents and deposit resources as income in the form of wages, the increase in the last one indirectly reflects the growing macroeconomic development (eg., (GRP) gross regional product). Therefore, the reduction of inequality in the distribution of income is not at the expense of social policy, but due to its economic growth;

b) changes of the number of economic agents involved in exchanges (N = 2500, 5000, 10 000 persons respectively). The initial income (m_0 = 5 conv. units.) and the maximum value of bank loan (D = 10 000 conv. units.) remain sustainable. As it can be seen from the figure, the increase in the number of participants of exchanges leads to the left-shift of the distribution function (which is logical because the same amount of total income is distributed among a large number of economic agents), and to the growth of inequality of agents income distribution. The last one caused by a large number of exchanges that take place between economic agents, i.e. the increase in the number of exchanges between agents, leads to the following: the bulk of economic agents becomes poor and income is concentrated in the hands of a small group of economic agents. This implies that under conditions of equal opportunities and coincidences market there is a sharp change in the initial distribution of income (when all agents are equally rich) to the final distribution (the vast majority of the poor, no middle class, a small number of the richest, long «tail» oligarchs);

c) changes of value of the maximum bank loan (D = 5000, 10 000, 20 000 conv. units. respectively). The initial income (m_0 = 10 conv. units.) and the number of economic agents (N = 5000 people) remain sustainable. The growth of total lending, as in case b), leads to a left-shift of the distribution function and the growth of inequality in income distribution of economic agents. This case well illustrates unjustified increase in the money supply in the economy to support socially vulnerable groups at a constant macroeconomic result (eg., (GRP) gross regional product), resulting in «wasting» the extra income received and not desired reduction in income inequality of economic agents.

Thus, there are considered homogeneous kinetic models of income distribution, in which there is the interaction of a large number of economic agents under specified initial conditions prescribed period and in exchanges.

Heterogeneous kinetic models of income distribution are distinguished in a class of statistical models of income distribution [Dragulescu, Yakovenko 2000; Scafetta et al., 2004]. Processes occurring in the economy are determined by the interaction of many heterogeneous agents that operate simultaneously. The action of any agent depends on the expected actions of a limited number of other agents and on the aggregated state of the agents. By «local» interaction it is understood that each agent interacts only with one of the other agents in its «neighborhood» – the economic field.

In exchanges the size of distribution of income is the difference between price and value of the asset, and all prices are agreed on both sides of agent pairs. Because the value of an asset is an exogenous variable, the net value of income distribution dominates the endogenous variable, established as a result of an agreement. Thus, the endogenous value of income is determined by the balance of income of both parties, and from a fixed amount of distribution $\Delta m = 1$ moving to the sum distribution, which is a random part of the difference in income for each pair of economic agents:

$$\Delta m_{ij,t} = \varepsilon \left| m_{i,t} - m_{j,t} \right|, \tag{14}$$

where the random variable changes in accordance with the even distribution, based on the stochastic nature of the transaction.

The time evolution of income distribution and its own form displays asymmetric Subbotin distribution at steady state, which includes, in special cases, the distribution of Gaussian and Laplace and has the following form:

$$P(x) = \begin{cases} \dfrac{1}{A_l} e^{-\frac{1}{b_l}\left|\frac{x-m}{a_l}\right|^{b_l}} & \text{for } x \le m, \\ \dfrac{1}{A_r} e^{-\frac{1}{b_r}\left|\frac{x-m}{a_r}\right|^{b_r}} & \text{for } x > m, \end{cases} \tag{15}$$

where $A_l = 2a_l b_l^{\frac{1}{b_l}} \Gamma\left(1 + \frac{1}{b_l}\right)$, $A_r = 2a_r b_r^{\frac{1}{b_r}} \Gamma\left(1 + \frac{1}{b_r}\right)$.

Function (15) depends on five parameters:m, a_l, b_l and a_r, b_r accordingly to describe the lower and upper tail. In Figure 3 The graphs of income distribution are reflected by asymmetric Subbotin distribution at steady state.

When building asymmetric Subbotin distribution there were used empirical data on changes in income in accordance with the data survey of households in Ukraine [*Costs and resources…*]. Money income of households consists of a sum of money and natural (in money value) incomes of household members in the form of: salary (excluding income tax and compulsory deductions), income from entrepreneurial activity and employment, property income from interest, dividends, sale of shares and other securities, income from the sale of real estate, personal and household goods, products of subsidiary farming and food, pensions, scholarships, social benefits (benefits and cash subsidies for housing and communal services, electricity and fuel, compensation for unused entitlement on spa treatment, travel privileges of certain categories of citizens, etc.), allowances from relatives and other persons and other income. According to [*Costs and resources…*] all households are divided by the average level of income (x) into groups.

It is necessary to compare the average monthly income of households with cash income sufficient to ensure normal functioning of the human body and maintaining healthy set of food and a minimal set of non-food items as well as a minimum set of services required to meet basic social and cultural needs of the individual parameter m fixed at the subsistence level, operating in the country at the time of analysis. According to [*The Law of Ukraine…*] in 2011 m = 911 UAH per person per month.

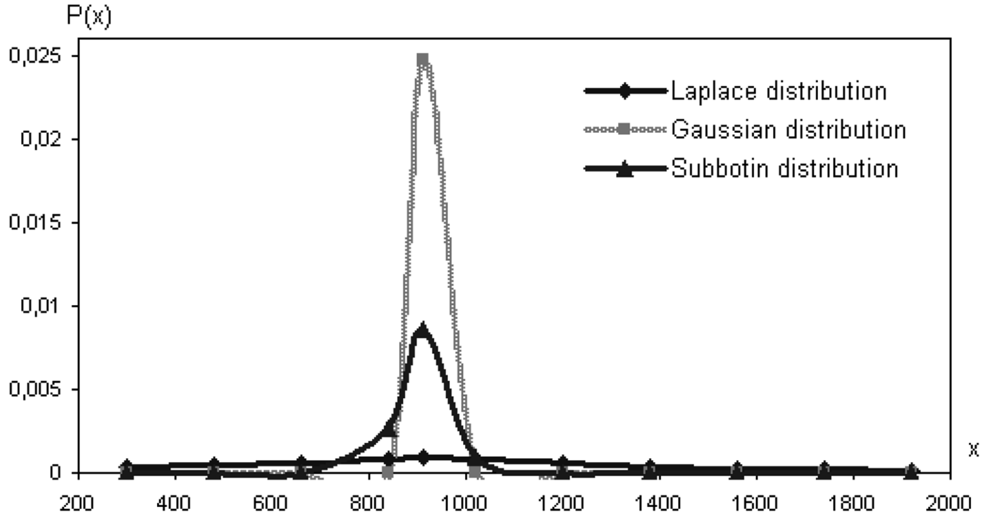

Figure 3. Probability of income distribution $P(x)$ with m = 911, $b_l = b_r$ = 1;1,5; 2, $a_l = a_r$

Source: http://www.ukr.stat.gov.ua.

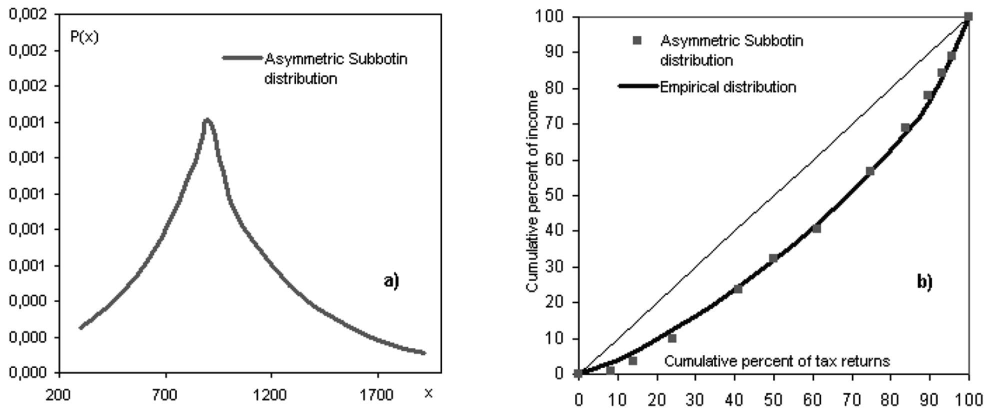

Figure 4. Asymmetric Subbotin distribution with m = 911, b_l = 1,06 and b_r = 1,03: a) probability distribution $P(x)$ of income; b) Lorenz curve

Source: http://www.ukr.stat.gov.ua.

The case of symmetric Subbotin distribution is when parameters b_l and b_r are equal, and accordingly $a_l = a_r$. As special cases there are considered Gaussian distributions with $b_l = b_r = 2$ and Laplace in $b_l = b_r = 1$.

As it is shown in Figure 3, the Laplace distribution reduces inequality in the distribution of income of households compared with two other distributions, and it can be seen in the formation of economic structure, which is close to the socialist one. Normal distribution (Gaussian) is the most desirable, according to which there can be the middle class, unless the distribution curve is more «drawn» to the x-axis. The distribution of Subbotin just reflects this «drawn» condition, but because of three distributions under consideration it is the most plausible.

For different parameters b_l = 1,06 and b_r = 1,03 asymmetric Subbotin distribution takes the form which is similar (compared with previous symmetric distributions) to the distribution of the empirical data (Figure 4).

As it is shown in Figure 4a), the cumulative distribution of population size and the corresponding number of this income in empirical data (solid line) and the values obtained from the asymmetric Subbotin distribution (points) coincide. Therefore testing the model on likelihood ratio by Laplace, Gaussian, and symmetric distributions, and by asymmetric Subbotin distributions allows to conclude that the asymmetric Subbotin distribution provides the best description of the income for the entire range of data.

4. Conclusion

There were built and analyzed statistical models, which are modifications of the random exchange models. It Has been established that the models of Laplace or Subbotin income distribution can be used to define the shape of income distribution in almost the entire volume of data range. However, for the top of the income distribution there have been confirmed significant and out of balance deviations, which are subject to forming the distribution of wealthy, upper class population. This is explained by the fact that in these models the exchanges were considered as a single process of changing income for all economic agents, while according to the empirical data the richest people get richer by investing. Therefore, investment is another economic tool change of income for the wealthy group of society. Accordingly, the definition of investment behavior as a factor of change in the accumulation of income through the formation of income may be subject to further research.

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MODELE STATYSTYCZNE DYSTRYBUCJI DOCHODÓW W SYSTEMACH SPOŁECZNO-GOSPODARCZYCH

Streszczenie: Niniejszy artykuł poświęcony jest wykorzystaniu podejścia opartego na agentach do budowy modeli dystrybucji dochodów, przy której systemy gospodarcze są modelowane jako suma autonomicznych, interaktywnych agentów. Rozpatrzone modele otwierają nową drogę do analizy rozkładu dochodów, która przejawia się we współdziałaniu pomiędzy dużą liczbą agentów gospodarczych, co pozwala na ustalenie pożądanych rozkładów stacjonarnych między nimi.

Słowa kluczowe: dystrybucja dochodów, podejście oparte na agentach, dystrybucja Laplace, asymetryczna dystrybucja Subbotina, model statystyczny, system społeczno-gospodarczy.