DIDACTICS
OF
MATHEMATICS

9(13)

The Publishing House
of Wrocław University of Economics
Wrocław 2012

Reviewers cooperating with the journal
Giovanna Carcano, Salvatore Federico, Marian Matłoka,
Włodzimierz Odyniec, Anatol Pilawski, Achille Vernizzi, Henryk Zawadzki

Copy-editing
Elżbieta Macauley, Tim Macauley, Marcin Orszulak

Proof reading
Barbara Łopusiewicz

Typesetting
Elżbieta Szlachcic

Cover design
Robert Mazurczyk

Front cover painting: W. Tank, Sower
(private collection)

This publication is available at: www. journal.ue.wroc.pl and www.ibuk.pl

Abstracts of published papers are available in the international database
The Central European Journal of Social Sciences and Humanities
http://cejsh.icm.edu.pl

Information of submitting and reviewing paper is available
on the Publishing House's website www.wydawnictwo.ue.wroc.pl

PL ISSN 1733-7941

The original version: printed

Printing: Printing House TOTEM
Print run: 200 copies

Table of contents

Marek Biernacki, Katarzyna Czesak-Woytala

Efficiency of mathematical education in Poland............................ 5

Marek Biernacki, Wiktor Ejsmont

Optimum class size. Testing Lazear's theorem with intermediate mathematics scores in Polish secondary schools 15

Katarzyna Cegiełka

Composition of the European Parliament in the 2014-2019 term... 25

Piotr Dniestrzański

Degressively proportional functions using the example of seat distribution in the European Parliament.. 35

Piotr Dniestrzański

Systems of linear equations and reduced matrix in a linear algebra course for economics studies... 43

Wojciech Rybicki

Some reasons why we should teach matrices to students of economics.. 55

Wojciech Rybicki

Further examples of the appearance of matrices (and the role they play) in the course of the economists' education 75

OPINIONS

Jacek Juzwiszyn

6th European Congress of Mathematics – report of the participation .. 91

D I D A C T I C S   O F   M A T H E M A T I C S

No. 9 (13) 2012

DEGRESSIVELY PROPORTIONAL FUNCTIONS USING THE EXAMPLE OF SEAT DISTRIBUTION IN THE EUROPEAN PARLIAMENT

Piotr Dniestrzański

Abstract. This article presents an example of the current problem from the borders of economics and politics, which can be used in mathematics courses for economics majors. This problem is the distribution of seats in the European Parliament among the member states of the European Union, and the principle of degressive proportionality formulated for this purpose in the legislation.

Keywords: degressive proportionality, mathematics teaching.

1. Introduction

Mathematics, one of many subjects in the curricula of economics' studies, is sometimes treated by students as a "necessary evil" you just have to deal with. At the same time more and more often, because of further cuts in the number of teaching hours, teachers are forced to reduce the scope of the taught content or the implementation of some parts of the material to an even more narrow range[1]. The best bet in this situation would be to reduce the application part which, in turn, sometimes is not very well received by the participants. Probably anyone who has run a course in mathematics (for studies other than mathematics) will have encountered, in connection with the presented material, questions like, "What do we need it for?", "When will we use it?", "Where will we be able to apply it?", etc. Presenting at an appropriate stage of the lecture (class/practice) a well-chosen example of

Piotr Dniestrzański
Department of Mathematics, Wrocław University of Economics, Komandorska Street 118/120, 53-345 Wrocław, Poland.
E-mail: piotr.dniestrzanski@ue.wroc.pl

[1] The deterioration of the results of teaching mathematics at the universities of economic developments in the software in high school and a decrease in the number of hours extensively in college teaching article treats Miśkiewicz (2011).

application is usually the key to a wider interest in the presented material among the participants. A great source from which mathematics (and not only) teachers can draw from in recent years is an issue which the European Union has not been able to settle for a long time – the distribution of seats in the European Parliament. On the basis of this issue, one can build a lot of potential problems, more or less complex, ideally suited for use in a mathematics course. This example can show how even simple tools of quantitative methods may be useful in the analysis, resolution and, above all, understanding of important issues related to the functioning of significantly large structures, including the entire European Union and, more specifically, the European Parliament which represents its citizens. Although parliamentary democracy has been in use in the majority of member states for many years, the EU still has not managed the creation of a precise mechanism for selecting members of the body that represents its citizens. This article does not aim at analyzing the problem itself (it is being dealt with by many mathematicians, economists, lawyers and politicians), but at building on its basis examples of the possibilities of using it in an academic course of mathematics.

2. Derivation of the issue

The concept of degressive proportionality became popular around 2005/6, in connection with the distribution of seats in the European Parliament among the member states of the European Union. The distribution of seats is based solely on the size (measured by population) of the member state. In this case no other parameters or characteristics of the merits of the state, such as its size, the size of GDP, the length of presence within the structures of the European Union, are relevant. This is of course consistent with the traditions of selecting national parliaments, and no other criteria for the allocation of seats were taken into account. Large variations of the member states in terms of population prevents the use of proportional divisions. Suffice to note that the ratio of currently the most populated (the largest) member of the EU (Germany) and the smallest (Malta)[2] is $81843743/416110 \approx 196.7$. So, if Malta had a minimum representation of the current 6 seats (assuming proportional distribution), Germany would have to be represented by 1180 members of Parliament and the Parliament

[2] Eurostat, on 1 December 2012.

would come up to about 7,260 members.[3] Assuming, possibly in the near future, the accession of Turkey to the EU structures, the situation would be even more drastic.

The first fine adjustment of the composition of the European Parliament took place at the Council of Europe meeting in Edinburgh (11-12 December 1992) and was connected with the unification of Germany. See below what was then established:[4]

1. Each state, regardless of population, receives six seats.

2. Countries with a population of between 1 and 25 million receive one additional seat for every 500,000 citizens.

3. Countries with a population of between 25 and 60 million receive one additional mandate per million citizens.

4. States with a population of over 60 million receive a mandate per each 2 million citizens.

The problem of allocating seats due to the dynamic fluctuations of the EU's composition and increase of its importance, caused the need for stricter regulation, especially in view of the accession of more countries and demographic changes in the member states. The existing legislation in this subject is the Treaty of Lisbon,[5] which states:[6]

*The European Parliament shall be composed of representatives of the Union's citizens. They shall not exceed seven hundred and fifty in number, plus the President. Representation of citizens shall be **degressively proportional**, with a minimum threshold of six members per Member State.*

No Member State shall be allocated more than ninety-six seats.

The ambiguous phrase "degressively proportional" and the problems associated with its use, resulted in an immediate adoption (prepared by the Committee on Constitutional Affairs) of the European Parliament resolution[7] containing six rules to clarify the rule of degressive proportionality. For further considerations two of these principles are crucial:

The principle of fair distribution: no state will have more seats than a larger member state or a smaller amount of seats than a smaller member state.

[3] A more detailed analysis can be found in the work of Cegiełka et al. (2010).

[4] The given algorithm is then stored in the Treaty of Amsterdam, together with the upper limit of the total number of deputies to 700.

[5] The Treaty of Lisbon reforming the Treaty on European Union and the Treaty Establishing the European Community, signed on 13 December 2007.

[6] Article 9a, the new Article 14 of the Treaty on European Union.

[7] At the meeting of the Parliament on 11 October 2007.

The principle of relative proportionality: the ratio of the population size to the number of seats is greater the larger the state, and smaller the smaller the state.

The principle of fair distribution (PFD) and the principle of relative proportionality (PRP) constitute the core of what is now called **degressively proportional distribution**. Other rules (not cited in the article) determine the minimum and maximum number of seats a member of the Union can get, with a maximum size of Parliament (750 MPs). It is also stated that smaller countries should have greater representation than that according to ordinary proportionality, and it mentions the flexibility for minor modifications aimed at "the most equitable distribution of seats".

It is worth noting that the findings of the Council of Europe meeting in Edinburgh (the four points mentioned earlier) are the prototype of degressively proportional divisions. The distribution of seats in line with these findings clearly meets the conditions of PFD and PRP. The only drawback in this case is the uncontrolled number of Members of Parliament, since in the case of the (expected) rapid development of the Union it could be troublesome.

3. Degressively proportional functions

Let x be population of a state and $y = f(x)$ the number of members representing the state of population x. PFD and PRP rules require the following limitations for the $f(x)$ function:

$$f(x) \text{ is non-decreasing,} \tag{1}$$

$$\frac{f(x)}{x} \text{ is decreasing.} \tag{2}$$

Apart from the necessary rounding issues with the allocation of seats (we are dealing with the allocation of the so-called indivisible goods), we will present an analysis of functions satisfying conditions (1) and (2). Of course, we can assume that

$$f : R_+ \to R_+ . \tag{3}$$

What is the conjunction of conditions (1)-(3), and can the considered issue be used in a mathematics course at a university of economics (and not only economics)? The function satisfying conditions (1)-(3) will be henceforth called the **degressively proportional function** or **allocation function**.

The analysis of the properties of the degressively proportional functions and their applications is an example of a nice "full-blooded problem", which can be presented to students. It has many attributes that can make the students interested in the issue:

1. It is easily definable.
2. It is closely related to an unresolved (at least until the time when these words are written) important issue.
3. The examined issue can be analyzed without the use of complex mathematical apparatus.
4. There is a wide space for stimulating students' creativity.
5. More mathematically proficient students have the opportunity to demonstrate their skills.

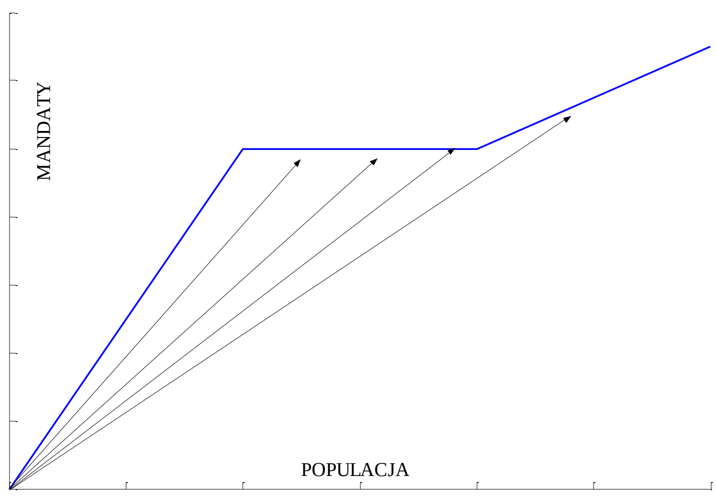

Fig 1. The function allocating seats according to population,
and a non-concave function satisfying the principle of relative proportionality

Source: own elaboration.

The first reaction after taking a look at the conditions of (1)-(3) is often to say that $f(x)$ is a concave function. This is a half-truth, which is not completely true. Concavity here is a sufficient condition but not a necessary one. The proof of the relevant facts can be found in the article Dniestrzański (2011). Figure 1 shows the corresponding geometric illustration, such as the function determining the distribution of seats according to population, which satisfies the principle of relative proportionality but is not a concave function.

Let us now move the considered issue to the level of a mathematics course for economics majors. What are the problems associated with the proposed issue of the division of seats the students may try to resolve? How to formulate a suitable task?

Proposed tasks for students

Exercise 1.

Give an example of a $f: R_+ \to R_+$ function that is a degressively proportional function and is (at least for a certain intervals) convex.

Exercise 2.

Give an example of a $f: R_+ \to R_+$ function which is a degressively proportional function and convex for the interval $(0, a)$, $a > 0$.

Exercise 3.

Determine intervals on which the function $f: R_+ \to R_+$ defined by $f(x) = x^n + a$ ($a > 0$, n is a natural number) is degressively proportional.

Exercise 4.

Prove that under conditions (1) and (3) concavity is a sufficient condition so that the given function becomes degressively proportional.

These tasks are just a few examples of the possible use of this issue in the course of mathematics. Along with purely mathematical considerations, there can (and should) appear the application issues. The concept of degressively proportional function has a strictly practical derivative that may help (on the other hand, it can also narrow down) inventing examples of applications. Here are two simple problems which the students can tackle.

P1. Give examples (real or theoretical) of implemented degressively proportional divisions.

P2. Define degressively proportional division. Where are (or could be) such divisions used?

4. Generalization for functions of multiple variables

The issue of distribution of seats in the European Parliament is a good example of broader opportunities to engage students to use quantitative methods to analyze economic and social issues. Going forward in the study of the degressively proportional function, one may go on to functions of multiple variables. In this case we are able to reverse the problem or, more precisely, one may raise the following questions:

1. How to generalize the notion of degressively proportional function in the case of functions of multiple variables?
2. What possible use could such functions have in the modeling of economic phenomena such as the distribution of wealth, or other?
3. Can one state where the divisions similar to the allocations based on degressively proportional functions are already in place?

Consider the $f(x, y)$ function. In order to generalize the notion of degressive proportionality to functions of two variables (going to any number of variables is a formality), we can expect that the function $f(x, y)$ should satisfy the conditions:

$$f(x, y) \text{ is non-decreasing with respect to } x \text{ (with fixed } y\text{)}, \tag{1a}$$

$$f(x, y) \text{ is non-decreasing with respect to } y \text{ (with fixed } x\text{)}, \tag{1b}$$

$$\frac{f(x, y)}{x} \text{ is decreasing with respect to } x \text{ (with fixed } y\text{)}, \tag{2a}$$

$$\frac{f(x, y)}{x} \text{ is decreasing with respect to } y \text{ (with fixed } x\text{)}, \tag{2b}$$

$$f : R_+ \to R_+ \tag{3a}$$

Are there other natural generalizations of the concept of degressively proportional function in the case of multiple variables? How one can reformulate the conditions (1a), (1b) and (2b), assuming the differentiability of function $f(x, y)$? These are examples of additional questions indicating other possible directions of exploitation of the topic.

Accurate mathematical analysis of the concept of degressive proportionality can be found, inter alia, in an article by Słomczyński, Życzkowski (2012) and the compilation by Misztal (2012). The proposal for the distribution of seats which seems to be the closest to be approved by the European Union (called the Cambridge compromise) is described in detail in the work of Grimmett et al. (2011).

5. Summary

The distribution of goods and "burdens", determining the relevant rules of such divisions, is one of the oldest dilemmas of humanity. The problems with the distribution of seats in the European Parliament show that still there are situations in which one does not really know how to carry out the proper allocation. In such cases, to some extent, a mathematical description of the problem is very useful. It allows for definition and clarification and thus introduces more flexibility in preparing appropriate analyzes. The problem of the allocation of seats in the Europarliament is one of the most interesting challenges of this type. We are dealing here with a simple (in the mathematical description) issue with great potential. The paper presents (or rather outlines) the educational potential of the problem. The focus is on selected items which can be used in the mathematics courses at universities of economics and may be one of the incentives to study the subject.

References

Cegiełka K., Dniestrzański P., Łyko J., Misztal A. (2010). *Skład Parlamentu Europejskiego w kontekście podziałów proporcjonalnych*. Prace Naukowe Uniwersytetu Ekonomicznego w Katowicach. Pp. 159-170.

Dniestrzański P. (2011). *Dywersyfikacja podziału mandatów w Parlamencie Europejskim*. In: *Ekonometria. Zastosowania metod ilościowych*. Prace Naukowe Uniwersytetu Ekonomicznego we Wrocławiu. Wydawnictwo UE. Pp. 32-41

Grimmett G., Laslier J.-F., Pukelsheim F., Ramirez-González V., Rose R., Słomczyński W., Zachariasen M., Życzkowski K. (2011). *The allocation between the EU Member States of seats in the European Parliament Studies*. PE 432.760.

Lamassoure A., Severin A. (2007). *European Parliament Resolution on "Proposal to amend the Treaty provisions concerning the composition of the European Parliament*" adopted on 11 October 2007 (INI/2007/2169).

Misztal A. (2012). *Degresywna proporcjonalność a kształtowanie składu Parlamentu Europejskiego*. Wydawnictwo Uniwersytetu Ekonomicznego we Wrocławiu. Wrocław.

Miśkiewicz M. (2011). *Wpływ nowego programu nauczania matematyki w szkołach średnich na wyniki nauczania matematyki w uczelniach ekonomicznych*. Didactics of Mathe-matics 8(12). Wroclaw University of Economics. Pp. 101-112.

Słomczyński W., Życzkowski K. (2012). *Mathematical aspects of degressive proportionality*. Mathematical Social Sciences 63. Pp. 94-101.

Traktat z Lizbony (2007). Dziennik Urzędowy Unii Europejskiej. C 306. Tom 50.

Young H. P. (2003). *Sprawiedliwy podział*. Scholar. Warszawa.