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EFFICIENCY OF MATHEMATICAL EDUCATION IN POLAND

Marek Biernacki, Katarzyna Czesak-Woytala

Abstract. Based on the analysis of international exam results, which was carried out with OECD-PISA in mathematics among Polish students of *gimnazjum* (secondary school) in 2003-2009, and based on the results of the exam in mathematics among students of the first year in the University of Economics in Wroclaw in 2009-2012; the article shows that Polish education in mathematics is, unfortunately. Currently at the level typical for a communist system, i.e. one which is not developing the ability to solve new or sophisticated problems and not practicing new methods for these problems, and as a result, is not open to the needs of a market economy.

Keywords: effectiveness of mathematics education, PISA – Programme for International Student Assessment.

1. Introduction

The goal of this paper is to analyze the efficiency of education in the perspective of economic growth. The authors of this paper claim that the Polish mathematical education remains at a level which is typical for a communist education system. This means that it does not meet the needs of a market economy. In a report published by the World Bank in 1996, the authors claimed that the communist educational system, when compared to other countries, provides a better basis of knowledge, but does not teach how to ask questions, solve new problems or apply new methods while working on the earlier defined problems (World Bank, 1996).

According to Zienkowski (2003), "Education is an organized process of acquiring scientific knowledge and its use". So "the goal of educational policy is to build student's knowledge by conveying information, developing skills, forming attitudes and values. Equality of access to education, which is the third goal, justifies both the justice and the fact that the waste

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of talents is not an effective activity" (Barro, Lee, 2001). The effectiveness of education can be external, which means that it helps to achieve economic and cultural goals, or internal, which means that it gives good educational results (results in exams). According to Barro and Lee (2001), the educational results in science and mathematics are the most significant for economic growth. This is why these areas will be the subject of our analysis.

If we look at the recent educational reforms in Poland through the prism of teaching science, we can notice that there are fewer teaching hours and the range of topics covered during the courses is more limited.

In junior high school, physics and chemistry are limited to four teaching hours per week, while there are three hours of arts and CSPE per week. For foreign languages five hours per week are planned. The high school syllabus in mathematical education has been reduced and such topics as, for example, differential calculus, have been excluded. Those high school students who choose physics in their *matura* exam are only a small percent of the student population.

In 2010, the written *matura* exam in mathematics at basic level became obligatory again. In that year, 87% of students passed this exam and the average score was almost 50% of the points available. In 2011, only 79% of high school graduates passed this exam and the average score was less than 45%. Last year, 85% of students passed the basic *matura* exam in mathematics with the average score of 56%. The exam is considered to be passed when the score equals at least 30% of the points available. It has to be mentioned that the tasks in which the commands contained phrases "prove that…" or "justify that…" were the most problematic ones for the students. This was because in such tasks one cannot apply standard, mechanical solutions as practiced on the examples from the workbook.

The consequence of the situation described is the fact that there are some students at the universities of economics or technical universities who have difficulties with… fractions. With such a level of education it will be difficult in Poland to build a knowledge-based economy of a truly European kind.

2. International comparison

The analysis here presented concerns 35 countries which are members of the OECD and which have been taking part in PISA (Programme for International Student Assessment) since the year 2000. The PISA research involves pupils aged 15 and is performed in three dimensions: reading, mathematics and science. The structure of this research makes it possible to compare the results not only between countries, but also between different editions of the test. For the purpose of this article, data concerning the results in mathematics are analyzed in detail. The PISA test in mathematics was conducted in Poland for the first time in 2003, which is why the analysis presented here will concern the period 2003-2012.

In terms of the average results in mathematics, Polish students are similar to the students from all the other OECD countries. This means that the differences between the Poles' average score in mathematics and the OECD average score is statistically insignificant.[1] Figure 1 presents the average score of students in Poland and in some selected countries which are similar to Poland in terms of expenditure on education (expressed as percentage of GDP *per capita*).

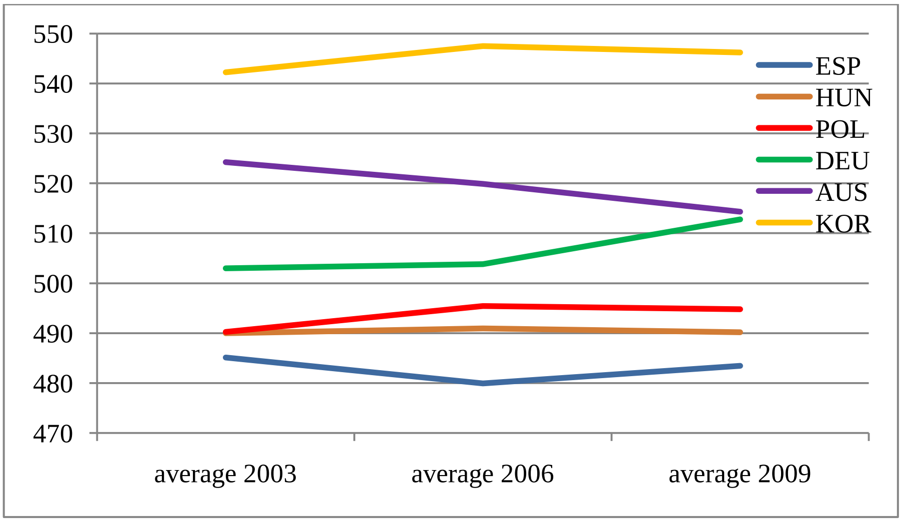

Fig. 1. Average score in mathematics in PISA in Poland and other countries which are similar to Poland in terms of expenditure on education (as percentage of GDP *per capita*)

Source: own study based on the OECD data.

In terms of average score in mathematics in 2003, Poland is in 21st place out of the 29 countries which took part in the assessment that year. A similar result (490 points) was also achieved by Hungary. In such countries as Spain, USA, Portugal, Italy, Greece, Turkey and Mexico, the results were poorer.

[1] As described in the report of the Ministry of National Education, 2009.

In 2006 Poland also came 21st. However, this time all 34 countries took part in the assessment. In comparison with the previous edition of PISA, Poland was better than such countries as Luxembourg, Norway or Slovakia. In the 2009 edition of PISA, Polish students again improved their position in the ranking. Their average score was about 494.8 points and this gave them 19th place from among all the OECD countries. That year Polish students were slightly better than students from Sweden, the Czech Republic and Great Britain. Still, the average score of students from Austria, Slovakia or Norway was even higher.

Analyzing Polish students' average score against the background of the results of students from other countries, it seems that in every subsequent edition of PISA, Poland has been achieving ever-higher positions in the OECD ranking. Yet a detailed analysis of the results shows that the higher position in the ranking does not come together with an improved knowledge of mathematics.

PISA research allows to assess the level of students' knowledge at six educational levels. In order to be classified at some level, the student needs to demonstrate a defined range of knowledge and skills. The first level represents the least advanced knowledge and the sixth the most advanced. However, they are some students who are not able to solve even the first level tasks.

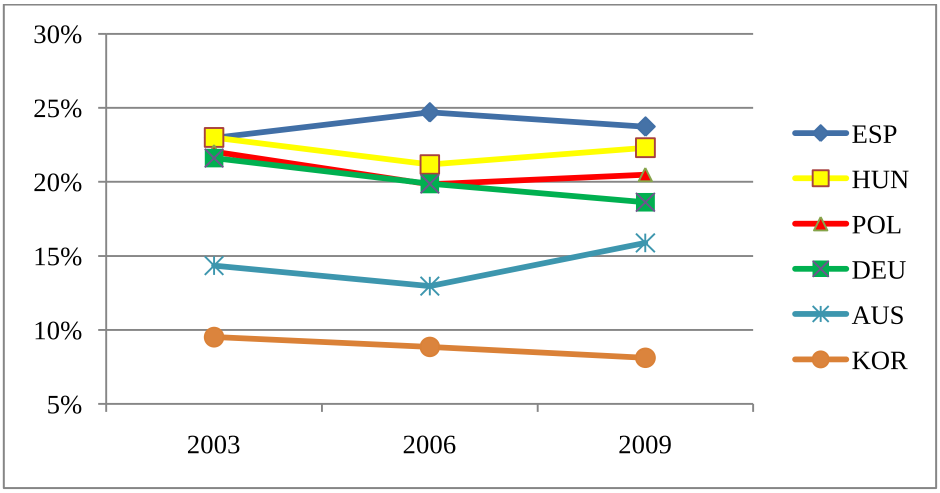

Fig. 2. Percentage of pupils who achieved at most 1st level in mathematics in Poland and in other countries similar to Poland in terms of expenditure on education

Source: own study based on the OECD data.

Figure 2 visualizes the percentage of students who were classified at the first level of PISA scores in mathematics in Poland and in other countries similar to Poland in terms of expenditure on education. In this figure we can see that the percentage of the weakest students in Poland is decreasing (the most noticeable is the decline between 2003 and 2006, and then the stagnation in 2009). A clearly decreasing trend can be noticed in Germany and Korea. In all the other countries it is difficult to observe a clear direction of change.

It should be noted that Poland is one of the countries with the greatest number of the weakest students (over 20%).

So the question is: does the percentage of the best students increase together with the decrease of the percentage of the weakest students? It turns out that such a relationship does not exist in Poland. Figure 3 presents the percentage of students who achieved one of the highest levels in mathematics (5^{th} or 6^{th} level) in the PISA test in Poland and in other countries which are similar to Poland in terms of expenditure on education.

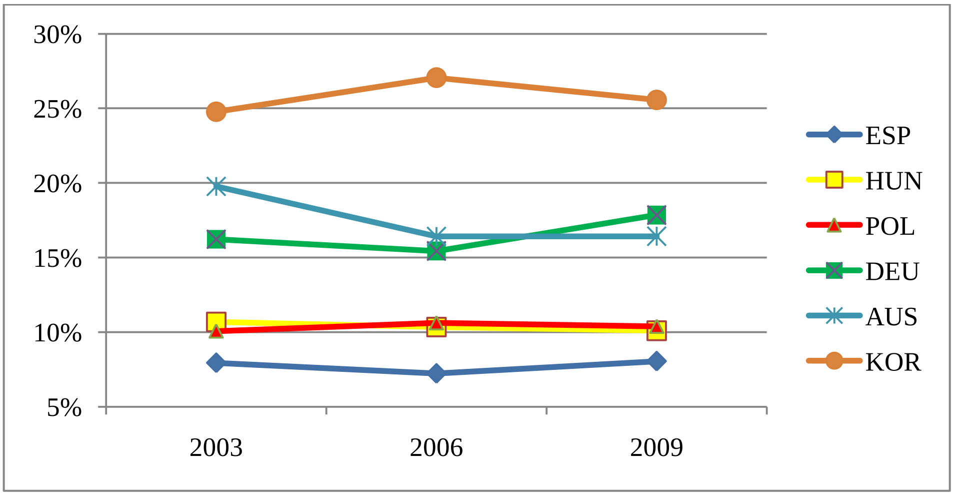

Fig. 3. Percentage of pupils who achieved 5th or 6th level in mathematics in Poland and in other countries similar to Poland in terms of expenditure on education

Source: own study based on the OECD data.

From this figure we can conclude that:

- when compared to other countries that are similar in terms of expenditure on education, Poland has one of the smallest numbers of students achieving the best results (similar to Hungary and Spain),

- despite the declining percentage of the weakest students in Poland, the percentage of the best pupils remains stable (and low).[2]

As has been stated before, the average results of Polish students in subsequent PISA editions are rising (or at least not decreasing), the percentage of the weakest pupils is decreasing slightly and the percentage of the best pupils is constant. Consequently, the hypothesis concerning enhancement of the level of mathematical education appears not to be true. It can be concluded, though, that Polish schools teach average students and do not concentrate on high-achievers who have the potential to score even higher. This means that the Polish educational system does not stimulate the development of the most talented students, but focuses on the weakest or average ones.

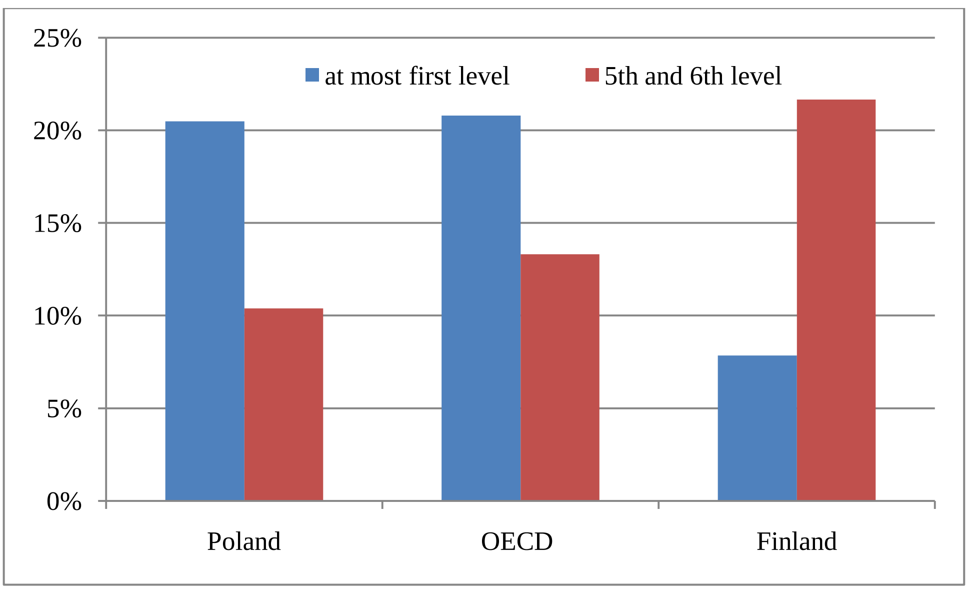

Fig. 4. Percentage of students who achieved the highest and the lowest levels in mathematics in Poland, all OECD countries and in Finland

Source: own study based on the OECD data.

Finland is the European country which achieves the highest positions (1^{st} or 2^{nd} place) in every edition of PISA. Therefore, it is worth analysing the distribution of Polish students' scores in comparison to the scores

[2] In the 2009 report of the Ministry of National Education the problem of stagnation of mathematical skills is discussed.

achieved by Finland and all the other OECD countries. This comparison is presented in Figure 4. In this figure we can see that the percentage of the weakest students in Poland is almost equal to this number in all the OECD countries, and is much higher than in Finland, while the percentage of pupils at the highest levels in mathematics is significantly lower in Poland than in Finland, however, it is lower than in all the OECD countries.

3. Results of Polish students

The educational effects are usually assessed on the basis of final exam scores as compared to the results achieved in previous years or in different schools. Based on the students' test results in mathematics in 2009 and 2012 at the Wroclaw University of Economics, an analysis of the percentage of best and worst students was performed (tail distribution analysis). In the tests analyzed (2009 and 2012), the tasks represented different level of difficulty. In 2009 the tasks in the exam were as follows:

1. *Draw a graph of the function:* $f(x) = (x+2) \cdot e^{\frac{1}{x}}$.

2. *Calculate the improper integral:* $\int_0^\infty \frac{1}{4+9x^2} dx$.

3. *Find the inverse transform* $f : R^3 \to R^3$

$$f(x, y, z) = (x + 2z, x + y + z, -2x + 2y - 5z).$$

4. *Solve the system of equations using the Gauss method (matrix transformation)*

$$\begin{cases} x - y + 2z = 1 \\ 2x - 2y + 5z = 2 \\ -x + y - 3z = -1 \end{cases}.$$

5. *Examine the vectors' linear independence* $\{I, A, A^2\}$, *where*

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

6. *Calculate* $\iint_D (x-y)^2 dxdy$ *, where D is an area bounded with coordinate axes* ($x=0$ i $y=0$) *and the line* $x + y = 1$.

7. *Consumer can spend 90 zl for two goods. Each of these goods costs 1 zl per unit. Consumer's utility function which describes how one values x units of the first good and y units of the second good looks as follows:*

$$u(x,y)=x^{\frac{1}{2}}y^{\frac{1}{3}}.$$

Find a basket of goods with the maximum usability.

In 2009 about 19% of the students passed their final exam with at least a good grade and there were about 50% of students who did not pass the exam in the first term.

In 2012, in the same faculty (the same syllabus, equal number of teaching hours, the same teacher and the same number of students) the final exam was similar to the previous one:

1. *Calculate the limit* $\lim_{n\to\infty}\left(\frac{n+3}{n-2}\right)^{4n}$.

2. *Draw a graph of the function:* $f(x)=\frac{x^2+x+2}{x-1}$.

3. *For the linear transformation:*

$f(x,y,z)=(x+y,x-z,y-z)$, $g(x,y,z)=(x+2z,x+y+z,-2x+2y-5z)$ *find* $g^{-1}\circ f(x,y,z)$.

4. *Calculate the local extremum of the function:* $f(x,y)=\frac{2}{3}x^3+2xy-y^2+1$.

5. *Calculate the gradient of function* $f(x,y)=\sqrt{x^2+y^2}$ *in the point* $P(3,4)$. *Draw this gradient and the level curve passing through this point.*

6. *Calculate the improper integral:* $\int_0^{\infty} x\cdot e^{-x}dx$.

In the first term, 53 out of 111 students passed this exam (46%). Only 10 students (9%) achieved a good or very good score. Both exams (in 2009 and 2012) were considered passed when a student achieved at least 50% of the points. Each task was worth an equal number of points.

The proportions of students who failed the test in 2009 and 2012 do not differ statistically significantly (p-value = 0.58), but the percentage of the best students (with a good or very good grade) decreased and the difference between 2009 and 2012 is statistically significant (p-value = 0.03).

4. Conclusion

Comparing the results from both these exams with the results from the international tests in mathematics we can conclude that:

– the percentage of students who passed their exam in the first term decreased noticeably (50% in 2009 and 46% in 2012), but the percentage of students who achieved the weakest results in the PISA exam equals 20% and is constant. This can be the result of the situation we are facing now, i.e. the problem of demographic decline as there are fewer students. So the universities are trying to attract candidates by lowering the score required to be admitted as a student.

– percentage of students who achieve good or very good grades in tests at university is decreasing and was equal to 19% in 2009 and 9% in 2012. There is also a similar (but stable) level of 10% of students achieving the best results in PISA (5^{th} or 6^{th} level).

As was mentioned in the introduction, the Polish economy needs to take better care of the national human capital in order to make it possible for Poland to catch up with the economies of the developed countries. The economy needs to focus not only on the equality of educational opportunities, but also on the quality of high school and university education, especially in the fields of science and mathematics. The optimal educational system should keep a good basis of general education, but in addition it should develop the ability of solving difficult, sophisticated and new problems. This means that it should develop the attitude of innovation promoted by economists. There is also another very important goal of the Polish educational system on every level: to develop the attitude of responsibility for ourselves and for the society we are living in.

References

Barro R.J., Lee J.-W. (2001). *International Data on Educational Attainment: Updates and Implications*. Oxford Economic Papers. Oxford University Press. Vol. 53 (3). Pp. 541-563.

Biernacki M. (2009). *Effectiveness of mathematical education*. Didactics of Mathematics 5-6(9-10). Wrocław University of Economics. Pp. 19-32.

Programme for international student assessment (2009). Wyniki Badania 2009 w Polsce. Ministerstwo Edukacji Narodowej. http://www.ifispan.waw.pl/pliki/pisa_2009.

Projekt EKOLOGIA (2010). Analiza przyczyn problemów w nauczaniu przedmiotów matematyczno-przyrodniczych.http://innowacyjnyekolog.pl/uploads/files/analiza_ przyczyn_problemow_nauczania.pdf

World Bank (1996). *World Bank – World Development Report: From Plan to Market*. Oxford University Press. New York.

Zienkowski L. (2003). *Gospodarka oparta na wiedzy – mit czy rzeczywistość?*. In: L. Zienkowski (Ed.). *Wiedza a wzrost gospodarczy*. Wydawnictwo Naukowe SCHOLAR. Warszawa.