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Contents

Foreword 5

Diana Bílková: TL-Moments: Analogy of Classical L-Moments 7

Dagmar Blatná: Application of Robust Regression in the Analysis of Internet Access in European Countries 17

Martin Boďa, Mária Kanderová: Rebalancing Issues in Tracking Error Variance Minimization 26

Martin Boďa, Viera Roháčová: Application of Six Sigma Ideas to Timing Decisions at Financial Markets 36

Anton Dekrét, Rudolf Zimka: On the Price Hartwick´s Task and Its Inverse in a Dynamic Model of an Economy with Exhaustible Resources 46

Joanna Dębicka, Agnieszka Marciniuk: Comparison of Reverse Annuity Contract and Reverse Mortgage on the Polish Market 55

Petra Dotlačilová, Jitka Langhamrová: The Influence of Mortality Models for the Expected Future Life-time of Older People 65

Marek Ďurica, Lucia Švábová: Delta and Gamma for Chooser Options 75

Vlastimil Farkašovský: New Concept of Pension Funds Performance Evaluation 85

Albert Gardoń: The Normality of Weekly Relative Changes of the Freight Rate in Container Shipping 95

Mária Grausová, Miroslav Hužvár, Jana Štrangfeldová: Healthcare Systems Efficiency in the Visegrád Group 104

Stanisław Heilpern: Multiple Life Insurance - Pension Calculation 114

Alena Kaščáková, Gabriela Nedelová: Changes in Slovak Households´ Economy 122

Igor Kollár, Pavol Kráľ, Peter Laco: Methodology for Assessing Website Improvement in Corporate Environment 131

Maciej Kostrzewski: Some Method of Detecting the Jump Clustering Phenomenon in Financial Time Series 141

Cyprian Kozyra, Beata Zmyślona, Katarzyna Madziarska: Complementary Objective and Subjective Measures of Hospital Services Quality 150

Pavol Kráľ, Mária Stachová, Lukáš Sobíšek: Utilization of Repeatedly Measured Financial Ratios in Corporate Financial Distress Prediction in Slovakia 156

Ivana Malá: The Use of Finite Mixture Model for Describing Differences in Unemployment Duration 164

Lukáš Malec: Studying Economics and Tourism Industry Relations by Smooth Partial Least Squares Method Depending on Parameter 173

Tomáš Marcinko: Consequences of Assumption Violations Regarding Classical Location Tests 180

Edyta Mazurek: The Income Tax Progression Depending on Social Insurance Contribution in Poland 190

Petr Musil, Jana Kramulová, Jan Zeman: Regional Consumption Expenditures: An Important Starting Point for Regional Input-output Tables 200

Katarzyna Ostasiewicz, Walenty Ostasiewicz: Good Life: From Political to Human Economy 208

Anna Sączewska-Piotrowska: Analysis of Poverty Transitions in Poland Using Multilevel Discrete-Time Event History Models 219

Martina Šimková, Petra Švarcová: Disadvantaged University Students in the Czech Republic 229

Michal Široký: The Use of Short-term Business Statistics for Quarterly GDP Flash Estimates in the Czech Republic 239

Zdeněk Šulc, Hana Řezanková: Evaluation of Recent Similarity Measures for Categorical Data 249

Lucia Švábová, Marek Ďurica: The Relationship Between the Finite Difference Method and Trinomial Trees 259

Kristýna Vltavská, Jaroslav Sixta: The Estimation of Final Consumption Expenditures 270

Lenka Vraná: Business Cycle Analysis: Tracking Turning Points 277

Janusz Wywiał: On Bayesian Testing in Auditing 284

Emília Zimková: Window Analysis of Supper-efficiency Change: Case of the Slovak Banking System 294

Beata Zmyślona: Statistical Modelling of the Impact of Diabetes on the Risk of Hospitalization 301

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THE INFLUENCE OF MORTALITY MODELS FOR THE EXPECTED FUTURE LIFE-TIME OF OLDER PEOPLE

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Abstract
Mortality always was one of the most interesting topics for demographers. The researchers conclude that human life is gradually extending and it has also resulted in an increasing number of oldest people. However, if we examine the evolution of mortality during the human life, we find out that the mortality throughout the human life is changing. But it is important to know that mortality of the oldest persons has a different character. Due to a smaller number of living persons at the highest age, mortality of these persons should be adjusted because of eliminating the systematic and random errors. We can use for it some of the available models which are used for the leveling of mortality curve at the highest ages. The aim of this paper is to analyze the differences in the life expectancy at the highest ages, which are obtained from the application of selected models. As an input data will be used the data about mortality of the Czech population in the period from 1920 to 2012.

Key words: *mortality, life expectancy, models of mortality.*

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1. Introduction

The cause of lengthening of human life is primarily decrease of mortality. There exist many reasons for this evolution. One of them is a higher level of health care but the first thing was a significant improvement in the care of infants. Later came to the fore healthcare about the oldest people. This positive trend will probably continue well into the future (see Kannisto, 1988).

If we examine how mortality evolves during the human life, so we find that it varies. The difference in mortality is obvious especially when we compare its level at the lowest and at the highest ages. The problem is that not so many people live until the highest ages. Then the data about deaths and living older people are loaded by systematic and random errors. For this is good to model the intensity of mortality by an appropriate function. To do this, we can use several available models.

2. Methodology

For the investigation of the evolution of mortality is the most frequently used an indicator known as life expectancy (LE). LE at age x is characteristic similar to average, it may be affected by extreme values (especially by high infant mortality). But there are other indicators such as probable or normal length of life (see Koschin, 2000 or Cipra, 1990). Due to the positive evolution of mortality is always better to publish more than one indicator, and then to compare them.

Inasmuch that the human life is constantly extending (this is also reflected at the highest ages) (see Horiuchi and Wilmoth, 1998 or Kannisto et al., 1994), it is good to have the best imagination about how mortality develops during the human life. But here we run into the problem of capturing of mortality for older people, where values of age-specific death rates show high random fluctuations (see Thatcher et al., 1998).

The basic indicators which are used for description of mortality are the age-specific deaths rates (or the intensity of mortality). Their values are then used for the calculation of other indicators. Its value is not affected by changes in the age structure (see Koschin, 2000):

$$m_x = \frac{M_x}{\bar{S}_x} \tag{1}$$

where M_x is the number of deaths at complete age x and $\bar{S}_x$ is the number of mid-year population at age x.

Between the age-specific deaths rates and the intensity of mortality is true the relationship:

$$m_x \doteq \mu(x + 0{,}5) \tag{2}$$

where μ $(x + 0{,}5)$ is the intensity of mortality at age $(x + 0{,}5)$.

This relationship is especially true for younger people (except the first year of life – age 0). For older people we will have problem with a small number of deaths and living persons, and therefore it is advisable to model their intensity of mortality or the age-specific deaths rates by suitable function. To do this, we can use several existing analytic functions (see Fiala, 2005).

One of the oldest functions (but still used very often) is the Gompertz-Makeham function (G-M). It includes two types of mortality. The first one is dependent on the age and the second one is not affected by age. This function is built on the assumption of a constant rate of increasing in mortality with an increasing age. But this assumption is not true for older people (assumption about the constant rate of increasing in mortality with an increasing age is correct approximately till 85 years). Therefore, this function overestimates the probability of dying (at the highest ages), and at the same time it underestimates values of life expectancy at these ages (see Dotlačilová et al., 2014 or Gavrilov and Gavrilova, 2001).

As the other one could be used the logistic function (in this article will be presented Kannisto (K) and Thatcher model (T)). In these models is expected slower growth in the probability of dying. It also means that they provide the highest values of life expectancy (they belong among the most optimistic ones). Furthermore, they are suitable for using for older people (see Thatcher at al., 1998).

Heligman-Pollard model (H-P) also belongs to the group of logistic models. And its values of life expectancy are closer to other two logistic functions. So it belongs among the most optimistic ones (see Thatcher at al., 1998).

An individual group forms the Coale-Kisker model (C-K), which is focused on the changes in age-specific deaths rates at two consecutive ages. This model essentially corresponds to an exponential quadratic function and it provides a rather lower values of LE (see Thatcher at al., 1998).

At comparing the values of LE gained from the using of particular models, we find that they are different. But here it is important to realize that some of these models are suitable for smoothing of mortality curve from 60 to 85 years and some of them for the ages greater than 80 years.

More detailed overview of each model is presented in appendix 1.

These models can be divided into three groups (on the basis of achieved values of the probability of dying). The first group includes models in which the probability of dying reaches values close to one at relatively low age (approximately around 105 years). In the second group are models for which the probability of dying achieved one rather the limit value - even at the age 120 years is not reached 1). The third one includes models that achieve value of probability of dying about 0.6, even at high age (well over 120 years old).

Another part of this paper will be about the presentation of the methodology, which is used for the calculating of mortality tables by the Czech Statistical Office (CZSO). For the smoothing of age-specific deaths rate it is necessary to combine several methods. For lower ages are used moving averages of 3 values, 9 or 19 values. At higher age is used the analytic function - G-M function (or its modification). Parameters of the G-M function and modified G-M function were estimated by using the procedure in MS Excel Solver (more detailed overview of the formulas is in the Appendix 2). For calculating of the final smoothing is necessary to combine all mentioned methods. The smooth transition from one type of smoothing to another was secured by weighted averages (for more details see Fiala, 2005).

In the last part of our contribution we will focus on the calculation of complete life tables. It was carried out using known formulas (more detailed overview for this formulas e.g. Fiala, 2005).

3. Results

For the analyzing of the evolution of the mortality was used data about the Czech population from 1920 to 2012 (for more details see ČSÚ, 2013 or Human mortality database, 2014). Different models, which are used for smoothing and extrapolation of mortality curve will be applied. For completeness, the obtained outputs are complemented by the values of life expectancy gained from the complete mortality tables and values of life expectancy obtained from the CZSO mortality tables.

In the Figure 1 is shown the evolution of LE of 80-years-old males in the Czech Republic obtained by using the different methods. It is obvious that the highest values are achieved for K model (except for the period 1953-1960 – here the most optimistic is C-K model), for the lowest values can't be made clear conclusion. The lowest values are obtained for these models: H-P, G-M.

For females (see Figure 2), can be made a similar conclusion, but trend of LE is not so clear. The highest values are achieved even for C-K model.

The greatest fluctuation in the values of LE of 80-year-old males and females experience from 1920 and around the time of the 2nd World War[1]. A further significant decrease occurred in the early 80s.

[1] Break in data series due to German occupation of Bohemia, separate statistics of German and Czech population, transfer of German population to Germany after the war etc.

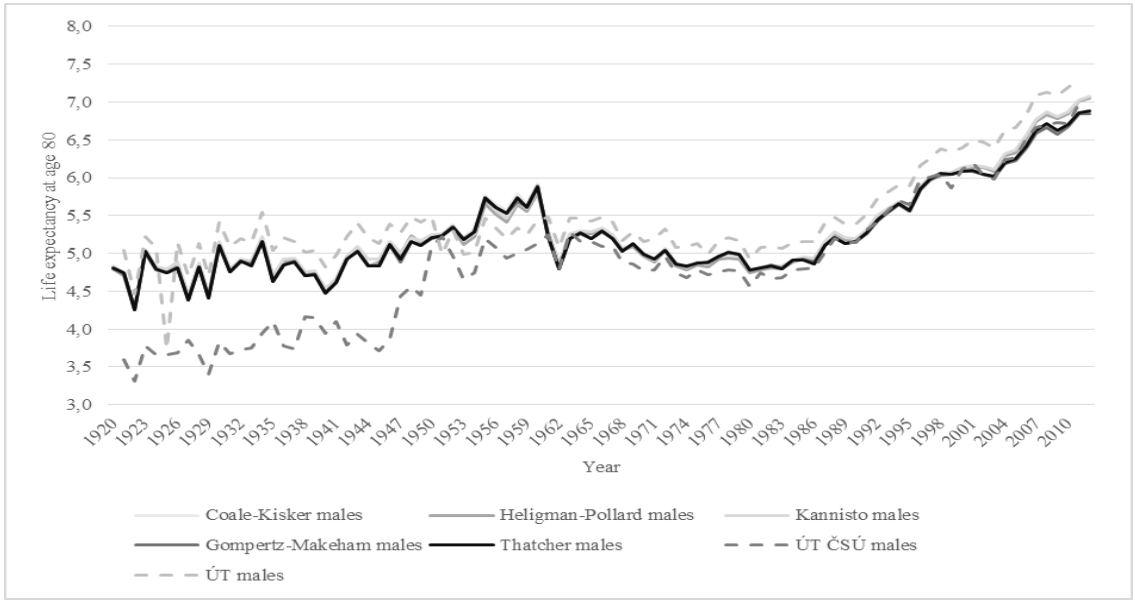

Figure 1 - The values of life expectancy of 80 years old males obtained by using different methods of smoothing
Source: data: ČSÚ (2013), authors' calculations (Burcin et al., 2012)

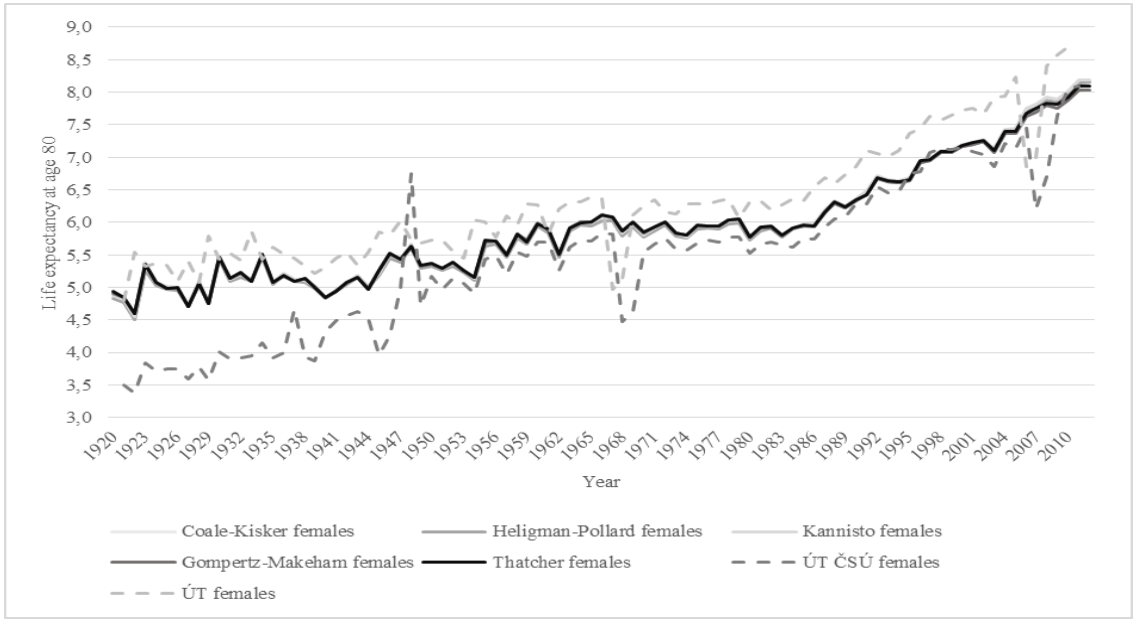

Figure 2 - The values of life expectancy of 80 years old females obtained by using different methods of smoothing
Source: data: ČSÚ (2013), authors' calculations (Burcin et al., 2012)

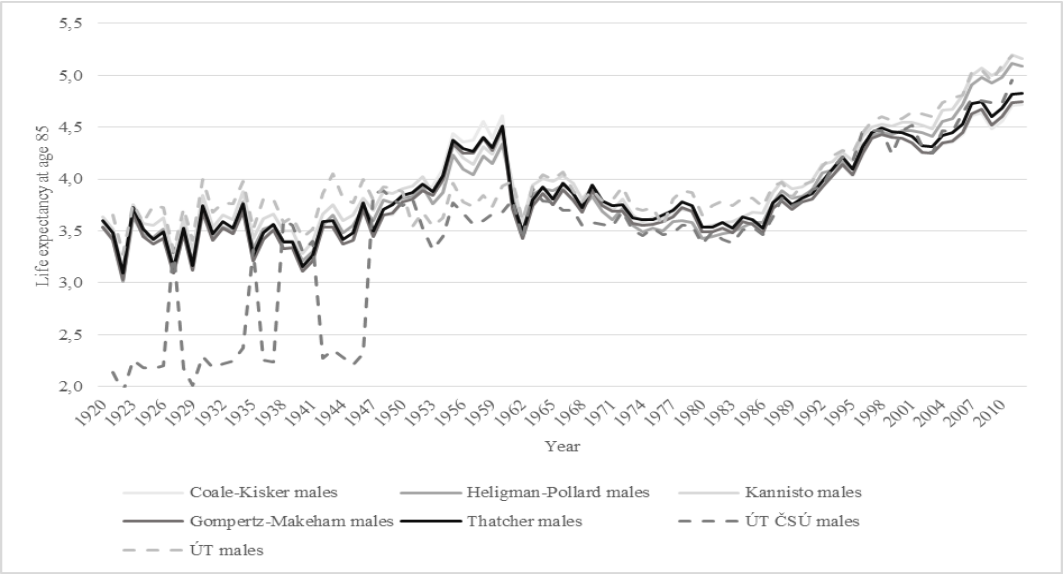

Figure 3 - The values of life expectancy 85 years old males obtained by using different methods of smoothing
Source: data: ČSÚ (2013), authors' calculations (Burcin et al., 2012)

In the Figure 3 is shown the evolution of LE of 85-years-old males in the Czech Republic. It is obvious that the biggest fluctuations occur mainly around and during the 2nd World War (or before this time).

If we examine the values obtained for different models, we find out that the lowest values are obtained by the G-M function or H-P model. On the other hand the highest values provide K or T model. For females, we come to the same conclusion for the lowest values, but the highest values are provided by K, T or C-K model.

The fourth figure shows the evolution of 85 years old females. Here we come to the same conclusion for the lowest values, but the highest values are provided by K, T or C-K model.

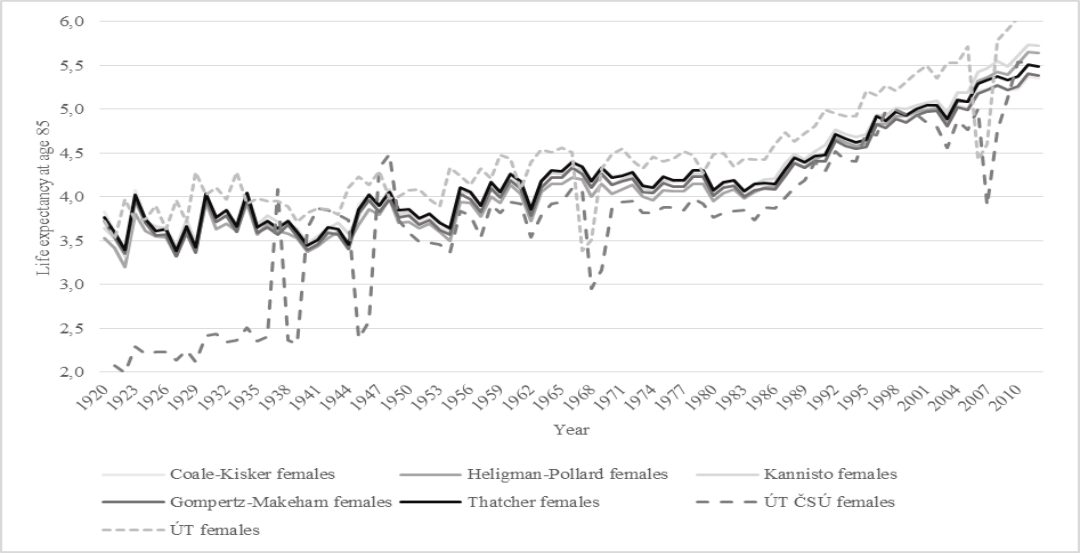

Figure 4 - The values of life expectancy 85 years old females obtained by using different methods of smoothing
Source: data: ČSÚ (2013), authors' calculations (Burcin et al., 2012)

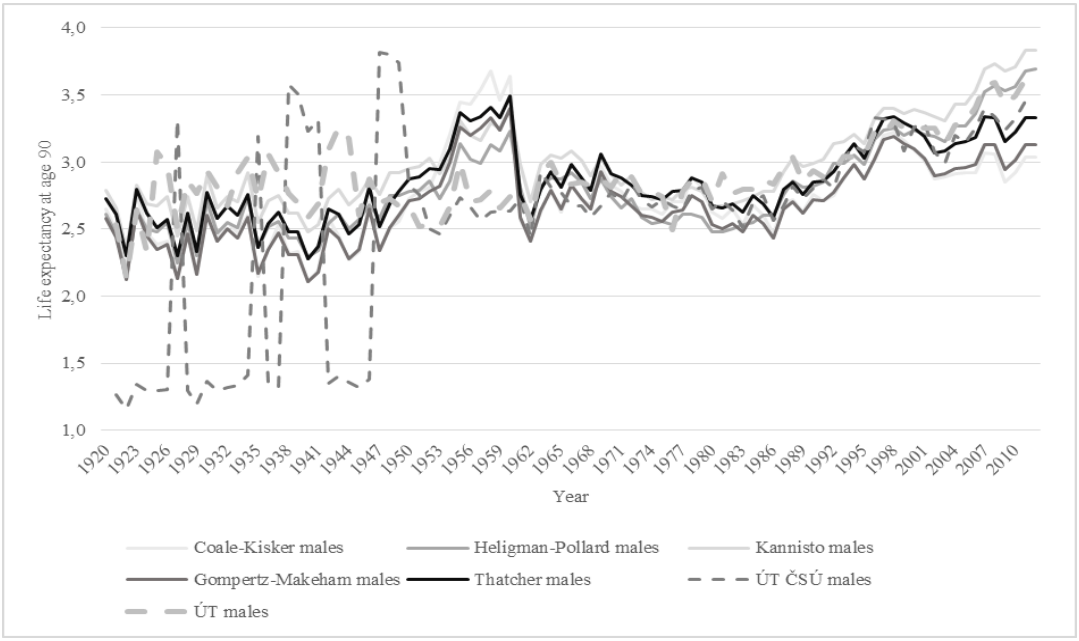

Figure 5 - The values of life expectancy 90 years old males obtained by using different methods of smoothing
Data: ČSÚ (2013), authors' calculations (Burcin et al., 2012)

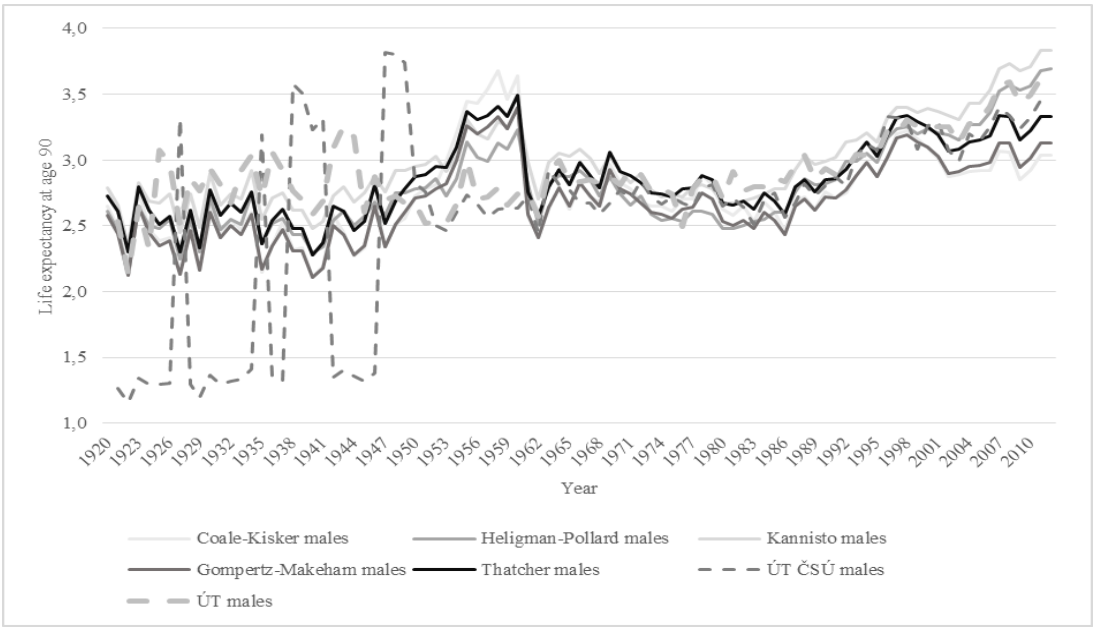

Figure 6 - The values of life expectancy 90 years old females obtained by using different methods of smoothing
Data: ČSÚ (2013), authors' calculations (Burcin et al., 2012)

The fifth and sixth figure shows the evolution of LE of 90-years-old males and females. Until the highest ages the highest values of LE as for males and as for females are obtained by K, T or C-K model. On the other hand the lowest values are obtained from the G-M function or H-P model.

It is also clear that the most significant fluctuations occur mainly around and during the 2nd World War (or before this time). Reasons for this evolution are the same like for the previous figures.

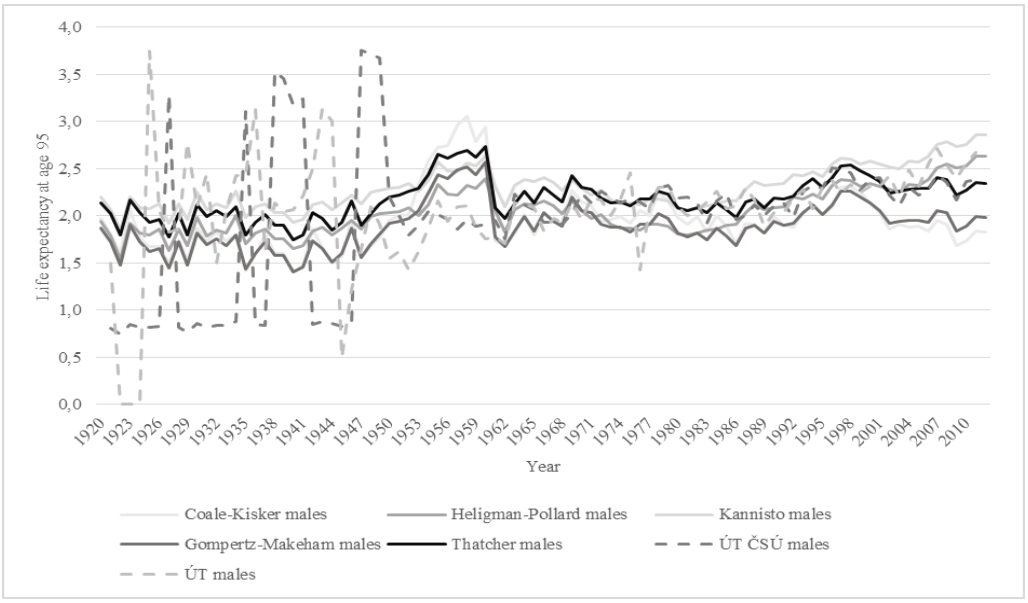

Figure 7 - The values of life expectancy 95 years old males obtained by using different ways of smoothing
Source: data: ČSÚ (2013), authors' calculations (Burcin et al., 2012)

In the Figures 7 and 8 is shown the evolution of males' and females' LE for at age 95 years. As in the previous ages, it is also clear here that the optimistic models are K or T model (for both sexes: males and females). On the contrary, we can include among the most pessimistic models G-M function, C-K or H-P model.
If we compare each graph, we can see that with increasing age there is a greater fluctuation. This is due to the small numbers of death and small number of living persons at the highest ages.

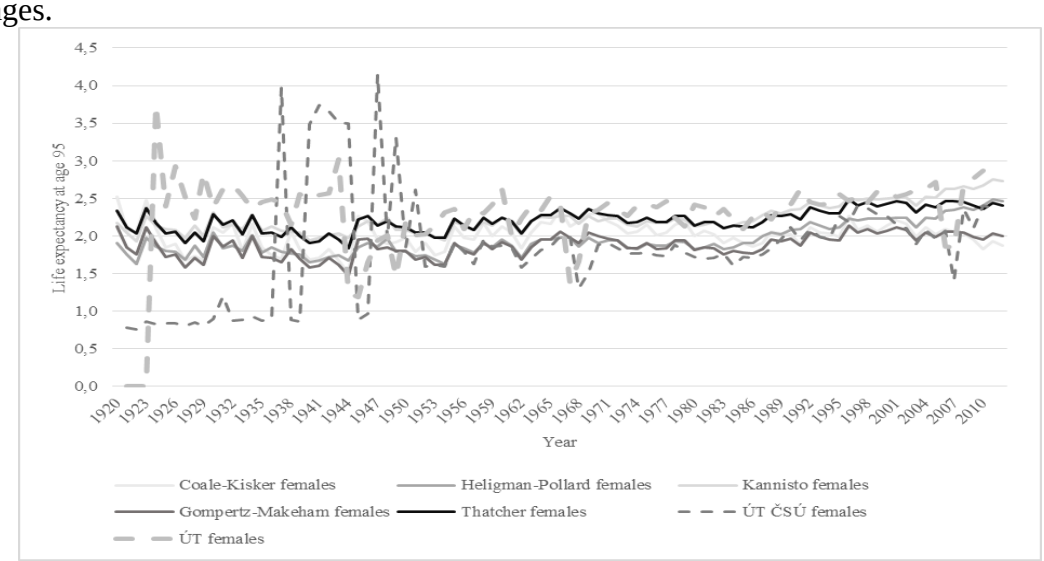

Figure 8 - The values of life expectancy 95 years old females obtained by using different ways of smoothing
Source: data: ČSÚ (2013), authors' calculations (Burcin et al., 2012)

4. Conclusion

From the obtained results it is clear that there are big fluctuations in the values of life expectancy at the highest ages. This is due to the systematic and random errors.

Significant fluctuations occurred (for males and for females) during and around the 2nd World War.

If we analyze the values of the life expectancy which are obtained after the application of selected methods of smoothing with regard to the individual ages we come at all ages to similar conclusions. This is mainly due to the nature character of each function. Application of K model give relatively low values of probabilities of dying that are below 1 around the age of 120 years (probability of dying in this case varies only about 0.6). This evolution means that we get the highest values of life expectancy from this model and basically for all ages. In contrast, the pessimistic model is for example the G-M function. This is due to the fact that here the probability of dying achieve one already around 105 years. Higher probabilities of dying cause lower values of the life expectancy for the oldest people.

If we examine the evolution of the probability of dying obtained after the application of various types of smoothing of mortality curve and compare them with empiric values we find out that in the most optimistic models the probability of dying is underestimated primarily at lower ages. On the other hand for the pessimistic models is true, that they lead to the underestimation of the probability of dying. This also means that they are suitable for smoothing at the lower ages.

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Appendix 1

Name of the model	Model	Typ of model	Parameters of model
Gompertz-Makeham function	$\mu_x = a + b.c^x$	exponencial	a, b, c
Heligman-Pollard model	$q_x = \frac{b.e^{a.x}}{1+b.e^{a.x}}$	logistic	a, b
Thatcher model	$\mu_x = \gamma + \frac{z}{1+z}$, kde $z = \alpha.e^{\beta.x}$	logistic	α, β, γ
Kannisto model	$\mu_x = \frac{\alpha.e^{\beta.x}}{1+\alpha.e^{\beta.x}}$	logistic	α, β
Coale-Kisker model	$m_x = e^{a.x^2+b.x+c}$	linear	a, b, c

(see Thatcher et al., 1998, Gompertz, 1825)

Appendix 2

Types of smoothing	Formula	Age range
Moving average from 3 values	$\tilde{m}^{(3)} = \frac{m_{x-1} + m_x + m_{x+1}}{x}$	<3, *z* - 1>
Moving average from 9 values	$\tilde{m}^{(9)} = 0{,}2.m_x + 0{,}16.(m_{x-1} + m_{x+1}) + 0{,}12.(m_{x-2} + m_{x+2}) + 0{,}08.(m_{x-3} + m_{x+3}) + 0{,}04.(m_{x-4} + m_{x+4})$	<6, *z* - 4>
Moving average from 19 values (a)	$\tilde{m}^{(19)} = 0{,}3333.m_x + 0{,}2963.(m_{x-1} + m_{x+1}) + 0{,}0741.(m_{x-2} + m_{x+2}) - 0{,}037.(m_{x-4} + m_{x+4})$	<6, 29>
Moving average from 19 values (b)	$\tilde{m}^{(19)} = 0{,}2.m_x + 0{,}1824.(m_{x-1} + m_{x+1}) + 0{,}1392.(m_{x-2} + m_{x+2}) + 0{,}0848.(m_{x-3} + m_{x+3}) + 0{,}0336.(m_{x-4} + m_{x+4}) - 0{,}0128.(m_{x-6} + m_{x+6}) - 0{,}0144.(m_{x-7} + m_{x+7}) - 0{,}0096.(m_{x-8} + m_{x+8}) - 0{,}0032.(m_{x-9} + m_{x+9})$	<30, *z* - 9>
Gompertz-Makeham function	$\mu_x = a + b.c^x$	<60;82>
modified Gompertz-Makeham function	$\mu_x = a + b.c^{x_0 + \frac{1}{d}.\ln[(x-x_0).d+1]}$	<83;110>

where x is age, x_0 is age, from which the smoothing is realized according to modified G-M function and z is age, for which is known the correct value of age-specific mortality rate.
(see Fiala, 2005, Koschin, 2000 or Šimpach, 2012)

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