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THE N-OPTIMALITY AND INCONSISTENCY OF KNOWLEDGE POSTULATES OF CONSENSUS DETERMINING IN DECISION SUPPORT SYSTEMS[1]

Abstract: This article presents the problem of consensus determining postulates defined in decision support systems. The consensus determining methods and the general postulates for consensus estimation and their characteristics are presented in the first part. Next, the definitions of new postulates pertaining to decisions support systems were elaborated, and the possibility of their use in practical solutions is presented. The consensus function granting these postulates is also characterized. The application of these postulates, as a consequence, can lead to the process of making decisions will be more flexible, and the risk involved in decisions will be significantly reduced.

Keywords: decision support systems, postulates of consensus, knowledge conflicts.

1. Introduction

Making decisions in economic matters has become a key component of any business activity. Problems in this area are typically associated with the highly volatile character of the market [Jajuga 2007]. Decisions must be made virtually in real time, since only prompt and accurate reaction to changing market conditions provides tangible benefits, for example a high return rate. Another important determinant is the high level of risk involved in economic decisions. Since the analysis of information and drawing valid conclusions is a time-consuming process, and since real-time computing is beyond human processing capabilities, the process of making decisions is typically supported by computer software, employing a range of computing methods such as artificial intelligence systems, capable of identifying relevant information and drawing conclusions based on input data. These systems integrate information that may be useful for decision-making purposes and provide mechanisms for fast and effective problem solving. Typical applications include expert systems

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used by banks for credit standing evaluation, neural network systems for real-time evaluation and the identification of economic phenomena and decision support systems using fuzzy logic to analyse non-binary data. Another important area of development in recent years is the use of agent and multi-agent systems [Korczak, Lipiński 2008; Soto et al. 2009] – these, unlike other AI systems, offer the capability of unaided operation and unaided decision-making, i.e. without user input and irrespective of any external factors.

At present, decision support systems (DSS) are typically distributed, i.e. integrating a set of individual computing machines over a telecommunication network (usually the Internet), with the help of distributed system applications [Coulouris et al. 1998]. These systems offer the potential of the fast processing of large amounts of data. However, in most cases, the distributed systems used for the support of economic decision-making processes tend to generate multiple variants of solutions which may result in knowledge conflict within the system. For example, in multi-agent systems each individual agent may utilize a different method of decision support and, consequently, arrive at a different solution. Users expect a unified variant – or to put it in simple terms, they want a single decision. The decision-making process is followed by implementation and only one decision can be implemented at any given time – ideally one that will bring a tangible benefit to the user while simultaneously limiting the level of risk involved. If a decision support system generates multiple variants, users face the problem of selecting the best possible variant – the ultimate decision. Since the task of selecting the best possible variant, as was already mentioned, should ideally be realized in (or close to) real time, it is expected that the DSS will automatically present a single variant that offers the best possible results for the user, thus solving the knowledge conflict. Professional literature presents a wealth of methods that can be used to this effect, such as negotiation methods [Dyk, Lenar 2006] and deductive computing methods [Barthlemy, Janowitz 1991]. Negotiation methods allow for determining a solution that best suits all parties involved, based on compromise, but it is burdened with the problem of the mass exchange of information between system components which makes the postulate of real-time computing particularly difficult to achieve – or even impossible. On the other hand, deductive methods of computing (such as those based on game theory, classical mechanics and selection methods) offer high computing power, but do not easily satisfy the requirement of identifying the best possible variant with simultaneously limiting the risk of inadequate selection.

It seems that the above inconveniences (and the resulting knowledge conflict) can be resolved with the use of consensus methods [Sobieska-Karpińska 2012; Hernes 2011]. Consensus methods offer the benefit of determining a single best variant (or a single decision, in this context) out of multiple possible variants. It must be noted that the single decision determined using consensus methods will not necessarily belong to the domain of the variants generated by the system in the first place. This is because consensus methods take into consideration all conflicting parties and interests. Each party is represented in the system, each input is taken into account, and the ultimate decision is generated so that each party 'loses' as little as possible in the process. The ultimate decision is endorsed by all modules (parties) and the decision represents the interests of all parties to a degree that satisfies all conflicting parties. Therefore consensus methods offer real-time solutions, a good compromise at a reduced risk level and, consequently, the selection of a decision that brings tangible benefit to the user. [Sobieska-Karpińska, Hernes 2012] argue that using consensus methods for the purpose of identifying and presenting a target solution to the user will offer a reduction of decision-making time, since users are not burdened with the task of analysing and selecting the best possible variant. It also reduces the risk involved in the process, since the variants identified and selected by the user may fail to bring the expected benefit, or even result in a loss.

It must be noted, however, that the consensus algorithms used in DSS must satisfy certain consensus postulates. These postulates represent conditions to be met by consensus-calculating functions. Only the proper definition of these conditions will ensure that decisions made with the help of consensus algorithms will bring tangible benefit to the user.

Professional literature (see, e.g. [Barthlemy, Janowitz 1991; McMoris et al. 2000; Nguyen 2002]) does provide some general (universal) postulates for consensus estimation, but these assumptions fail to take into account some important aspects of decision-making, such as the risk and uncertainty involved. Therefore it seems necessary to broaden the list of postulated parameters.

The purpose of this paper is to present the general postulates for consensus estimation (that need to be included, regardless of the problem they are meant to address via consensus calculation) and their characteristics, as well as to suggest new postulates pertaining to economic decisions and present the consensus functions granting these postulates. This will allow for the more accurate construction of consensus algorithms and, consequently, the development of IT solutions able to calculate consensus results automatically, based on a set of solutions generated by the system. In this approach, the system will present the user with one ultimate decision that may be implemented to the best effect [Sobieska-Karpińska, Hernes 2013]. Consequently, the process of making economic decisions will be more flexible, since the system will suggest the most appropriate solution in (or close to) real-time. In addition, the risk involved in decisions will be significantly reduced, since users will not be able to select manually a decision that may be burdened with such risk at implementation phase.

2. General postulates of consensus determining

The purpose of introducing the postulates is the determination on their basis the classes of the functions of consensus or, in other words, different methods of consensus determining.

In addition, because the postulates are conditions which are expected to meet on a consensus function, one can make it justify the use of these functions in practice.

In the later part of the article we will use the following symbols:

$\Gamma(U)$ − set of all not empty subsets of universe U (e.g. set of objects-elements of decision, such as credit's features, financial instruments),

$\Gamma'(U)$ − set of all not empty subsets with repetitions of universe U,

$\cup'$ – sum of set with repetitions.

Let $X, X_1, X_2 \in \Gamma'(U), x \in U$. In the later part of article we will use the following parameters:

$o(x,X) = \Sigma_{y\in X}\, o(x,y)$,

$o^n(x,X) = \Sigma_{y\in X} [\, o(x,y)]^n$ for $n\in$N.

Let us notice that parameter $o(x,X)$ represents the sum of the distance from element x that belongs to universe U for elements of profiles X, but the largeness $o^n(x,X)$ represents the sum of n-powers of its distance. This value can be interpreted as the measure of evenness of the distance from element x for elements of profiles (e.g. set of decisions) X. if value n is greatest memorial then n, the distances are more even.

In the work [Sobieska-Karpińska, Hernes 2011] consensus function is defined as follows:

Definition 1

Consensus function at space (U,o) we call optional functions of forms:

$$c : \Gamma'(U) \rightarrow \Gamma(U)\,. \tag{1}$$

For profile $X\in\Gamma'(U)$ each of the elements set $c(X)$ we call its *consensus,* however all the set $c(X)$ we call *representation of profile X.* Let C be the set of all consensus functions in a space (U, o).

Using the overall function of the consensus, one can then define the more detailed class consensus functions, however in order to define these classes of functions, it can use the postulates for consensus defined as follows (on the basis of [Barthlemy, Janowitz 1991; McMorris et al. 2000; Nguyen 2002]):

Definition 2

Let X be optional profile, then we can say that consensus function $c\in C$ grants postulate:

1. Reliability (*Re*), *if*

$$C(X) \neq \varnothing \tag{2}$$

2. Consistency (*Co*)*, if*

$$(x\in C(x)) \Rightarrow (x\in c(X\cup' \{x\})) \tag{3}$$

3. Quasi-unanimous (*Qu*), *if*

$$(x\notin C(x)) \Rightarrow ((\exists n\in \mathrm{N})\; x\in c\; (X\cup' \{n{*}x\})) \tag{4}$$

4. Proportional (*Pr*), *if*

$$(X_1 \subseteq X_2 \wedge x \in c(X_1) \wedge y \in c(X_2)) \Rightarrow (o(x,X_1) \leq o(y,X_2)) \quad (5)$$

5. 1-Optimality (O_1), *if*

$$(x \in C(x)) \Rightarrow (o(x,X) = \min_{y \in U} o(y,X)) \quad (6)$$

6. 2-Optimality (O_2), *if*

$$(x \in C(x)) \Rightarrow (o^2(x,X) = \min_{y \in U} o^2(y,X)). \quad (7)$$

These postulates for the function of consensus express the primary condition defining different method consensus. The first postulate (*reliability*), sets up that it is always possible to appoint a consensus for each profile. It answers the optimistic attitude that every conflict has a solution. Reliability is the known criterion in the theory of choice [Daniłowicz, Nguyen 1992].

The postulate *consistency* requires the implementation of the condition that if some element x is a consensus for profile X, then after expansion this profile about x ($X \cup$' $\{x\}$), this element should be the consensus for the new profile. Consistency is an important property of consensus, because it allows users to forecast the behavior of the rule of appointment of a consensus, when premise of independent choices are joined.

According to postulate *quasi-unanimous,* if a certain element x is not a consensus for profile X, then it will be a consensus for profile X^1 inclusive of X and n protrude element x for certain n. In other words, each of the elements of universe U should be chosen as a consensus for such a profile, if the number of its pronouncement is sufficiently large.

The Proportionality postulate is a quite natural property, because if the profile is the greatest memorial then the difference between its elements and the consensus is greatest.

The last two postulates are very particular. The first of them, postulate *1-Optimality* requires that the consensus is nearest (most similar) to the elements of the profile. This postulate, very well –known in literature, defines concrete function classes called medians. Postulate *2-Optimality*, on the other hand, requires that the sumof the square of distance from the consensus for elements of profiles was the smallest. The cause of introduction of this postulate results from (the also very natural) following condition concerning the determination function consensus: the consensus has to be ”fair”, this means that its distance for elements of profiles should be the most even. Let us notice that the number $o^n(x,X)$ defined earlier, can be treated as a measure of evenness of the distance between certain object x and elements of profiles X. Therefore, the above-mentioned condition requires that, in order to value o^n (consensus, X) is minimal. As work [6, 7] shows functions granting postulate *2-optimality* are better than the function granting postulate *1-optimality*, by reasons of greatest evenness, but they differ from the other function of the consensus' greatest similarity for elements of profiles. From this results that postulate *2-optimality* is a good criterion of the appointment of a consensus.

Let us note that the first three postulates, namely *Re*, *Co* and *Qu*, are independent of the structure of universe U, represented by distance function o (used to establish the consensus function class in methods based on Boolean reasoning), while the last three postulates (*Pr*, O_1 and O_2) are formulated on the basis of o function (these postulates are employed in optimization methods). Postulates *Re*, *Co* and *Qu* are also used in cases when the distance function (or in more general terms – the macro-structure) for universe *U* cannot be specified. For economic decisions, the function of distance can always be reliably defined, therefore all postulates can be employed allowing for the use of both constructive and optimization methods of consensus estimation.

The above general postulates of consensus estimation, as already mentioned, are not sufficient for economic purposes. For this reason the author puts forward two postulates to supplement the above list: the n-optimality and the inconsistency of knowledge postulates of consensus determining.

3. The n-optimality and the inconsistency of knowledge postulates

A good approach in estimating the best possible decisions in economic matters, i.e. when dealing with problems typically burdened with risk and uncertainty, is to employ an evenly distributed consensus – that is, one that takes into account all possible solutions, with each solution estimated at equal measure. This helps to minimize the risk of the ultimate decision, since the potential of putting more weight to an incorrect decision is eliminated. Therefore, if *2-Optimality* postulate offers more even distribution that *1-Optimality* postulate, then a postulate of *n-Optimality* should be defined, to offer even smoother distribution than *2-Optimality* for $n>2$. Consequently, a definition for such a new postulate will take the following form:

Definition 3

The consensus function $c \in C$ *grants an n-Optimality postulate* (O_n), *if*

$$(x \in C(x)) \Rightarrow (o^n(x, X) = \min_{y \in U} o^n(y, X)). \tag{8}$$

This postulate is a generalization of postulates *1-Optimality* and *2-Optimality*.

Another extended postulate on consensus determining in economic decision support systems is the inconsistency of the knowledge postulate:

Definition 4

Consensus function $c \in C$ *grants an inconsistency of knowledge postulate (Uk), if*

$$(x \in C(x)) \Rightarrow (o^n(x, X) > \min_{y \in U} o^n(\{X \backslash y\}, X)). \tag{9}$$

The above postulate allows for the determination of an element for which the distance to consensus is larger than the sum of consensus distances of all the remaining elements (in other words, one of the profile elements is markedly more distant from the consensus than the others). Such a situation may result from inadequate knowledge on the part of one of the conflicting parties (for example, a software agent). If this is the case, then decisions generated by the defaulting party should not be taken into account. This problem may be solved by adopting a multi-stage process of consensus estimation, as suggested in [Sobieska-Karpińska, Hernes 2012]. Such a method is implemented in A-Trader multi-agent system for FOREX platform [Korczak et al. 2012]. These systems consist of a large number (hundreds) of processing agents, which on the basis of the FOREX signals, take the specified decision to buy/sell. The paper [Korczak et al. 2013] presents using the consensus methods to reduce the level of the investment risk, as a strategy of Supervisor Agent in a-Trader System.

Building consensus algorithms on the above postulates is not necessarily a guarantee of success in arriving at the best possible solution. For example, the algorithms may find a consensus solution for which a given element of the decision-making process is at the same time adopted and rejected, leading to a contradiction. Some authors also take into account the profile's consensus susceptibility [Hernes, Sobieska-Karpińska 2009]. If a profile (a set of decisions) is not prone to consensus, methods of satisfying its susceptibility may be adopted, such as the inclusion of decisions generated by new parties (e.g. new software agents).

However, it must be noted that consensus estimation functions used for the purpose of supporting economic decisions must meet both general consensus postulates and the expanded postulates (these postulates may also be used for other than only economic problems). Otherwise the estimated consensus may not warrant tangible benefits to the user, for example – by placing more weight on an inappropriate decision contained in the profile.

4. An example of consensus function

On the basis of the postulates elaborated in a previous part of article, the consensus function granting particularly *n-optimality* and *inconsistency of knowledge* postulates, can be defined. This function also specifies the criteria for determining uniformity of consensus. These criteria determine the extent to which consensus is similar to the opinion of the agents. If consensus is very similar to the opinion of one of the agents, and in a small degree similar to the opinion of the other agents, then the consensus is non-uniformity. However, if the consensus is equally similar to the opinions of all agents, then it is very uniform.

The function granting *n-optimality* and *inconsistency of knowledge* postulates and specifying the criteria for determining the uniformity of consensus is function $C_{n,}$ where $n = 1, 2,....,$ denoted as:

$$C_n : 2^{\Gamma'(U)} \rightarrow \Gamma'(U)\,. \tag{10}$$

Given a profile (set of agents decisions) $A \in 2^{\Gamma'(U)}$ and function:

$$o^n(x,A) = \Sigma_{y\hat{I}X}[\,o(x,y)]^n, \tag{11}$$

For $n = 1, 2, \ldots,$ $x \in U, y \in A,$.
Definition of C_n function is the following:

Definition 5

$$e \in C_n(A) \leftrightarrow o(x,A) = \min_{y\ \in \Gamma'(U)} o^n(y,A)\,. \tag{12}$$

A very important problem is the determination for which value of *n* consensus algorithms will be elaborated. Because in the work [Hernes, Nguyen 2007] it was found that above a certain value of *n,* a consensus will have the same character, then often the consensus is determined for *n* = 1 and *n* = 2. For larger *n,* the consensus algorithms may be elaborate, but they are very complex, so in practice it is hard to use them because they require more CPU processing power. This situation does not guarantee the required speed of the taken decision. For *n* = 1 the consensus should be the nearest to the profile elements. For *n* = 2 the distances among consensus and profile elements have a larger sum but they are more uniform. The decision as to which criterion is better should be based on the characteristics of the decision which will be made. If one needs to make a decision similar to decisions designated by one of the agents and, at the same time, take into account decisions designated by other agents, then one should use *n* = 1. However, if one needs to make a decision, which uniformly takes into consideration decisions designated by all of the agents, which must not be a decision very close to one of these agents', then one should use *n* = 2. The level of risk associated with the decision-making process is also decreased, consequently.

5. Conclusion

Making decisions in economic matters is a complicated process, particularly in the face of high risk and uncertainty associated with this form of activity, since it may lead to unpredictable results. Improper decisions may be detrimental to the functioning of a whole organization. Distributed systems offering support for economic decisions are a viable solution, provided that they are able to generate a single, reliable recommendation. However, if individual nodes of the system (such as software agents) generate multiple instances of solutions, the overall reliability of the system is considerably lower. Therefore proper care should be taken to ensure that the user receives the best possible solution generated automatically by the system, so that he or she can make a correct decision that will benefit the organization. The use of consensus methods provides the potential of arriving at a single best decision – one that does not necessarily belong to the original domain of decisions generated by individual nodes, but suitably similar. Consequently, the level of risk involved is considerably lower. If users were to perform their own analyses and manually select from decisions generated by the system under time pressure, their choices would be potentially burdened with error – the more so if we take into account the time pressure involved. In addition, consensus methods allow for a considerable reduction of decision time, since the system presents the user with a single best solution determined automatically on the basis of variants generated by individual nodes.

For obvious reasons, consensus methods do not warrant absolute accuracy of the resultant decision, but they do warrant some degree of satisfaction. Some of the individual variants generated by system nodes may prove more appropriate than the automatic suggestion determined using consensus methods, but one can never be certain that the user would have selected the best variant (if he/she were to analyse and select it manually). In such cases, selecting the worst possible variant is also possible, which can only increase the risk involved.

However, the correct function for consensus estimation should incorporate and take into account all the postulates presented above, particularly the *n-optimality* and *inconsistency of knowledge* postulates, as the function presented in this article. Negligence in this respect may result in ill-advised suggestions with negative consequences.

The proper implementation of consensus postulates is, therefore, a prerequisite for the correct design of consensus algorithms to be used in DSS.

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POSTULATY N-OPTYMALNOŚCI ORAZ NIEZGODNOŚCI WIEDZY W WYZNACZANIU CONSENSUSU W SYSTEMACH WSPOMAGANIA DECYZJI

Streszczenie: W artykule omówiono problem zdefiniowania dodatkowych postulatów wyznaczania consensusu w systemach wspomagania decyzji. W pierwszej części scharakteryzowano stosowane dotychczas metody i postulaty wyznaczania consensusu. Następnie opracowano nowe postulaty, odnoszące się w szczególności do systemów wspomagania decyzji, a także zaprezentowano możliwości ich wykorzystania w rozwiązaniach praktycznych. Przedstawiono również funkcję consensusu spełniającą te postulaty. Wykorzystanie opracowanych postulatów może w konsekwencji mieć pozytywny wpływ na elastyczność procesu podejmowania decyzji, jak również ograniczyć poziom ryzyka związanego z tym procesem.

Słowa kluczowe: systemy wspomagania decyzji, postulaty wyznaczania consensusu, konflikty wiedzy.