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**MODELING DEMAND FORECAST VARIANCE  
IN A DISTRIBUTED SUPPLY CHAIN NETWORK USING  
GENERALIZED STOCHASTIC PETRI NETS**

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**Abstract:** There is a trade-off in Supply Chain Management Systems between efficiency and demand variability. When no variation occurs in consumer need, order cycle, product portfolios, and in distribution lead time, then the supply chain would be just a routine business process. Unfortunately, in practice this is not often the case. Thus, ranking demand variability is one of the prime challenges to reduce safety stock without affecting customer demand. This paper studies supply chain demand variability with multiple suppliers, manufacturers, distributors, wholesalers, retailers, and customers as tiers, and each stage as an echelon that faces stochastic demand volatility. A Generalized Stochastic Petri-Net (GSPN) model is proposed in a distributed scenario to synchronize the response capabilities among the players in the chain, and to lower down the supplier demand variance with scheduled ordering policies. Maintaining a uniform inventory stock throughout the chain has two main effects: the bullwhip effect (BWE) will be negligible, and uncertainty in decision making at each echelon will be reduced substantially.

**Keywords:** Supply Chain Management, Bullwhip Effect, Generalized Stochastic Petri Nets, Retail Network.

## 1. Introduction

Demand Variability is the difference between what we insist on and what actually happens. It may be a statistical distribution of happenings expected from a process. Variability in any form will obstruct the supply chain efficiency. The Bullwhip Effect (BWE) is one such critical disadvantage for SCM performance. Sustainability for

small and medium size enterprises becomes an issue when the measure of BWE is on the higher side. A single stage and single tier chain with lone customer is the simple form of SCM without any BWE as cases of collaborative chains like food, health and beauty products, garments, etc., where the success of the chain depends on customer pleasure and contentment.

The primary objective of this research work is to investigate the reasons of BWE at every stage of Supply Chain (SC) performance and measure it by accessing profit, average fill rate, response time, and capacity utilization. For example, the average fill rate can be improvised by carrying excess inventory to guard against the stock out situations. However, the SC operations must make a balance between inventory cost and lost profits due to stock outs. Some of such common causes of BWE are communication gap, information twisting and gaming and the corresponding effects are inconsistent inventory, fund obstruction, loss of income and incompetent customer services [Hau et al. 1997].

Increased demand variability has multiple shocks on SC performance. It increases inventory variability and the corresponding safety stocks. Lead times will also go up due to the increased order variability, which emphasizes the increase in safety stocks [Anderson, Morrice 2000]. In this paper we have studied the autocorrelation in demand, rather than its variability. Due to demand variability and the existence of BWE, prompt decision making at each echelon of SCM is difficult to workout. The first quantitative measure of BWE was evaluated using the BWE index [Chen et al. 2000]. However, for a realistic demand order processing and monitoring of delivery logistics, a large distributed multi-dimensional real time data repository is necessary [Chaki, Nabendu, Sarkar 2010]. The present work provides experimental evidence of BWE due to demand variability using a simulation technique. The impact of BWE is examined using a generalized stochastic Petri Net model augmented with data marts at each echelon and a central warehouse for the entire SCM operations. Routine transaction management activities, like ordering fresh stock or the issuance of material by a stakeholder in a SC, are always dependent either on its successor or its predecessor. Hence, if a stake holder has the option to observe the stock and demand status of the entire chain or at a specific point of interest, then one can exercise better judgment in placing the order. This would also help with reducing the lead time to maintain minimum safety stock.

Our objective is to analyze the cause and effect relationship throughout the chain in a transparent way such that there should be a free flow of information, irrespective of the stakeholder location and place. Besides, functionalities like ordering a product or monitoring the stock at a particular echelon or over the entire chain are time dependent activities. In order to analyze the dynamics in SCM and such non definitive nature of activities, an appropriate dynamic model for the system would be useful. In this paper, we first proposed the Generalized Stochastic Petri Nets (GSPN) model for SCM in section 4 of the paper. In section 5, the proposed GSPN model has been revised to control BWE in a supply-chain. We proposed using central data warehouse

integrating multiple data marts maintained locally to ensure information flow to the stakeholders. The central coordination is planned with separate data marts at each echelon and a warehouse as an apex server. Online Analytical Processing (OLAP) queries are used for material and information flow. The large repository is placed to maintain real time demand analysis, market forecast, strategic decision making, and query processing for business intelligence (BI) applications.

## 2. Bullwhip effect in Supply Chain with change in demand

SCM Systems use forecasting for the purpose of planning the production, requirement of material and control of the inventory. The forecast usually differs from what is actually happening. Uncertainty rules the supply chain. As we move upwards along the chain from the end customer to the raw material supplier, forecast errors become higher and higher at every echelon. In order to balance the forecast error, more and more safety stock becomes the necessity at each echelon. Again, during periods of higher demand the downstream participants will increase their orders over some period of time and will stop or reduce orders at some point of time and will force the upstream participants to procure and produce more.

Let us consider some numerical observations on the effects of BWE among manufacturers and the corresponding three tiers of suppliers in a SC with an initial market demand of a hundred units at some point of time over a particular item.

**Table 1.** 5% Reduced Market Demand and Change in Stock Level

Period	Demand (0)	Manufacturer			Supplier 1			Supplier 2			Supplier 3		
		Prod. Rate (1)	Start Stock (2)	Fin. Stock (3)	Prod. Rate (4)	Start Stock (5)	Fin. Stock (6)	Prod. Rate (7)	Start Stock (8)	Fin. Stock (9)	Prod. Rate (10)	Start Stock (11)	Fin. Stock (12)
1	100	100	100	100	100	100	100	100	100	100	100	100	100
2	95	90	100	95	80	100	90	60	100	80	20	100	60
3	95	95	95	95	100	90	95	120	80	100	180	60	120
4	95	95	95	95	95	95	95	90	100	95	60	120	90
5	95	95	95	95	95	95	95	95	95	95	100	90	95
6	95	95	95	95	95	95	95	95	95	95	95	95	95

Source: own elaboration.

Table 1 show the figures when demand is reduced by 5% per period and Table 2 depict the figures when demand is enhanced by 5% per period with the constraint that the process will maintain one period stock as demand. Col. (0) the demand at the start of period 2 is 95 due to the 5% demand reduction. The finished stock with the manufacturer at the close of period 1 is the opening stock col. (2) of the manufacturer at the start of period 2 which is 100. Finished stock col. (3) at the end of period 2 should be 95 as we have to maintain one period demand as closing stock. Thus the

new production rate of the manufacturer in col. (1) should be 90 to hold 95 as the finished stock and can be derived by the expression:

$$2 * \text{Demand} - \text{Finished Stock} \tag{1}$$

Hereafter, the demand for supplier 1 is raised by the manufacturer, which is 90 the data value of col. (1) of period 2. The start stock (5) of period 2 for supplier 1 should be brought from col. (6) of period 1, and is equal to 100. In order to maintain the finished stock as one period demand, the finished stock col. (6) of period 2 of supplier 1 should be 90. This can be sustained by producing 80 units by supplier 1 and can be obtained by applying the equation (1) with demand figure as 90 (manufacturer production rate at period 2). Similarly, demand for period 2 of supplier 2 is raised by supplier 1, which is the production rate at period 2 of supplier 1 of col. (4), equals 80 and the corresponding finished stock col. (9) of supplier 2 at the end of period 2 should be 80, which is the one month demand figure and can be derived by using equation (2). The periodic data for supplier 3 is built similarly.

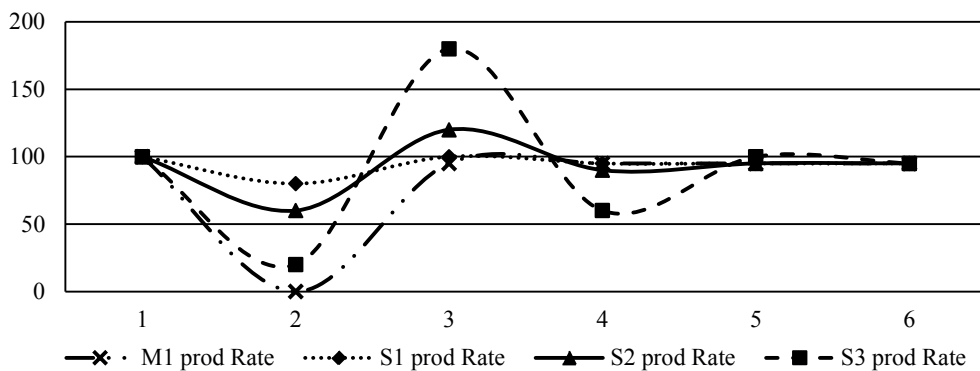


Figure 1. Stock Variation at 5% demand reduction on a single Item

Source: own elaboration.

Computations of start stock of a particular period is replaced with the finished stock of the previous period, i.e. columns (2), (5), (8) and (11) of period 3, are replaced with (3), (6), (9), and (12) of period (2). The finished stocks (3), (6), (9), and (12) of the respective SC member is computed as:

$$\text{Production rate} + \text{start stock} - \text{Demand} \tag{2}$$

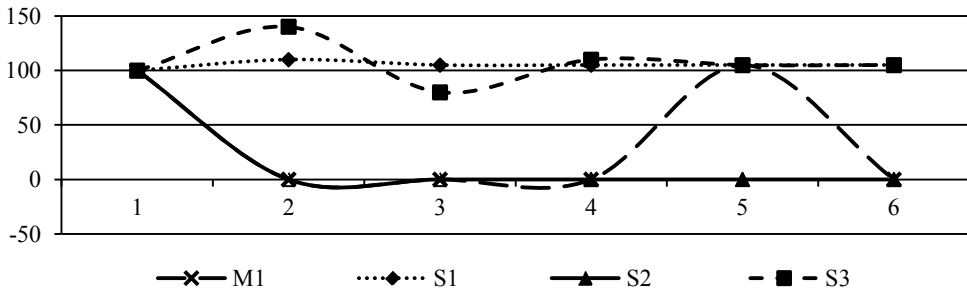
The graph of Figure 1 clearly shows that for a 5% demand reduction there is a huge oscillation of stock over the six periods of operations for 100 units of an item, where the demand figure cascades down from the immediately preceding SC partner and the amplitude of oscillation is maximum for supplier 3 followed by supplier 2 and supplier 1. This confirms the existence of BWE.

**Table 2.** 5% Enhanced Market Demand and Change in Stock Level

Period	Demand (0)	Manufacturer			Supplier 1			Supplier 2			Supplier 3		
		Prod Rate (1)	Start Stock (2)	Fin Stock (3)	Prod Rate (4)	Start Stock (5)	Fin Stock (6)	Prod Rate (7)	Start Stock (8)	Fin Stock (9)	Prod Rate (10)	Start Stock (11)	Fin Stock (12)
1	100	100	100	100	100	100	100	100	100	100	100	100	100
2	105	110	100	105	120	100	110	140	100	120	180	100	140
3	105	105	105	105	100	110	105	80	120	100	20	140	80
4	105	105	105	105	105	105	105	110	100	105	140	80	110
5	105	105	105	105	105	105	105	105	105	105	100	110	105
6	105	105	105	105	105	105	105	105	105	105	105	105	105

Source: own elaboration.

Let us now observe the other instance, when market demand is enhanced by 5% over 100 items of a particular item. The data in columns (0) to (12) of Table 1 is compiled the same way as that of Table 2 using equation (1) equation (2).



**Figure 2.** Stock Variation at 5% demand increase on a single item

Source: own elaboration.

The graph of Figure 2 shows that for a 5% demand increase, there is still an oscillation of stock over the six periods. The manufacturer maintains a uniform production from the 3<sup>rd</sup> period onwards. Supplier 1 suffers smaller stock variations compared to S2, whereas S3 suffers from the maximum oscillations in stock. If there had been a greater number of suppliers down the chain the most distant supplier would have suffered huge stock overheads. Thus, once again we confirm the existence of BWE due to demand enhancement in the process [Buchmeister 2008].

### 3. Literature survey

An imperative message in SCM research is that organizations should consider global Supply Chain performance and not just the performance of their portion the chain [Lee, Billington 1992]. If we look back along the echelon stages of an SC, then oscillations in inventory in response to variations in demand for a product is known as the Forrester effect or bullwhip effect [Forrester 1961]. A notable number of works published in leading journals dealing with BWE are briefly reviewed and analyzed in this section. SCM performance is measured as a ratio of demand against delivery. It is observed that a ratio greater than unity implies that BWE prevails [Chen et al. 2000]. The research on BWE can be broadly categorized as Descriptive, Investigative, Industrialized, and dynamic approaches.

#### Descriptive approach

Demand forecasting and order lead times are identified as the root causes of BWE. Uneven customer demand is identified and capacity adjustment to maintain unchanged inventory level is studied in [Lee, Billington 1992]. Occasionally seasonal variations dominate demand uncertainty and, as a result, the demand to order variance ratio becomes insignificant for measuring BWE [Chen, Lee 2009]. Experiments on seasonal data using three different forecasting levels were used to monitor lead times, order batching and control the BWE. No unique demand management strategy was suggested to control the seasonal effects. Improvements through collaborative strategies are advised under low seasonality [Centeno, Pérez 2008]. Order variance ratio and demand distortions are studied by the US census bureau data to observe the BWE [Cachon, Randall, Schmidt 2007]. In this work the cause of the BWE due to rationing, gaming, and order batching in SCM are addressed. A virtually managed inventory (VMI) is introduced, compared with traditionally managed SCM and demonstrated how VMI reduce the BWE [Disney, Towill 2003]. This study highlights the practical problems of data incompleteness in measuring the BWE over amplified variable demand in a retail chain from the lowest to the highest echelon. The experimental results of measurements of the bullwhip effect in two different supply chains are discussed [Fransoo, Wouters-Marc 2000]. The Supply Chain Operations Reference (SCOR) model was developed by the Supply Chain council (SCC). It maps five basic SC functions: plan, source, make, deliver and return with customer's demand. It is continuously used to measure the SC performance to reduce the BWE throughout the chain [Huan, Sheoran, Wang 2004]. The Beer Game is an empirical model to study the BWE used by the students of MIT. The role players, like customer, retailer, whole seller, distributor and manufacturer do not communicate with each other. The minimum cost bearing stockholder is declared the winner of the game. No formalism is suggested to control the BWE, but it was established that as the game progresses the problem becomes non-linear in nature [Geary, Disney, Towill 2003].

### **Investigative approach**

In order to quantify the occurrence of BWE, transfer function frequency response plots are used as the guiding principle to monitor the SC replenishment [Dejonckheere et al. 2004]. Propagation of BWE is measured on the ready inventory stock using Fourier Transform and a mathematical formalism is suggested to maintain the excess stock during the SC operations at an increased cost [Wang 2006]. A model of a multi-stage SC issued to study the impact of 'worst-case' scenario of BWE at the fourth echelon using the Analysis of Variance (ANOVA) method over five design parameters (information enrichment percentage (IEP), time to adjust inventory ( $T_i$ ); time to adjust work-in-progress ( $T_w$ ); production or pipeline delay ( $T_p$ ); sales exponential smoothing (forecasting) constant  $\alpha$ ) [Hussain et al. 2007]. Fifteen manufacturing organizations were surveyed about time delays at different stages of a stochastic multi-stage SC and measured the major causes of BWE and the process losses. A mixed integer linear model was proposed to analyze the dynamic time delays. The study concludes with recommendations for continuous improvement and loss reductions due to time delays [Puigjaner, Lainez 2008]. This work focuses on logistic cost minimization, information sharing and lead time monitoring between two echelons; warehouse and retailer in a serial SC to measure the impact of BWE on respective inventory holding using the Minimum Mean Squared Error (MMSE) method. The work proposed for a forecasting lead time demand (LTD) and an adaptive base-stock inventory policy to determine their respective order quantities and finally put forward an opinion that a reduction of lead time is more advantageous than information sharing to control BWE [Agrawal, Sengupta, Shanker 2009]. A nonlinear optimization model is proposed to reduce BWE by using a four stage echelon system on demand distortion, misperception of feedback, price variation, batch ordering, and strategic decisions [Hohmann, Zelewski 2011].

### **Industrial approach**

Thirty fast moving consumer goods (FMCG) were considered to measure the quantitative presence of BWE in SCM. Discrete event simulation models were used to show the impact of demand variability on BWE. The design parameters are item wise reorder levels, economic order quantity (EOQ), and demand information in shared and unshared mode at each echelon. At each configuration, the total logistic costs and the resulting demand variance amplifications are computed to evaluate the BWE. The study also suggested eleven key design guidelines to optimize the SC [Bottani, Montanari 2010]. BWE on perishable products are considered where customer demand varies, retailers and wholesalers order cycle varies, and a delay in delivery at various touch points occurs. A dynamic model of the SC of perishable products with a set of convergence parameters are suggested by adjusting the order cycle and delivery delay time. The model can be optimized by stabilizing the convergence function and thus control the BWE [Wang 2011]. The packaging

business used to stock products based on forecasting and inventory policies. A stochastic demand reduction SC models is suggested to control the BWE. The modeling parameters used are supplier order frequency, order received rate, shipment rate, backlog order status, present inventory holding, backlog inventory, acquisition rate, and present order quantity. The model inferred that the reduction of delivery lead time should be four weeks for a drastic reduction of BWE. The study also confirms that the shorter the lead time, the lesser will be the Bullwhip effect and the quicker will be the rate of stabilization of the system [Lewlyn et. al. 2011].

### Dynamic approach

This category presents a survey work among the companies who have implemented ERP or some integrated software to manage the SCM functions. A framework is recommended to understand all types of enterprise applications that maintain the SC and its corresponding BWE at each echelon [Sherer 2010]. The integration of SCM activities using the IT enabled Service Oriented Architecture (SOA) is discussed. The primary advantages are avoidance of data inconsistencies and automatic multiple channel updates over different technologies. It realizes better supply chain performance to support agile reconfigurable architectures. The study also suggests some performance metric to evaluate SC performance and the corresponding BWE effects over multiple channels [Obeidat, Zaatreh 2010]. In this work, BWE is studied over a four-stage tree structured SC model. A software simulation model was designed with a four stage tree structure. Three empirical observations were framed to study the diverse effects of order allocation on BWE at different instances. BWE can be improved to some extent applying the appropriate order allocation mechanism. The study revealed some interesting observations on BWE, like that it could produce a reverse bullwhip effect under certain conditions [Zhuoqun 2011]. Let us now summarize the survey over the four broad categories as in Table 3.

**Table 3.** Review on BWE Approaches

Author	1: descriptive 2: investigative 3: industrial 4: dynamic				Comments
	1	2	3	4	
1	2	3	4	5	6
[Lee, Billington 1992]	√				Study identifies uneven customer demand
[Fransoo et.al 2000]	√				Experimental results of BWE are discussed
[Disney, Towill 2003]	√				Measuring problems for incomplete data sets
[Geary, Disney, Towill 2003]	√				Beer game to study the BWE
[Huan et. al 2004]	√				SCOR performance measures to monitor BWE



1	2	3	4	5	6
[Cachon et. al. 2007]	√				BWE by Order variation and demand distortions
[Centeno, Pérez 2008]	√				BWE on Seasonality
[Chen, Lee 2009]	√				Seasonal variation GT demand ambiguity implies order variance ratio trivial for measuring BWE.
[Dejonckheere et al. 2004]		√			Transfer function plot to monitor the f BWE
[Wang 2006]		√			Measured BWE propagation using Fourier Tr.
[Hussain et al. 2007]		√			statistical method ANOVA used to measure BWE
[Puigjaner, Lainez 2008]		√			Significant causes of BWE is measured using a mixed integer model
[Agrawal, Sengupta, Shanker 2009]		√			Lead time reduction is suggested over information sharing to monitor the BWE.
[Hohmann, Zalewski 2011]		√			4 stage echelon system used to measure the BWE
[Bottani, Montanari 2010]			√		Surveyed thirty fast moving consumer goods to emphasize quantitative presence of BWE in SCM
[Lewlyn et al. 2011]			√		Packaging industry confirms shorter the lead time, lesser will be the BWE
[Wang 2011]			√		BWE studied over perishable products. Can be optimized by adjusting order cycle and delivery delay time
[Obeidat, Zaatreh 2010]				√	A large amount of ERP organizations are surveyed. A framework is recommended for understanding all types of enterprise applications that maintain the SC and its corresponding BWE at each echelon.
[Sherer 2010]				√	Reviewed on the integration of SCM activities using the IT enabled Service Oriented Architecture (SOA). Suggested performance measurement (PM) matrices to evaluate SC performance and corresponding BWE effects over multiple channels.
[Zhuoqun 2011]				√	BWE is studied through a four stage tree structured SC model. A software simulation model was designed with four stage tree structure and suggested causes of reverse BWE.

Source: own elaboration.

The synopsis of the study on demand variability and BWE can clearly draw the conclusion that there is not much of holistic approach to control the BWE in a dynamic, multi-aspect, decentralized, stochastic and non-linear environment. Controlling the BWE in a distributed environment requires an efficient modeling tool to embed the stochastic behavior of demands in the model.

#### 4. GSPN model of the multi stage multi tier retail chain

Petri nets are directed bipartite graphs used for the modeling of discrete dynamic even-driven systems developed by Carl Adam Petri. It is a meta-modeling tool to study and analyze the complexity among the large number of collaborative devices,

business processes, distributed communications, and various process simulations [Krogstie 2001]. Petri Nets have power over a number of specific functional properties of the system under design. Two types of properties that can be differentiated are behavioral and structural. The behavioral properties depend on the initial state or marking of a Petri net, whereas the structural properties depend on the topology of a Petri Net. A Structural model is a directed graph representing the static part of the system. There are two kinds of nodes, places and transitions, represented by circles and rectangles. Places represent state variables and transitions represent transformers. The net is said to be ordinary when all arc weights are equal to one, i.e. each occurrence of adjacent transitions consumes one token from input to output place. Behavioral Models capture the dynamics of the system behavior using evolution rules for the marking. Markings are represented by tokens. The token at a place is its state value. The values get changed to adjacent states with the occurrence of transitions [Ramchandani 1974].

An ordinary Petri net cannot model services like the simultaneous arrival of tokens to a queue. In such concurrent situations an extension of the ordinary Petri Net is needed. A transition will be enabled when its pre-conditions hold, but not post-conditions. In order to tackle this and many other aspects, like temporal issues, stochastic analysis, a reduction of state-space explosion for large models GSPN's was developed [Tado Murata 1989]. The need for timing variables in the models of various types of dynamic systems is apparent since these systems are real time in nature. A Timed Petri Net [Wang 1998] has been devised to serve the purpose. The firing rules defined for a Timed PN control the process of moving the tokens around. Time Petri Net model (TPN) is another powerful formalism and conciliation between modeling power and verification complexity [Ajmone-Marsan, Conte, Balbo 1984]. The timed nets are classified as timed place Petri Nets (TPPN) and timed transition Petri Nets (TTPN), depending on whether the timing bounds annotate places or transitions [Wang 1998].

A Colored Petri Net (CPN) obeys the same firing rules as that of the basic Petri net with the exception of a functional dependency between the color of the transition and the colors of the tokens [Jensen 1996]. Each token in a CPN is attached with a color to identify the token. Each place and transition is attached to a set of colors. A transition will fire with respect to each of its colors.

Stochastic timed Petri nets (STPNs) are Petri nets in which stochastic firing times are associated with transitions that automatically generate the stochastic process which governs the system's behavior.

#### **4.1. Generalized Stochastic Petri Net (GSPN)**

A GSPN with initial markings can be described uniquely by a 6-tuple: GSPN (P, T, I, O, M,  $\lambda$ ), where, (P, T, I, O, M) is a marked PN. GSPN supports both immediate and timed transitions [Ajmone-Marsan, Conte, Balbo 1984]. Here, P is the set of

Places and T is the set of Transitions connecting the places in T. The sets P and T are mutually exclusive. I is the Input function, where the value  $I(p, t)$  is the number of directed arcs from the place p to the transition t. O is the output function, where the value  $O(t, p)$  is the number of arcs from the transition t to the place p. M is the Initial marking of places, where the value  $M(p)$  is the number of tokens that are located in the place p.

The marking dependent firing rates associated with transitions are represented as  $\lambda = (\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n)$ . Firing delay is the elapse time associated with every transition. This delay is a random variable with a negative exponential probability density function. For any marking dependent transition  $t_i$  with an associated firing rate  $\lambda_i$ , can be expressed as  $\lambda_i(m_i)$  and the average firing delay of transition  $t_i$  in marking  $m_i$  is  $[\lambda_i(m_i)]^{-1}$  [Chaki, Nabendu, Bhattacharya 2006]. The non-definitive nature of the large retail networks is perfectly suitable for modeling such systems using GSPN.

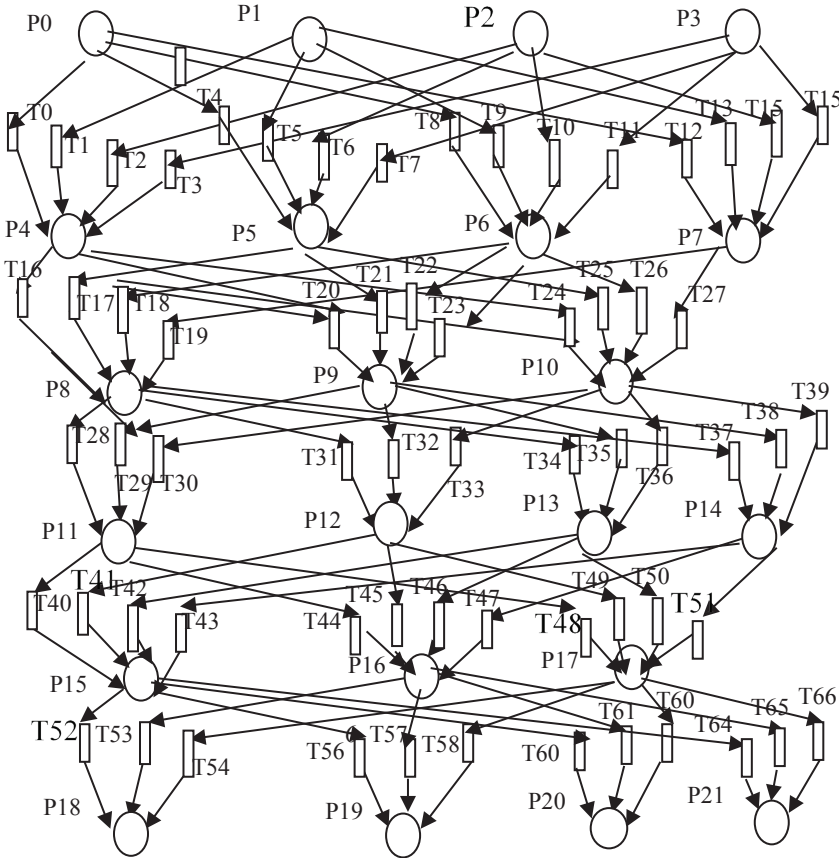


Figure 3. GSPN model of Retail Network

Source: own elaboration.

There are some frequently used SCM models like SCOR [Huan, Sheoran, Wang 2004]. Epicor is a SOA based SCM software, AMT is a SCM ERP application known as FlexRFP, which is a web based ERP application.

Figure 3 depicts general scheme of the multi echelon and multi-tier system to deal with the multiple products and multiple suppliers in a location independent manner. Managing the inventory in a stochastic natured multi-echelon model is a complex process consisting of a set of virtually linked upstream and downstream flows of products, services, finances and information meeting the customer's demand. The SC echelon of the proposed model is composed of customer, retailer, wholesaler, distributor, manufacturer, and supplier. The multiple tiers are horizontally sequenced from left to right and multi stages are represented vertically down. This scheme aims at simulating and analyzing the BWE by the proposed GSPN model by using the PIPE 2.5 simulation tool [Bonet 2007]. Let us now model the multi stage multi-tier SC using GSPN with a constraint that material flow will be to any of the role players in the succeeding layer and the information flow will be to its preceding layer only.

List of places are: P0, P1, P2, ..., P21

List of transitions are: T0, T1, T2, ..., T66

The GSPN model in Figure 3 symbolizes P0, P1, P2, and P3 as suppliers, P4, P5, P6, and P7 as manufacturers, P8, P9, and P10 as distributors, P11, P12, P13, P14 as wholesalers, P15, P16, and P17 as Retailers and P18, P19, P20, P21 as customers. {T0 to T66} are the corresponding timed transitions with rate=1. This is an extended simple type ordinary net as all arc weights are unity.

**Table 4.** Token Accumulation in the GSPN model of Retail Network

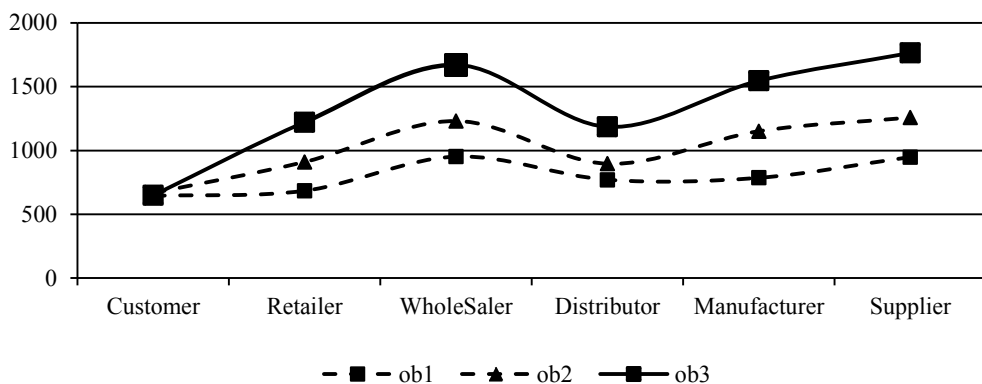
Place	OB1	OB2	OB3
1	2	3	4
<b>P0</b>	192.0198	287.2392	437.239
<b>P1</b>	245.86139	337.9369	432.937
<b>P2</b>	241.9703	341.3156	431.316
<b>P3</b>	268.56436	291.3189	462.319
<b>Supplier</b>	<b>948.41585</b>	<b>1257.811</b>	<b>1763.81</b>
<b>P4</b>	228.9802	321.814	421.214
<b>P5</b>	193.53465	328.9701	428.97
<b>P6</b>	138.72277	296.0997	391.5
<b>P7</b>	224.55446	204.1894	304.189
<b>Manufacturer</b>	<b>785.79208</b>	<b>1151.073</b>	<b>1545.87</b>
<b>P8</b>	250.0297	263.5515	363.552
<b>P9</b>	261.85149	362.9668	451.967
<b>P10</b>	258.70297	269.7375	369.738
<b>Distributor</b>	<b>770.58416</b>	<b>896.2558</b>	<b>1185.26</b>
<b>P11</b>	149.87129	350.2326	390.273
<b>P12</b>	211.40594	306.3522	406.312

1	2	3	4
<b>P13</b>	320.0495	298.0432	398.343
<b>P14</b>	270.51485	275.9136	475.214
<b>Wholesaler</b>	<b>951.84158</b>	<b>1230.542</b>	<b>1670.14</b>
<b>P15</b>	211.48515	311.2791	443.279
<b>P16</b>	251.91089	308.1096	427.11
<b>P17</b>	220.91089	290.8439	351.233
<b>Retailer</b>	<b>684.30693</b>	<b>910.2326</b>	<b>1221.62</b>
<b>P18</b>	150.436	242.478	173.162
<b>P19</b>	161.01	140.4718	154.472
<b>P20</b>	151.396	122.9269	162.927
<b>P21</b>	181.36634	160.9239	157.635
<b>Customer</b>	<b>644.20834</b>	<b>666.80057</b>	<b>648.196</b>

Source: own elaboration.

A state space analysis says that the GSPN model is not safe. It is not strongly connected as all the places in the net are not directly attached to each other. There are no live markings and no non-zero safe marking for the proposed model. Moreover, Figure 3 is not bounded so there can be deadlocks. This is because the number of tokens at any place is a positive integer and each transition gets an output place in the immediate succeeding layer [Buchmeister 2008; Tado Murata 1989].

There is no restriction of number tokens at any of the places. A random number of continuous observations were performed on the GSPN model of Figure 3 over three independent simulations, OB1 to OB3, using the PIPE2 tool. These are recorded in Table 4. Figure 4 clearly shows that as the operation progresses the performance of the system becomes non-linear.



**Figure 4.** Simulated Stock Status at different SC Partners

Source: own elaboration.

The Bullwhip Effect indexes are measured between the pair of communicating stages like supplier and manufacturer, manufacturer and distributor, distributor and wholesaler, wholesaler and retailer, and retailer and customer. The fact that the BWE index is greater than 1 shows the presence of BWE in the system. For example, the ratio between the supplier and the manufacturer  $\frac{948.41585}{785.79208} > 1$  as per OB1, which implies that BWE exists. Again, the ratio between the retailer and the customer is  $\frac{1221.62}{648.19} > 1$  as per OB3.

Our objective is to model the system in such a way that the BWE should be the bare minimum [Chen et al. 2000; Torres, Maltz 2010]. At every echelon in Figure 4 the existence of BWE persists and there is an increasing trend. This can be controlled if there exist communication and transparency among the adjacent operating echelons and their respective tiers [Sterman 1989]. However, there may be some business risk which may not prompt the operating echelons to communicate and share information. But by establishing strategic partnerships, information can be shared and exchanged by reducing sizeable risks. Again, just establishing communication and transparency between two adjacent echelons will not successfully eliminate BWE completely, as information in a supply chain is highly decentralized and mostly without coordination and communication. There should be some mechanism to have central coordination and transparency along the downstream to upstream of the chain while issuing an order for an item or a change in planning at any echelon of the system. With this conceptual understanding, we propose a revised model in section 5 that better allows us to control BWE.

## 5. A revised GSPN model

We have observed that central coordination will reduce the variability impacts by improving forecasts and reducing lead times. The demand signals can be processed in advance for faster stock replenishments. The other causes of BWE are like inflated orders, batch ordering and back orders, can be controlled and synchronized by comparing the demand information at each echelon with the central coordination source [Simchi-Levi, Kaminsky, Simchi-Levi 2008]. Let us now revise the process flow of the GSPN model of section 4 (as depicted in Figure 3) with a view to remove the non-linearity from the SCM process and control the BWE.

Functions like ordering a product, monitoring the stock movement and demand variability at a particular echelon or over the entire chain are time-dependent activities. In order to model such non-definitive nature of activities, GSPN is useful. It supports both immediate and timed transitions with time delays. Lead time reduction is one of the primary issues of the stakeholder to guard against stock-out situations by reducing safety stock and saving money on being blocked with the stock. The availability of the updated information flow at the point of decision

making about a product at any instant is necessary from the immediate preceding and succeeding layers, which can be accessed from local data marts. However, a particular stakeholder may require moving beyond these two layers demands for a central repository or a DW.

We propose to use a central data warehouse in tandem with the GSPN model. Each dimension of the multi-dimensional data archive represents one stage of the chain or, in other words, each stage is a SC player. The places  $P_i$   $\{I=1..n\}$  in the GSPN model of Figure 6 are representing SC players at the respective echelons e.g.  $\{P8, P9, \text{ and } P10\}$  are the distributors. The source of material flow towards the distributor is from the manufacturers  $\{P4, P5, P6, \text{ and } P7\}$  and the source for information flow towards the distributor is from the wholesalers  $\{P11, P12, P13, \text{ and } P14\}$ . In a supply-chain there exists a cause and effect association among the SC players.

Our objective is to analyze the cause and effect relationship throughout the chain in a transparent way such that there should be a free flow of information, irrespective of the stakeholder location and place. Besides transparency, the exact status of the demand against availability and issuance of fresh order should also be explicit to the place holder. In order to achieve this, the local data marts as well as the central warehouse are updated in parallel. As an example, location  $\{P23\}$  is mapped to data mart  $\{D2\}$ .

In the event of a fresh or revised demand from a particular echelon, the place holders of the GSPN will access the central warehouse through a simple OLAP query to access the particular cell of the lattice, e.g. the second layer from the top  $\{(12-16), (23-26), (34-36), (45-56)\}$  of Figure 6 is mapping the manufacturers  $\{P4, P5, P6, \text{ and } P7\}$ . The OLAP queries are used for material and information flow.

The central coordination is planned with separate data marts at each echelon and a warehouse as an apex server. In order to control the entire SC operation along with customer relationship management (CRM) on a real time basis, the central server can be placed and maintained in the cloud for efficient distributed applications.

The revised GSPN model is characterized by three additional data marts and a central warehouse. Places  $P0, P1, P2, \text{ and } P3$  denote suppliers,  $P4, P5, P6, \text{ and } P7$  denote manufacturers,  $P8, P9, \text{ and } P10$  denote distributors,  $P11, P12, P13, P14$  denote wholesalers,  $P15, P16, \text{ and } P17$  denote Retailers and  $P18, P19, P20, P21$  denote customers. Place  $P22$  is the source for the data mart  $D1$ , a central resource server between customer and retailer,  $P23$  is the source for data mart  $D2$ , which is a central information source between manufacturer and distributor, and  $P24$  is the source for data mart  $D3$ , a central information source for data mart  $D1, D2$ .

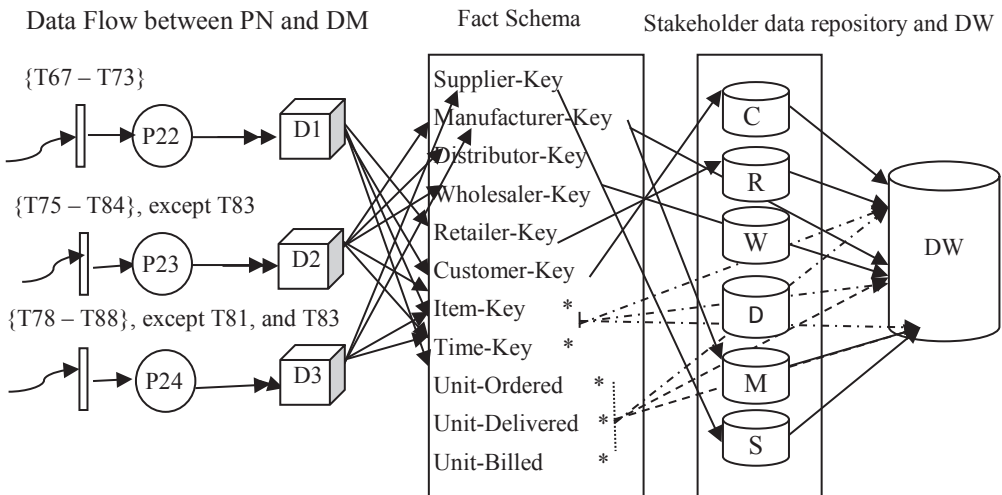
$O'$  is the apex warehouse server.  $\{T0 \text{ to } T88\}$  are the corresponding timed transitions with rate=1. The properties of the GSPN model of Figure 6 remain same as that of Figure analyzed by the tool [Bonet 2007; Zhuoqun 2011].

The model can be generalized by using the incidence matrix and state equations for a Petri net with  $n$  transitions and  $m$  places can be represented as  $A = [a_{ij}]$  an  $nm$





layer  $\{(L1234) \text{ through } (L3456)\}$  represents the total available stock of item between any four of the SC partners. As an example, L3456 represents the available stock with the distributor, wholesaler, retailer and customer. The next layer down the line is  $\{(L12345) \text{ through } (L23456)\}$  represents the total available stock of item between any five of the SC partners. As an example, L23456 represents the available stock with the manufacturer, distributor, wholesaler, retailer and customer. The last layer of the lattice of Figure 5 represents  $\{(L123456)\}$  equal to  $({}^6C_6 = 1)$  implies the total available stock with the supplier, manufacturer, distributor, wholesaler, retailer and customer at any instant of time. The information boundary of the layer wise lattices  $\{D1, D2, \text{ and } D3\}$  is restricted to the respective layers only and it is necessary for some supply chains which are territory based and regional.



**Figure 5.** The Meta Data Dependency diagram of the GSPN model of Figure 6

Source: own elaboration.

Later on an attempt is made to explain the novelty of the model by defining precisely the interactions between the Petri Net transactions, Data Marts and the DW. The centralized architecture is built with one data warehouse, into which all transactions of SCM are imported along the context hierarchies consisting of six dimensions, namely C: customer, R: retailer, W: wholesaler, D: distributor, M: manufacturer, and S: supplier. A data mart is used to act as a kind of materialized view on the data warehouse. A simple repository dependency diagram is presented in Figure 5 to illustrate the relationships among the data repositories with the PN.

Three distinct functions are Data Flow between PN and DM, accessing the fact schema, and the stakeholder data repository and DW are depicted in Figure 5. On activation of a transaction, like a fresh order raised for an item by a customer, the PN

will pass the transaction request through the location P22 and will access the data mart D1 using the fact schema to the customer and retailers data repository. Finally, the specific location of the DW is updated with the information like Item-Key, Time-Key, Unit-Ordered, Unit-Delivered, and unit-Billed, subject to the reality that the existing stock will be monitored through OLAP queries. Similarly, if the transaction is raised by the distributor, wholesaler, and retailer the request will be routed through the location P23 and the Data mart D2 will be accessed. When transactions commence from manufacturer and supplier, the data mart D3 will be accessed.

The novelty of the scheme is the use of concept hierarchy and OLAP operations instead of just online transaction processing (OLTP). The OLAP operations on data mart and DW is monitoring and controlling the attempt of excess stock generation with the echelons. Besides information sharing, the warehouse establishes the complete transparency of the chain and enables all CRM related BI applications and instant decision making, which enables the GSPN model to be efficient and effective.

As the GSPN model of Figure 3 does not hold such real-time controlling mechanism and data repositories for OLAP operations, it invites sufficient BWE at every echelon, demonstrated in Figure 4.

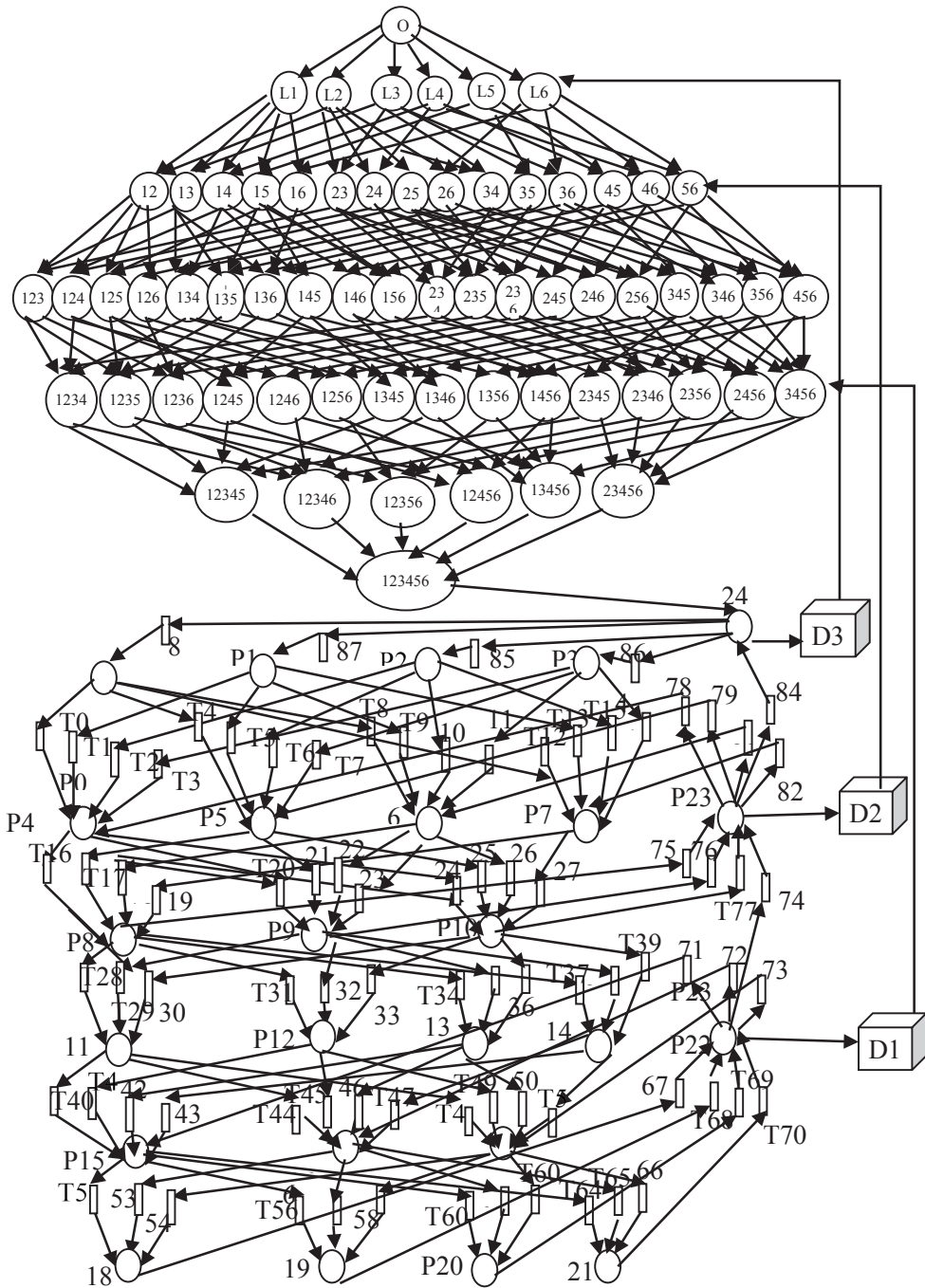
Table 5 shows the simulated stock status after three independent observations OB1, OB2, and OB3, using Pipe 2 over the revised GSPN model of Figure 6 depicted in Figure 7. Players at different echelons are in line with the central requirements. However, the value of the BWE Index is slightly alarming during OB2 and OB3 between distributor and wholesaler as we have not placed any local data marts with the wholesaler in Figure 6. Thus, a proper deployment of central resources is necessary to control the BWE.

**Table 5.** Stock with the Revised GSPN Model

Echelons	OB1	OB2	OB3
Supplier	496.118	1278.18	2270.4
Manufacturer	502.881	1295.88	2301.64
<b>BWEIndex (S/M)</b>	<b>0.98655</b>	<b>0.98634</b>	<b>0.98643</b>
Distributor	497.742	1295.33	2295.17
Wholesaler	497.980	1252.93	2290.89
<b>BWEIndex(D/W)</b>	<b>0.99952</b>	<b>1.03384</b>	<b>1.00187</b>
Retailer		1190.59	2297.70
Customer	505.792	1195.06	2206.65
<b>BWEIndex(D/W)</b>	<b>0.99557</b>	<b>0.99625</b>	<b>1.04126</b>

Source: own elaboration.

These central sources can be deployed to reduce the excess orders from any of the SC players and CRM queries can be generated from the warehouse {O} using the concept hierarchies through online analytical processing (OLAP). The most valuable



**Figure 6.** Revised GSPN model of Retail Network

Source: own elaboration.

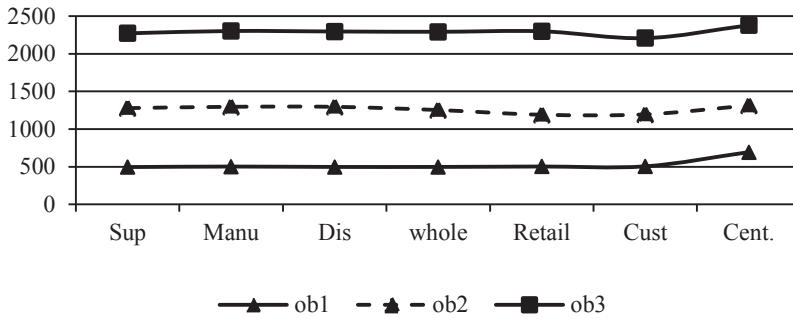


Figure 7. Stock at Echelons with Fig. 6

Source: own elaboration.

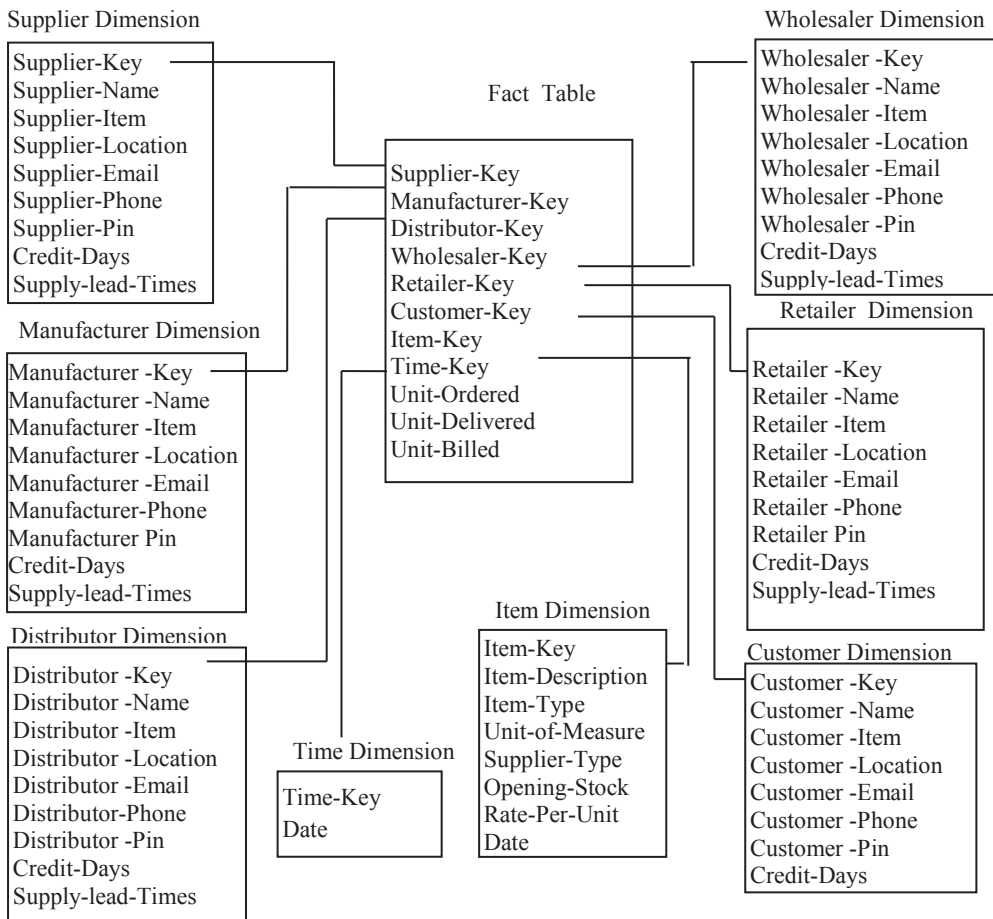


Figure 8. Star schema for the Lattice in Figure 6

Source: own elaboration.

customer through Recent access, Frequency, and Monetary value (RFM) for CRM activities can be defined as a strategy over time to enable the organization to be proactive and profitable. Some of such CRM queries are:

- Which distribution channels contribute the highest revenue and gross margin?
- Which customers are most profitable based upon gross margin and revenue?
- How many unique customers are purchasing this year compared to last year?
- Which are the highest selling items by SC echelons and forecasts?
- XYZ analysis of the block item status by the respective SC players.
- ABC analysis of the consumed items and their pattern of consumption.

It helps the organizations to have the right focus and allocate sufficient resources to where it is needed [Cunningham, Colleen et al. 2003]. The star schema of the model for query processing is presented in Figure 8.

The star schema of Figure 8 is the schematic of the lattice presented in Figure 6. It consists of all the six SC players of our revised model of Figure 6 with the additional dimensions of Time and Item. The derivable are Unit-Ordered, Unit-Delivered, and Unit-Billed. The Fact Table holds the keys and the derivable. All the key values are connected to the respective dimension tables. This structure is a generic multidimensional model for a data warehouse. A data normalization issues are weak in this structure and is independent from the vendor specific models and languages. The interactive data analysis by OLAP operations over the multidimensional model can be used for forecasting and trend analysis for cost reduction, either by lowering the price or by improving the lead time. The other statistical analyses at different granularities can be performed to process business query from the viewpoint of the customer as the SCM starts with the customer and ends also with the customer only [Han, Kamber 2004].

## 6. Conclusions

Variable demands in a SC invite BWE. Information visibility, communication among the SC partners, and stage-wise centralized repositories reduce the BWE even if demand variability persists. The proposed framework in section 5 is capable of handling demand driven real time activities over the SC. Operational soundness can be pioneered with the augmentation of stage wise data marts and a central warehouse, such components help in monitoring real time CRM applications. To start with, a multi-echelon, multi-tier retail network was proposed in section 4. After a set of simulation runs, it was observed that the inventory is unevenly getting piled among the SC partners. This invites demand uncertainty risks and the threat of product obsolescence. The proposed SC net model presented in section V is collaborative, data intensive and distributed. A large repository is placed to support real time demand analysis, market forecast, strategic decision making, and query processing for BI applications. There are only a few global, larger size retailers who may be interested in maintaining an integrated chain. However, for the sake of scalability of

the proposed model, stage wise data marts are also employed for medium and small size retailers. It will not be out of place to mention that ERP, and Electronic Data Interchange (EDI) based SC can handle transactions and control BWE. Yet the CRM applications are way ahead of simple transaction management in applications in a SCM. This work may further be extended for the downstream applications of the SCM through the GSPN model towards improving the economic aspects of the chain.

## References

- Agrawal S., Sengupta R.N., Shanker K., *Impact of information sharing and lead time on bullwhip effect and on-hand inventory*, "European Journal of Operational Research" 2009, vol. 192(2), pp. 576-593.
- Ajmone-Marsan M., Conte G., Balbo G., *A Class of generalized Stochastic Petri Nets for the Performance Evaluation of Multiprocessor Systems*, "ACM Transactions on Computer Systems" 1984, vol. 2(2), pp. 93-122.
- Anderson Jr. E.G., Morrice D.J., *A simulation game for service-oriented supply chain management: Does information sharing help managers with service capacity decisions. Production and Operations Management*, "International Journal of Logistics Management" 2000, vol. 9(1), pp. 40-55.
- Bonet P., *A Petri Net Tool for Performance Modelling*, PIPE (CLEI), 2007, vol. 2(5).
- Bottani E., Montanari R., *Supply chain design and cost Analysis through simulation*, "International Journal of Production Research" 2010, vol. 48(10), pp. 2859-2886.
- Route R.N., Lambrecht M.R., Van Houdt B., *Performance evaluation of a production/inventory system with periodic review and endogenous lead times*, "Willey Journal of Naval Research Logistics" 2007, 54(4), pp. 462-473, DOI: 10.1002/nav.20222.
- Buchmeister B., *Investigation of the Bullwhip effect using spread sheet simulation*, "International Journal of Simulation Model" 2008, 7(1), pp. 29-41.
- Cachon G.P., Randall T., Schmidt G.M., *In search of the bullwhip effect manufacturing & service*, "Journal of Operations Management" 2007, vol. 9(4), pp. 457-479.
- Centeno M.A., Pérez J.E., *Quantifying the Bullwhip Effect in the Supply Chain of small-sized companies*, Sixth LACCEI International Latin American and Caribbean Conference for Engineering and Technology (LACCEI'2008), 2008.
- Chaki, Nabendu, Bhattacharya S., *Performance analysis of multistage interconnection networks with a new high-level net model*, "Journal of Systems Architecture" 2006, vol. 52(1), pp. 56-70.
- Chaki, Nabendu, Sarkar B.B., *Virtual data warehouse modeling using petri nets for distributed decision making*, "Journal of Convergence Information Technology (JCIT)" 2010, vol. 5(5), pp. 8-21.
- Chen F., Drezner Z., Ryan J.K., Simchi-Levi D., *Quantifying the bullwhip effect in a simple supply chain: The impact of forecasting, lead times, and information*, "Management Science" 2000, vol. 4(3), pp. 436-443.
- Chen L., Lee H.L., *Bullwhip effect measurement and its implications*, Duke University Working Papers Series (Currently under revision for Management Science), 2009.
- Cunningham, Colleen et al., *Design and Research Implications of Customer relationship Management on Data Warehousing and CRM Decisions*, [in:] *Proceedings of the 2003 Information Resources Management Association International Conference (IRMA 2003)*, 2003, pp. 82-85.
- Dejonckheere J., Disney S.M., Lambrecht M.R., Towill D.R., *The impact of information enrichment on the Bullwhip effect in supply chains: A control engineering perspective*, "European Journal of Operational Research" 2004, vol. 153(3), pp. 727-750.

- Disney S.M., Towill D.R., *Vendor-managed inventory and bullwhip reduction in a two-level supply chain*, "International Journal of Operation and Production Management" 2003, vol. 23(6), pp. 625-651.
- Forrester Jay Wright, *Industrial Dynamics*, MIT Press, 1961, ISBN-10: 1883823366.
- Fransoo J.C., Wouters-Marc J.F., *Measuring the bullwhip effect in the supply chain*, "Supply Chain Management: An International Journal" 2000, vol. 5(2), pp.78-89.
- Geary S., Disney S.M., Towill D.R., *Bullwhip in Supply Chains: Past, Present and Future*, 17th International Conference on Production Research, Virginia, 3-7 August 2003.
- Han J., Kamber M., *Data Mining Concepts and Techniques*, Morgan Kaufman Publishers, 2004, ISBN: 81-8147=049-4, pp. 39-106.
- Hau L.L., Padmanabhan V., Whang, Seungjin, *The bullwhip effect in supply chains*, "Sloan Management Review" 1997, vol. 38(3), pp. 93-102.
- Hohmann S., Zelewski S., *Effects of vendor-managed inventory on the bullwhip effect*, "International Journal of Information Systems and Supply Chain Management (IJISSCM)" 2011, vol. 4(3), pp. 1-17.
- Huan S., Sheoran S., Wang G., *A review and analysis of supply chain operations reference (SCOR) model*, "Supply Chain Management: An International Journal" 2004, vol. 9(1), pp. 23-29.
- Hussain, Matloub P., Drake R., Dong M.L., *Quantifying the Impact of a Supply Chain's Design Parameters on the Bullwhip Effect*, 7th Global IEE Conference on Business & Economics, Italy, 2007, pp. 210-213.
- Jensen K., *Colored Petri Nets – Basic Concepts, Analysis Methods and Practical Use: Basic Concepts*, vol. 1, Springer-Verlag, London 1996, p. 234.
- Krogstie J., *Using a semiotic framework to evaluate UML for the development of models of high quality*, Idea Group Publishing, chapter 5, 2001, pp.89-106.
- Lee H., Billington C., *Managing supply chain inventory: Pitfalls and opportunities*, "Sloan Management Review" 1992, 33, pp. 65-73.
- Padmanabhan P., Whang S., *Information distortion in a supply chain: The bullwhip effect*, "Management Sci." 1997, 43, pp. 546-558.
- Lewlyn L., Rodrigues R., Hebbar, Sunith, Herle, Ramdev, *Bullwhip effect mitigation in trading system: A system dynamics approach*, [in:] *Proceedings of the World Congress on Engineering*, UK, 2011, vol. I, ISSN: 2078-0958.
- Obeidat R., Zaatreh Z., *Motivation for Integrating Supply Chains Using Service Oriented Architecture Approach*, Proceedings of NGMAST, 2010, pp.102-105.
- Puigjaner L., Lainez J.M., *Capturing dynamics in integrated supply chain management*, "Computers and Chemical Engineering" 2008, vol. 32 (11), pp. 2582-2605.
- Ramchandani C., *Analysis of Asynchronous Concurrent Systems by Petri Nets*, Project MAC, TR-120, M.I.T., Cambridge 1974.
- Sherer S.A., *Enterprise applications for supply chain management*, "International Journal of Information Systems and Supply Chain Management (IJISSCM)" 2010, vol. 3(3), pp.18-28.
- Simchi-Levi D., Kaminsky P., Simchi-Levi E., *Designing and Managing the Supply Chain*, 3rd edition, McGraw-Hill, Irwin, New York 2008, ISBN: 9780071198967.
- Sterman J.D., *Modeling managerial behaviour: Misperceptions of feedback in dynamic decision making experiment*, "Journal of Management Science" 1989, vol. 35(3), pp. 321-339.
- Tado Murata, *Petri Nets: Properties, Analysis and Applications*, Proceedings of the IEEE, vol. 77, no. 4, April 1989, pp. 541-580, ISBN: 0-387-13723.
- Torres O.C., Maltz A.B., *Understanding the financial consequences of the bullwhip effect in a multi-echolon supply chain*, "Journal of Business Logistic" 2010, vol. 31(1).
- Wang E., *A Virtual Integration Theory of Improved Supply Chain Performance*, "Journal of Management Information Systems" 2006, vol. 23(2), pp. 41-66.

Wang J., *Timed Petri Nets: Theory and application*, chapter 1 & 2, vol. 3(32), Kluwer Academic Publishers, 1998, pp. 1-134.

Wang W., *Analysis of Bullwhip effects in Perishable Product Supply Chain Based on System Dynamics Model*, IEEE, International Conference on Intelligent Computation Technology and Automation, 2011, vol. 1, pp. 1018-1021.

Zhuoqun LI., *A Multi-stages and tree-shape supply chain model based simulation system for bullwhip effect*, "Journal of Computational Information Systems" 2011, vol. 7(1), pp. 281-289.

## MODELOWANIE PROGNOZ ZMIENNOŚCI POPYTU W ROZPROSZONEJ SIECI LOGISTYCZNEJ Z UŻYCIEM UOGÓLNIONYCH STOCHASTYCZNYCH SIECI PETRIEGO

**Streszczenie:** Jednym z podstawowych problemów w systemach zarządzania łańcuchem dostaw jest rozwiązanie kompromisu między efektywnością a zmiennością potrzeb. W przypadku braku zmienności potrzeb klientów, cykliczności zamówień, portfela zamówień i czasu dystrybucji zarządzanie łańcuchem dostaw jest rutynowym procesem biznesowym. Niestety, w praktyce zarządzania taka sytuacja rzadko występuje, rozwiązanie problemu zmienności zapotrzebowania jest zatem jednym z głównych aktualnych wyzwań, mających na celu zmniejszenie ryzyka bezpiecznego zapasu bez wpływu na realizację potrzeb klientów. W artykule zbadano zmienność łańcucha dostaw przy założeniu wielu dostawców, producentów, dystrybutorów, hurtowników, sprzedawców i klientów, wykorzystując uogólnioną stochastyczną sieć (GSPN). Model pozwala na zachowanie jednolitego stanu zapasów magazynowych w łańcuchu dostaw; rozpatrzono efekt *bullwhip* (BWE) i niepewności w podejmowaniu decyzji.

**Słowa kluczowe:** zarządzanie łańcuchem dostaw, efekt *bullwhip*, uogólnione sieci Petriego (GSPN).