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SOME REMARKS ON THE ORIGINAL PRICE INDEX INSPIRED BY THE NOTES OF PETER VON DER LIPPE

Abstract: We discuss an original price index formula presented in Białek [2012a] and Białek [2012b]. The inspiration for this discussion are the correlated notes of Prof. Peter von der Lippe. This article presents the author's point of view on the problems considered by Prof. Peter von der Lippe and some other properties of the discussed formula.

Keywords: price index, generalized price index, Laspeyres index, Paasche's index, Fisher's index, Białek's index.

1. Introduction

From a theoretical point of view, an index should satisfy a group of postulates (tests) resulting from the axiomatic index theory (see [Balk 1995]). The systems of minimum requirements of an index come from Marco Martini [1992], Eichhorn and Voeller [1976] and Bernhard Olt [1996]. In the paper by Białek [2012c], we consider some general classes of indices that satisfy the postulates coming from Eichhorn and Voeller: *strict monotonicity, price dimensionality*, *commensurability, identity* and (optionally) *linear homogeneity* (see [Von der Lippe 2007]). In the papers of [Białek 2012a] and [Białek 2012b], we present and discuss certain special price index formula that belongs to the mentioned class. Prof. Peter von der Lippe in his notes [2012] about this price index wrote: "The formula is a bit unusual and unorthodox, yet quite interesting from a theoretic point of view. Some properties of this index are astonishing and unexpected, however, as a whole the index does not seem to be useful for the practical work of a statistical agency". In these notes, we can find some interesting remarks on Białek's index. Some of these remarks show some defects of this price index or need reinterpretation of the known tests (like *time reversibility*). In this paper we would like to discuss some of these remarks once again and extend the list of properties of our price index formula. We also show some possibilities of the practical use of Białek's index.

2. Definition of Białek's price index

Let us consider a group of N commodities observed at times s , t and let us denote[1]:

$P^s = [p_1^s, p_2^s, ..., p_N^s]'$ – a vector of commodities' prices at time s ;

$P^t = [p_1^t, p_2^t, ..., p_N^t]'$ – a vector of commodities' prices at time t;

$Q^s = [q_1^s, q_2^s, ..., q_N^s]'$ – a vector of commodities' quantities at time s ;

$Q^t = [q_1^t, q_2^t, ..., q_N^t]'$ – a vector of commodities' quantities at time t .

Let us define functions f_1 and f_2 as follows

$$f_1(Q^s, Q^t) = [\min(q_1^s, q_1^t), \min(q_2^s, q_2^t), ..., \min(q_N^s, q_N^t)]', \tag{1}$$

$$f_2(Q^s, Q^t) = [\max(q_1^s, q_1^t), \max(q_2^s, q_2^t), ..., \max(q_N^s, q_N^t)]'. \tag{2}$$

Using the above significations we can write Białek's index (see [Białek 2012a]) as follows

$$\begin{aligned} I_B^P(Q^s, Q^t, P^s, P^t) &= \prod_{j=1}^{2} \left(\frac{f_j(Q^s, Q^t) \circ P^t}{f_j(Q^s, Q^t) \circ P^s}\right)^{\frac{1}{2}} = \\ &= \sqrt{\frac{f_1(Q^s, Q^t) \circ P^t}{f_1(Q^s, Q^t) \circ P^s} \cdot \frac{f_2(Q^s, Q^t) \circ P^t}{f_2(Q^s, Q^t) \circ P^s}} \end{aligned}, \tag{3}$$

where " $\circ$ " denotes a scalar product of two vectors.

From Theorem 1 proved in the paper of Białek (2012c), we conclude that the index I_B^P satisfies[2] *price dimensionality*, *commensurability, identity,* and *linear homogeneity*. The formula (3) can be written in another, convenient form, used in the next part of the paper. For this purpose let us define the *lower* (I_L^P) and *upper index* (I_U^P) as follows:

$$I_L^P(Q^s, Q^t, P^s, P^t) = \frac{f_1(Q^s, Q^t) \circ P^t}{f_2(Q^s, Q^t) \circ P^s} = \frac{\sum_{i=1}^{N} \min(q_i^s, q_i^t) p_i^t}{\sum_{i=1}^{N} \max(q_i^s, q_i^t) p_i^s}, \tag{4}$$

$$I_U^P(Q^s, Q^t, P^s, P^t) = \frac{f_2(Q^s, Q^t) \circ P^t}{f_1(Q^s, Q^t) \circ P^s} = \frac{\sum_{i=1}^{N} \max(q_i^s, q_i^t) p_i^t}{\sum_{i=1}^{N} \min(q_i^s, q_i^t) p_i^s}. \tag{5}$$

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[1] The time moment S we consider as the *basis*, i.e. the reference situation, for the comparison.

[2] In the mentioned paper we show that Białek's index belongs to the general class of indices.

From (4) and (5) we can define the index I_B^P as a geometric mean of the *lower* and *upper* index, namely

$$I_B^P(Q^s,Q^t,P^s,P^t) = \sqrt{I_L^P(Q^s,Q^t,P^s,P^t) \cdot I_U^P(Q^s,Q^t,P^s,P^t)} \,. \tag{6}$$

Remark 1

The so called *mean value test* for prices means that the value of the index lies between the minimum and maximum price relative. In the paper of Peter von der Lippe [2012], the author is right in writing[3]: "I_L^P and I_U^P are not reasonable index formulas.(…) I_L^P and I_U^P do not possess the *mean value property.* It is quite obvious that neither I_L^P nor I_U^P can be written as (weighted) arithmetic mean of price relatives". But let us notice, that we have

$$I_L^P \cdot I_U^P = I_1^P \cdot I_2^P \,, \tag{7}$$

where

$$I_1^P = \frac{\sum_{i=1}^N \min(q_i^s,q_i^t)p_i^t}{\sum_{i=1}^N \min(q_i^s,q_i^t)p_i^s} = \sum_{i=1}^N w_i^1 \frac{p_i^t}{p_i^s} \,, \tag{8}$$

$$I_2^P = \frac{\sum_{i=1}^N \max(q_i^s,q_i^t)p_i^t}{\sum_{i=1}^N \max(q_i^s,q_i^t)p_i^s} = \sum_{i=1}^N w_i^2 \frac{p_i^t}{p_i^s} \,, \tag{9}$$

with weights that sum up to one as follows

$$w_i^1 = \frac{\min(q_i^s,q_i^t)p_i^s}{\sum_{i=1}^N \min(q_i^s,q_i^t)p_i^s} \,, \tag{10}$$

$$w_i^2 = \frac{\max(q_i^s,q_i^t)p_i^s}{\sum_{i=1}^N \max(q_i^s,q_i^t)p_i^s} \,. \tag{11}$$

[3] For convenience we drop all arguments of index functions and write for example I_L^P instead $I_L^P(Q^s,Q^t,P^s,P^t)$.

As we can notice, although I_L^P and I_U^P do not possess the mean value property, the I_B^P index can still be expressed as a product of two indices that satisfy the *mean value test*. The final conclusion is that the I_B^P formula fulfills the *mean value property.*

Remark 2

Certainly, as was noticed by Peter von der Lippe [2012], it can happen that I_L^P is smaller than the smallest price relative, and I_U^P exceeds the greatest price relative. Nevertheless, that is just the idea of the *lower* and *upper* index. In general, the following theorem can be proved

Theorem

Let us signify by $\tilde{\Lambda}(m)$ the class of price indices with the following form

$$I^P = [\prod_{j=1}^{m} \frac{f_j(Q^s, Q^t) \circ P^t}{f_j(Q^s, Q^t) \circ P^s}]^{\frac{1}{m}}, \tag{12}$$

for some fixed $m \in N$, where for each function

$$f_j(Q^s, Q^t) = [f_j^1(q_1^s, q_1^t), f_j^2(q_2^s, q_2^t), ..., f_j^N(q_N^s, q_N^t)]' \tag{13}$$

it holds[4]

$$\forall k \in \{1, 2, ..., N\} \ \min(q_k^s, q_k^t) \le f_j^k(q_k^s, q_k^t) \le \max(q_k^s, q_k^t). \tag{14}$$

Then, the following relation is true

$$\forall I^P \in \tilde{\Lambda}(m) \ \ I_L^P \le I^P \le I_U^P. \tag{15}$$

Proof

Let us assume that $I^P \in \tilde{\Lambda}(m)$ for some fixed $m \in N$. Thus we have from (12) and (14)

$$(\frac{I^P}{I_L^P})^m = \prod_{j=1}^{m} \frac{1}{(I_L^P)^m} \frac{f_j(Q^s, Q^t) \circ P^t}{f_j(Q^s, Q^t) \circ P^s} =$$

$$= \prod_{j=1}^{m} [\frac{[\sum_{i=1}^{N} \max(q_i^s, q_i^t) p_i^s]^m}{\sum_{i=1}^{N} f_j^i(q_i^s, q_i^t) p_i^s} \cdot \frac{\sum_{i=1}^{N} f_j^i(q_i^s, q_i^t) p_i^t}{[\sum_{i=1}^{N} \min(q_i^s, q_i^t) p_i^t]^m}] > 1, \tag{16}$$

[4] The assumption (14) seems to be quite natural, because as a rule functions f_j^k are defined as some kind of mean of q_j^s and q_j^t values. For instance, in the case of the Fisher, Laspeyres, Paasche, Marshall-Edgeworth, Walsh or Geary-Khamis indices this assumption is satisfied (in other words these price indices belong to the $\tilde{\Lambda}(m)$ class).

and hence

$$I_L^P \le I^P . \tag{17}$$

Analogically it can be shown that $I^P \le I_U^P$.

Concluding – we are aware, that I_L^P and I_U^P formulas represent rather some kind of limits than the price index formulas. In the next part of the paper we call these formulas "formulas" instead of "price indexes".

3. Properties of the Białek's price index

Remark 3

Profesor von der Lippe in his notes, shows that the Marshall-Edgeworth price index can be written as the weighted arithmetic mean of I_L^P and I_U^P formulas and also the weighted arithmetic mean of the Laspeyres price index (I_{La}^P) and the Paasche price index (I_{Pa}^P). It is easy to prove that the Fisher price index (I_F^P) can also be expressed by the weighted arithmetic mean of I_{La}^P and I_{Pa}^P (see Köves (1983)) and, in general, cannot be written as the weighted mean of I_L^P and I_U^P. It is interesting, that although I_{La}^P and I_{Pa}^P can be expressed by the weighted mean of I_L^P and I_U^P (see [Białek 2012b]), the index I_B^P, in general, cannot be written as the weighted arithmetic mean of I_{La}^P and I_{Pa}^P. In general, the following relation is true (see [Białek 2012b])

$$I_B^P = \alpha_1 I_{La}^P + \alpha_2 I_{Pa}^P , \tag{18}$$

where

$$\alpha_1 = \frac{I_B^P}{I_F^P}\left(\frac{I_F^P - I_{Pa}^P}{I_{La}^P - I_{Pa}^P}\right), \tag{19}$$

$$\alpha_2 = \frac{I_B^P}{I_F^P}\left[\frac{I_{La}^P - I_F^P}{I_{La}^P - I_{Pa}^P}\right], \tag{20}$$

and

$$\alpha_1 + \alpha_2 = \frac{I_B^P}{I_F^P} . \tag{21}$$

As it was noticed by Prof. von der Lippe, I_B^P may (in general) differ from I_F^P, thus it is possible to obtain $\alpha_1 + \alpha_2 \ne 1$. Nevertheless, in our simulation study (see sec. 4) we almost always observe I_B^P index between I_{La}^P and I_{Pa}^P (over 99% cases), thus the *Paasche and Laspeyres bounding test* (see von der Lippe (2007)) as a rule is satisfied (despite of Eq. (18)). It is also easy to see that (see [Białek 2012b])

$$\sqrt{\frac{I_L^P}{I_U^P}} \leq \frac{I_F^P}{I_B^P} \leq \sqrt{\frac{I_U^P}{I_L^P}}\,, \tag{22}$$

and thus, in case of small differences between quantities in the compared moments we have $\min(q_i^s, q_i^t) \approx \max(q_i^s, q_i^t)$ and thus $I_L^P \approx I_U^P$ and $I_F^B \approx I_B^P$. Finally, if we observe

$$\forall i \in \{1,2,...,N\} \quad q_i^t \geq q_i^s \text{ or } \forall i \in \{1,2,...,N\} \quad q_i^t \leq q_i^s\,, \tag{23}$$

then $I_B^P = I_F^P$ and in this case the index I_B^P can be written as weighted arithmetic mean of I_{La}^P and I_{Pa}^P.

Remark 4

Let us assume that the price of *k*-th component increases and generates the new value

$$\tilde{p}_k^t = p_k^t + p_k^\Delta \text{, where } p_k^\Delta > 0\,. \tag{24}$$

We get

$$I_B^P = \sqrt{\frac{\min(q_k^s, q_k^t)(p_k^t + p_k^\Delta) + \sum_{i \neq k} \min(q_i^s, q_i^t) p_i^t}{\sum_{i=1}^N \max(q_i^s, q_i^t) p_i^s} \cdot \frac{\max(q_k^s, q_k^t)(p_k^t + p_k^\Delta) + \sum_{i \neq k} \max(q_i^s, q_i^t) p_i^t}{\sum_{i=1}^N \min(q_i^s, q_i^t) p_i^s}} =$$

$$= \sqrt{\frac{\sum_{i=1}^N \min(q_i^s, q_i^t) p_i^t}{\sum_{i=1}^N \max(q_i^s, q_i^t) p_i^s} \cdot \frac{\sum_{i=1}^N \max(q_i^s, q_i^t) p_i^t}{\sum_{i=1}^N \min(q_i^s, q_i^t) p_i^s} + p_k^\Delta \frac{\min(q_k^s, q_k^t) \sum_{i \neq k} \max(q_i^s, q_i^t) + q_k^s q_k^t p_k^\Delta + \max(q_k^s, q_k^t) \sum_{i \neq k} \min(q_i^s, q_i^t)}{\sum_{i=1}^N \max(q_i^s, q_i^t) p_i^s \sum_{i=1}^N \min(q_i^s, q_i^t) p_i^s}} >$$

$$> \sqrt{\frac{\sum_{i=1}^N \min(q_i^s, q_i^t) p_i^t}{\sum_{i=1}^N \max(q_i^s, q_i^t) p_i^s} \cdot \frac{\sum_{i=1}^N \max(q_i^s, q_i^t) p_i^t}{\sum_{i=1}^N \min(q_i^s, q_i^t) p_i^s}} = I_B^P\,. \tag{25}$$

From (25) we conclude that the I_B^P index satisfies a *strict monotonicity.* Monotonicity is the most fundamental characteristic of all reasonable means. If a single price rises (declines), everything else being constant, there should be a reaction of the index formula also showing a change in the proper direction, that is an increase or a decrease in the price level.

Remark 5

Profesor Von der Lippe [2012] wrote: "Moreover, there does not exist a quantity index of Białek. Defined analogously to the price index, it should read as follows be the geometric mean of

$$I_B^Q = \left(\frac{f_1(P^s, P^t) \circ Q^t}{f_2(P^s, P^t) \circ Q^s} \cdot \frac{f_2(P^s, P^t) \circ Q^t}{f_1(P^s, P^t) \circ Q^s}\right)^{\frac{1}{2}}, \tag{26}$$

which definitely is not the same as[5] V_{st} / I_B^P".

We cannot agree with this statement. For instance, the Laspeyres quantity index I_{La}^Q exists and it is used in practice although we have $I_{La}^P \cdot I_{La}^Q \neq V_{st}$ which means that the Laspeyres formula does not fulfill *the factor reversal test*. In the case of Paasche indices, we also observe $I_{Pa}^P \cdot I_{Pa}^Q \neq V_{st}$. As we know, V_{st} divided by I_{Pa}^P gives the Laspeyres quantity index (or V_{st} divided by I_{La}^P gives the Paasche quantity index). In our simulation study (see sec. 4) we obtain the following, quite good approximation (27) not only for small differences between prices and quantities in the compared periods. This means it holds that

$$I_B^P \cdot I_B^Q \approx V_{st}. \tag{27}$$

It should be added that *the factor reversal test* is very restrictive[6] and it is out of the mentioned systems of minimum requirements.

Remark 6

In the paper of Białek (2012b), it is proved that the general formula ($m \geq 1$)

$$I^P(Q^s, Q^t, P^s, P^t) = \left[\prod_{j=1}^{m} \frac{f_j(Q^s, Q^t) \circ P^t}{f_j(Q^s, Q^t) \circ P^s}\right]^{\frac{1}{m}} \tag{28}$$

satisfies *time reversibility* if for any $X, Y, Z \in R_+^N$ it holds

$$\prod_{j=1}^{m} \frac{f_j(X, Y) \circ Z}{f_j(Y, X) \circ Z} = 1. \tag{29}$$

In the case of definition (3) and functions (1) and (2) we have the above condition (29) fulfilled and thus the I_B^P satisfies *time reversibility.* In our opinion, there is no need for symmetry of functions $f_j(Q^s, Q^t)$ as was suggested in the note by Peter von

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[5] The value index V_{st} is defined as $\sum_{i=1}^{N} p_i^t q_i^t / \sum_{i=1}^{N} p_i^s q_i^s$.

[6] M. Balk [1985] proves that the Fisher index is the only formula that satisfies: *factor reversal test*, *linear homogeneity* and *value dependence*.

der Lippe [2012]. We claim that the postulate of symmetry is stronger than the condition (29). In fact, let us consider the case of $m = 1$, $N = 2$ and $f = (f^1, f^2) : R^2 \times R^2 \to R^2$. For some fixed $X = (x_1, x_2)$, $Y = (y_1, y_2)$ and $Z = (z_1, z_2)$ (where $z_1 \cdot z_2 \neq 0$) we certainly have the following implication:

$$f(X,Y) = f(Y,X) \Rightarrow \frac{f(X,Y) \circ Z}{f(Y,X) \circ Z} = 1. \tag{30}$$

Now, let us assume that the function *f* is not symmetrical and for instance

$$f_1(X,Y) = z_2 + f_1(Y,X), \; f_2(X,Y) = -z_1 + f_2(Y,X). \tag{31}$$

This implies that

$$\frac{z_1}{z_2} = \frac{f_2(Y,X) - f_2(X,Y)}{f_1(X,Y) - f_1(Y,X)}. \tag{32}$$

Hence we have

$$z_1(f_1(X,Y) - f_1(Y,X)) = z_2(f_2(Y,X) - f_2(X,Y)) \tag{33}$$

and equivalently

$$[f_1(X,Y), f_2(X,Y)] \circ [z_1, z_2] = [f_1(Y,X), f_2(Y,X)] \circ [z_1, z_2]. \tag{34}$$

Thus we have $\frac{f(X,Y) \circ Z}{f(Y,X) \circ Z} = 1$ though $f(X,Y) \neq f(Y,X)$.

The conclusion is that the opposite implication to (30) may not be true and the assumption $f(X,Y) = f(Y,X)$ seems to be stronger than the condition (29).

Moreover: in the case of Laspeyres ($f_1(Q_s, Q_t) = Q_s$) and Paasche ($f_2(Q_s, Q_t) = Q_t$) formulas we do not have functions $f_j(Q^s, Q^t)$ that are invariant upon interchanging q_i^s and q_i^t. But in this case we have

$$\prod_{j=1}^{2} \frac{f_j(Q_s, Q_t) \circ P_s}{f_j(Q_s, Q_t) \circ P_s} = \prod_{j=1}^{2} \frac{f_j(Q_s, Q_t) \circ P_t}{f_j(Q_s, Q_t) \circ P_t} = 1, \tag{35}$$

and that gives us an argument that the Fisher index

$$I_F(Q^s, Q^t, P^s, P^t) = \sqrt{\prod_{j=1}^{2} \frac{f_j(Q^s, Q^t) \circ P^t}{f_j(Q^s, Q^t) \circ P^s}]}, \tag{36}$$

satisfies *time reversibility.*

4. Simulation study

Let us take into consideration a group of $N=5$ commodities, where the random vectors of prices and quantities are as follows (P^t and Q^t are defined identically as respectively P^s and Q^s)

$$P^s=[U(300,700),U(1000,5000),U(3,9),U(4000,7000),U(100,450)]',$$

$$Q^s=[U(30000,60000),U(200,500),U(300,900),U(20000,50000),U(400,900)]',$$

where $U(a,b)$ denotes a random variable with uniform distribution that has values in $[a,b]$ interval.[7] We generate values of these vectors in $n=10000$ repetitions. Let us denote additionally for each of $k-th$ repetition

$$\Delta 1_k = I_F^P(Q^s,Q^t,P^s,P^t)_k - I_B^P(Q^s,Q^t,P^s,P^t)_k, \tag{37}$$

$$\Delta 2_k = I_B^P(Q^s,Q^t,P^s,P^t)_k - \min(I_{La}^P(Q^s,Q^t,P^s,P^t)_k, I_{Pa}^P(Q^s,Q^t,P^s,P^t)_k), \tag{38}$$

$$\Delta 3_k = \max(I_{La}^P(Q^s,Q^t,P^s,P^t)_k, I_{Pa}^P(Q^s,Q^t,P^s,P^t)_k) - I_B^P(Q^s,Q^t,P^s,P^t)_k, \tag{39}$$

After calculations we get the results presented in Table 1.

Table 1. Minimum and maximum observed values of $\Delta 1$, $\Delta 2$ and $\Delta 3$ variables

Variable	Minimum value	Maximum value
$\Delta 1$	–0.1099	0.1427
$\Delta 2$	–0.0246	0.4020
$\Delta 3$	–0.0092	0.6826

Source: own calculations in Mathematica 6.0.

Histograms of $\Delta 1$, $\Delta 2$ and $\Delta 3$ variables are as follows (see Figure 1, Figure 2 and Figure 3).

We can notice that almost always we have $\Delta 1 \approx 0$, $\Delta 2 \geq 0$ and $\Delta 3 \geq 0$. However we observe situations when the values of formulas I_F^P and I_B^P differ from each other considerably (see Table 1). In more than 99% cases we observe the following relation

$$\min(I_{La}^P(Q^s,Q^t,P^s,P^t), I_{Pa}^P(Q^s,Q^t,P^s,P^t)) \leq I_B^P(Q^s,Q^t,P^s,P^t) \leq$$
$$\leq \max(I_{La}^P(Q^s,Q^t,P^s,P^t), I_{Pa}^P(Q^s,Q^t,P^s,P^t))$$

[7] Our experiment corresponds to the simulation presented in the paper of Białek [2012b] but some parameters of uniform distribution are changed to obtain a large volatility of these random variables.

which means equivalently that $\Delta 2 \geq 0$ and $\Delta 3 \geq 0$ (see Figure 2 and Figure 3). Nevertheless we observe $\Delta 2 < 0$ in (only) 55 cases and $\Delta 3 < 0$ in 59 cases, when the differences within prices and within quantities in the compared periods are extremely large. This leads to the conclusion that in "normal" real cases the I_B^P index satisfies the *Paasche and Laspeyres bounding test*. Some of the characteristics of all the considered price indices generated in our experiment are presented in Table 2.

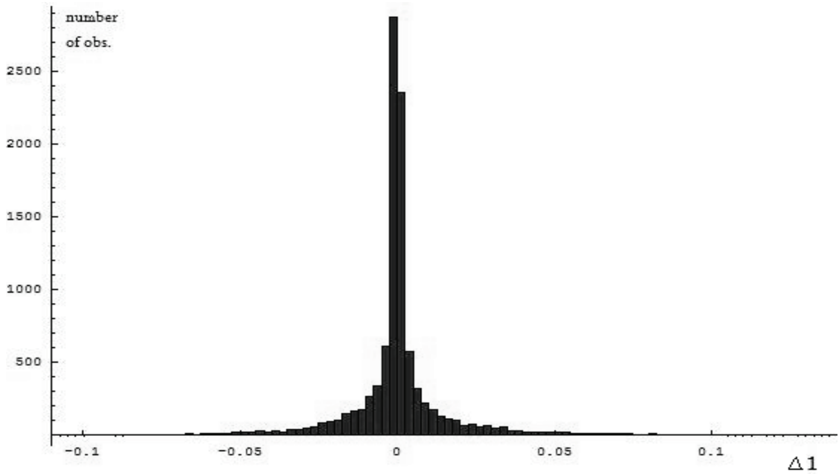

Figure 1. Histogram of $\Delta 1$ variable

Source: own calculations in Mathematica 6.0.

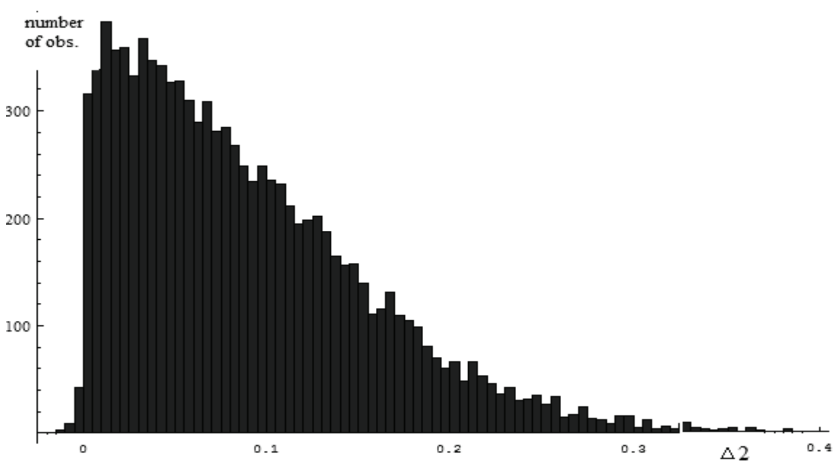

Figure 2. Histogram of $\Delta 2$ variable

Source: own calculations in Mathematica 6.0.

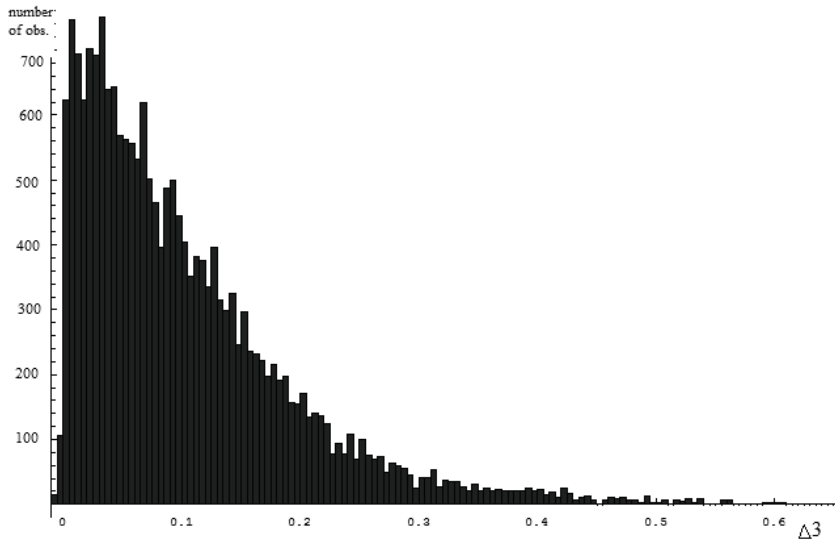

Figure 3. Histogram of $\Delta 3$ variable

Source: own calculations in Mathematica 6.0.

Table 2. Basic characteristics of the Laspeyres, Paasche, Fisher and Białek indices

Parameter	Laspeyres index	Paasche index	Fisher index	Białek index
Minimum	0.5881	0.3974	0.4973	0.5229
Maximum	1.7464	2.2510	1.9317	1.9210
Mean	1.1659	1.2755	1.2194	1.2103
Standard deviation	0.2863	0.3189	0.2835	0.2754
Volatility coeficient	0.2347	0.2635	0.2325	0.2276

Source: own calculations in Mathematica 6.0.

We obtain all the expected values of price indices over one and this fact is natural since, for the *i.i.d* bounded variables X and Y (P^t and Q^t are defined identically as respectively P^s and Q^s), from Jensen's inequality we get

$$E[Log(\frac{X}{Y})] = E[Log(X)] - E[Log(Y)] = 0 \leq Log[E(\frac{X}{Y})] \qquad (40)$$

and consequently

$$1 \leq E(\frac{X}{Y}) . \qquad (41)$$

It is interesting that the distance between a maximum and minimum value of the Fisher index equals 1,4344 and this is greater than the analogical value for the Białek index that equals 1,3981. Moreover, a standard deviation and a volatility coefficient are minimally smaller in the case of the Białek index in comparison with the rest of indices.

In our experiment we also verify whether we can write $I_B^P \cdot I_B^Q \approx V_{st}$ (see Remark 5, formula (27)). Our results for the a variable $\Delta 4 = \left| V_{st} - I_B^P I_B^Q \right|$ and $n = 10000$ repetitions are as follows (see Table 3 and Figure 4).

Table 3. Basic characteristics of $\Delta 4$ variable

Parameter	$\Delta 4$ variable
Mean	0.024
Standard deviation	0.022
Volatility coeficient	0.916

Source: own calculations in Mathematica 6.0.

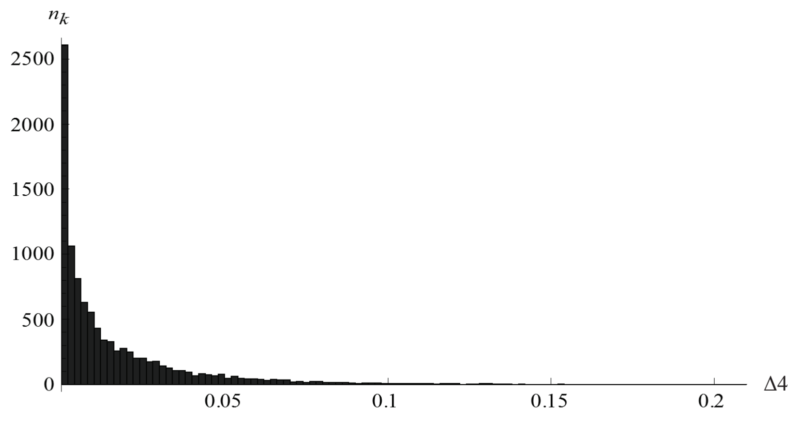

Figure 4. Histogram of $\Delta 4$ variable

Source: own calculations in Mathematica 6.0.

We repeat our experiment in the case of smaller differences within prices and within quantities in compared periods, this means for

$$P^s = P^t = [U(300,400), U(1000,1500), U(3,5), U(4000,6000), U(100,200)]',$$

$$Q^s = Q^t = [U(30000,50000), U(200,300), U(300,500), U(20000,30000), U(400,600)]'.$$

Our results for this case are as follows (see Table 4 and Figure 5).

Table 4. Basic characteristics of $\Delta 4$ variable

Parameter	$\Delta 4$ variable
Mean	0,00064
Standard deviation	0,00296
Volatility coeficient	4,597

Source: own calculations in Mathematica 6.0.

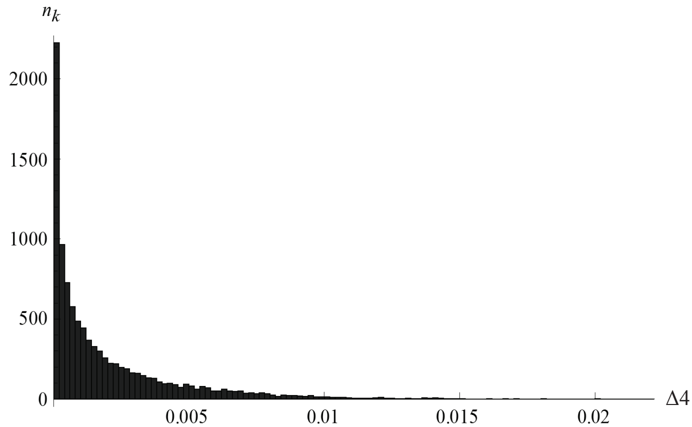

Figure 5. Histogram of $\Delta 4$ variable

Source: own calculations in Mathematica 6.0.

In both cases we can agree that it really holds that $I_B^P \cdot I_B^Q \approx V_{st}$ (in *normal* cases the difference is not higher that 0,01 – see Figure 5).

5. Conclusions

There is no doubt that price index numbers play an important role in economic and business decision-making. Thus, it is very important to use well constructed formulas of indices. The literature on axiomatic index theory is very extensive (see [Balk 1995]; Von der Lippe 2007]). In our opinion the proposed formula I_B^P is well constructed since it satisfies additionally *strict monotonicity, price reversal test, mean value test* and even *time reversibility.* From the simulation study we also know that the proposed index almost always also satisfies the *Paasche and Laspeyres* *bounding test.* In our simulation study the random variable I_B^P has also the smallest standard deviation and volatility coefficient. We can agree with Prof. von der Lippe that the I_B^P index is not eligible for serving as a deflator. We also agree that the usefulness of this index for the practice of official price statistics is limited, because the index does not have an "average of price ratios" interpretation.[8] However, the I_B^P could be, as an alternative to the Fisher index (see [Crawford 1998]), used for the calculation of, for example, biases in *CPI* measurement (see also [Cunningham 1996]). Moreover, I_B^P can be used for the measurement of pension (or investment) funds' units price dynamics (see [Białek 2009]).

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[8] The Fisher index has the same drawback.

PEWNE UWAGI O AUTORSKIM INDEKSIE CEN, ZAINSPIROWANE NOTATKAMI PETERA VON DER LIPPEGO

Streszczenie: W pracy dyskutujemy nad autorskim indeksem cen zaprezentowanym w pracach [Białek 2012a; Białek 2012b]. Niniejszy artykuł prezentuje punkt widzenia autora na kwestie, które w swojej pracy poruszył prof. Peter von der Lippe, omawia także nowe własności omawianego indeksu.

Słowa kluczowe: indeks cen, uogólniony indeks cen, indeks Laspeyresa, indeks Paaschego, indeks Fishera, indeks Białka.