Optica Applicata, Vol. XLII, No. 3, 2012
DOI: 10.5277/oa120301

Optical soliton perturbation with log law nonlinearity

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This paper studies the perturbation theory of optical solitons with log law nonlinearity. The adiabatic dynamics of the Gausson parameters are determined in the presence of the perturbation terms. The fixed point is also found and finally the numerical simulation is carried out.

Keywords: fiber optics, optical communications, pulse propagation and temporal solitons, Kerr effect, nonlinear optics.

1. Introduction

The dynamics of solitons in optical fibers is one of the most important areas of research in nonlinear optics that has been going on for the past few decades [1–25]. There has been a lot of work done in this areas particularly the dynamics of Thirring solitons [10], Hirota's method [16], photorefractive materials and Gaussian beams [13]. The dynamical system approach has also been used to study the perturbed solitons with Kerr law nonlinearity [24]. In this paper, the adiabatic dynamics of optical solitons with log law nonlinearity will be studied in the presence of perturbation terms. The fixed point of the established dynamical system will be determined.

The governing equation for studying the dynamics of optical soliton with log law nonlinearity is the nonlinear Schrödinger's equation (NLSE). The soliton perturbation theory will be utilized to study the dynamics of the solitons. It needs to be noted that for NLSE with log law nonlinearity, the solitons are known as Gaussons. In the case of $(1 + 2)$ dimensions, these are known as Gaussian beams.

2. Mathematical analysis

The dimensionless form of NLSE with log law nonlinearity is given by [2, 3]

$$iq_t + aq_{xx} + bq\log|q|^2 = 0 \tag{1}$$

where a is the coefficient of group velocity dispersion while b is the coefficient of log law nonlinearity. The dependent variable $q(x, t)$ is the wave profile while the independent variables t and x are the spatial and temporal variables, respectively. The 1-soliton solution to (1), also known as Gaussons, is given by [2, 11]

$$q(x, t) = A\exp\left[-B^2(x - vt)^2\right]\exp\left[i(-\kappa x + \omega t + \sigma)\right] \tag{2}$$

where A is the amplitude of the Gaussons, while B is its inverse width given by [2]

$$B = \sqrt{\frac{b}{2a}} \tag{3}$$

From the phase component, the wave number ω is given by

$$\omega = 2b\log(A) - a\kappa^2 - b \tag{4}$$

and the Gausson velocity v is related to its frequency as

$$v = -2a\kappa \tag{5}$$

Also, σ is the center of the phase of the Gausson. Thus, Eq. (3) prompts the constraint condition as

$$ab > 0 \tag{6}$$

2.1. Conservation laws

The NLSE with log law nonlinearity has at least two conserved quantities that are energy E and linear momentum M. They are respectively given by [2, 3]

$$E = \int_{-\infty}^{\infty} |q|^2 \mathrm{d}x = \frac{A^2}{B}\sqrt{\frac{\pi}{2}} \tag{7}$$

and

$$M = i\int_{-\infty}^{\infty} \left(q^* q_x - q q_x^*\right) \mathrm{d}x = -\frac{2\kappa A^2}{B}\sqrt{\frac{\pi}{2}} \tag{8}$$

These conservation laws are evaluated with the aid of the Gaussian solution that is given by (2).

2.2. Perturbation terms

The perturbed NLSE that is going to be considered is given by

$$iq_t + aq_{xx} + bq\log|q|^2 = i\varepsilon R \tag{9}$$

Here, R represents the perturbation terms and the perturbation parameter ε that is related to the relative width of the spectrum arises due to the quasi-monochromaticity [12].

When perturbation terms are turned on, the conserved quantities undergo an adiabatic deformation that are given by [12]

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \varepsilon\int_{-\infty}^{\infty}\left(q^*R + qR^*\right)\mathrm{d}x \tag{10}$$

$$\frac{\mathrm{d}M}{\mathrm{d}t} = i\varepsilon\int_{-\infty}^{\infty}\left(q_x^*R - q_xR^*\right)\mathrm{d}x \tag{11}$$

Thus from (10) and (11), it is possible to obtain the adiabatic variation of the soliton frequency as

$$\frac{\mathrm{d}\kappa}{dt} = \frac{\varepsilon}{2E}\left[i\int_{-\infty}^{\infty}\left(q_x^*R - q_xR^*\right)\mathrm{d}x + 2\kappa\int_{-\infty}^{\infty}\left(q^*R + qR^*\right)\mathrm{d}x\right] \tag{12}$$

Finally, the change of the soliton velocity is given by [12]

$$v = -2a\kappa + \frac{\varepsilon}{E}\int_{-\infty}^{\infty}x\left(q^*R + qR^*\right)\mathrm{d}x \tag{13}$$

3. Soliton perturbation theory

This section will study the adiabatic dynamics of optical soliton in the presence of perturbation terms. The adiabatic dynamics of the soliton energy and frequency will be obtained. Also, the change of soliton velocity will be determined. It needs to be noted that the case of Kerr law, power law, parabolic law as well as the dual-power law has already been studied in 2008 [12].

3.1. Perturbation terms

In this paper, the following perturbation terms that are considered are all exhaustively studied in the context of fiber optics and optical solitons [12],

$$R = \delta |q|^{2m} q + \alpha q_x + \beta q_{xx} - \gamma q_{xxx} + \lambda\left(|q|^{2m} q\right)_x + \theta\left(|q|^{2m}\right)_x q +$$

$$+ \rho |q_x|^{2m} q - i\xi (q^2 q_x^*)_x - i\eta q_x^2 q^* - i\zeta q^* (q^2)_{xx} - i\mu\left(|q|^{2m}\right)_x q +$$

$$- i\chi q_{xxxx} - i\psi q_{xxxxxx} + (\sigma_1 q + \sigma_2 q_x)\int_{-\infty}^{x} |q|^{2m} \, \mathrm{d}s \tag{14}$$

In (14), δ is the coefficient of nonlinear damping or multi-photon absorption [12]. Also, α is the intermodal dispersion, β is the coefficient of filters and λ is the self--steepening coefficient to avoid the formation of shock waves while θ is the nonlinear dispersion coefficient [12]. Moreover, μ is the coefficient of Raman scattering [20] and therefore ρ represents the coefficient of nonlinear dissipation induced by Raman scattering [20]. The coefficients of ξ, η and ζ arise due to quasi-solitons [12]. The integro-differential perturbation terms with σ_1 and σ_2 are due to saturable amplifiers [12]. The higher order dispersion terms are given by γ, χ and ψ which arises whenever the group velocity dispersion is close to zero.

The parameter m, in (14), represents full nonlinearity, where the intensity terms are generalized to an arbitrary exponent which is typically a positive integer [5–8]. Usually, $m = 0$, 1 or 2. For example, in the case of multi-photon absorption, if $m = 0$, δ represents the coefficient of linear attenuation. If, however, $m = 1$, δ represents the two-photon absorption (or a nonlinear gain if $\delta > 0$). Again, if $m = 2$, δ introduces a higher order correction (saturation or loss) to the nonlinear amplification or absorption [12].

Thus, the perturbed NLSE, with log law nonlinearity that is going to be studied in this paper is therefore given by

$$\begin{aligned} iq_t + aq_{xx} + bq \log|q|^2 = i\varepsilon\Big[& \delta |q|^{2m} q + \alpha q_x + \beta q_{xx} - \gamma q_{xxx} + \lambda\left(|q|^{2m} q\right)_x + \\ & + \theta\left(|q|^{2m}\right)_x q + \rho |q_x|^{2m} q - i\xi (q^2 q_x^*)_x - i\eta q_x^2 q^* + \\ & - i\zeta q^* (q^2)_{xx} - i\mu\left(|q|^{2m}\right)_x q - i\chi q_{xxxx} - i\psi q_{xxxxxx} + \\ & + (\sigma_1 q + \sigma_2 q_x)\int_{-\infty}^{x} |q|^{2m} \, \mathrm{d}s \Big] \end{aligned} \tag{15}$$

It needs to be noted that the exact solution of the perturbed NLSE with log law nonlinearity in the presence of time-dependent coefficients was obtained in 2010 [3]. In addition, exact solutions of the perturbed NLSE with Kerr law nonlinearity were also obtained in 2010 and 2011 by the traveling wave method, trigonometric series method and other methods [23–25]. In this paper, however, the study will stay focused on the adiabatic parameter dynamics of Gaussons in the presence of such perturbation terms.

3.2. Adiabatic parameter dynamics

Using relations (10) and (12), the adiabatic dynamics of the energy and frequency of the Gaussons, for the perturbation terms in (14) is given by

$$\frac{\mathrm{d}E}{\mathrm{d}t} = 2\varepsilon A^{2m+2}\sqrt{\frac{2a}{b}}\times$$
$$\times\left[\frac{\delta\sqrt{\pi}}{\sqrt{2m+2}} + \rho\sum_{r=0}^{m}\binom{m}{r}\left(\frac{2b}{a}\right)^{m-r}\frac{\kappa^{2r}}{(2m+2)^{\frac{2m-2r+1}{2}}}\Gamma\left(\frac{2m-2r+1}{2}\right)+\right.$$
$$\left.+\frac{\sigma_1\pi}{4\sqrt{m}} + \frac{\sigma_2}{2}\sqrt{\frac{\pi b}{12am}} - \frac{\beta\sqrt{\pi}}{2\sqrt{2}\,aA^{2m}}\left(2a\kappa^2+b\right)\right] \tag{16}$$

$$\frac{\mathrm{d}\kappa}{\mathrm{d}t} = \varepsilon\left[\frac{\mu m b A^{2m}}{a(m+1)^{3/2}} + \frac{\sigma_2\kappa A^{2m}\sqrt{b}}{\sqrt{6ma}} - \beta\kappa^3\right] \tag{17}$$

while the velocity change from (13) is

$$v = -2a\kappa - \varepsilon\sqrt{2}\left\{\alpha + \frac{3\gamma}{2\sqrt{2}\,a}\left(2a\kappa^2+b\right) + \frac{\kappa A^2}{2}(\xi-\eta-4\zeta)+\right.$$
$$\left.+\frac{A^{2m}}{\sqrt{2}\,(m+1)^{3/2}}\left[2m\theta+(2m+1)\lambda\right] - \frac{A^{2m}}{4\sqrt{6m}}\left(2\sigma_1-\sqrt{6\pi}\,\sigma_2\right)\right\} \tag{18}$$

In the absence of Raman scattering term, namely when $\mu = 0$, the dynamical system given by (16) and (17) has a fixed point $(\bar{A}, \bar{\kappa})$, where

$$\bar{A} = \left\{\frac{2b\beta\sqrt{3am(m+1)}}{a\left[4\delta\sqrt{3am} + \sigma_1\sqrt{6a\pi(m+1)} + \sigma_2\sqrt{2b(m+1)}\right]}\right\}^{\frac{1}{2m}} \tag{19}$$

and

$$\bar{\kappa} = 0 \tag{20}$$

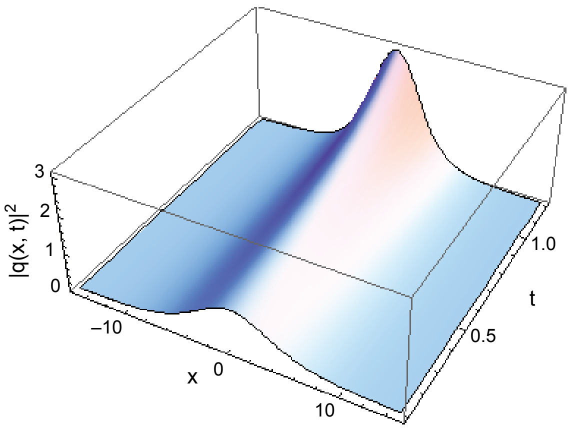

Fig. 1. Profile of a log law soliton with perturbation terms.

This shows that Gaussons, in the presence of these perturbation terms, will travel through the optical fiber with a fixed amplitude given by (19) and zero frequency shift as given by (20). This fixed point is a sink [12]. Thus, optical solitons travel down the fiber, after their amplitude and frequency get locked to these fixed values. This is known as *optical soliton cooling* [4]. In other words, Gaussons can start with a wide range of initial amplitudes and frequency and emerge at the end of the fiber with a constant amplitude, given by (19), and a constant velocity, given by (18), for $\kappa = 0$. In this context, it needs to be noted that this is not a video-pulse.

Figure 1 shows the profile of an optical soliton in the presence of some of the perturbation terms. Here $\varepsilon = 0.1$, $m = 1$, $\alpha = \beta = \sigma_1 = \sigma_2 = 1$, while $\rho = \xi = \eta = \zeta = 0$, and $\gamma = \theta = \lambda = 1$.

Figure 2 illustrates the idea of soliton cooling. The solitons start with an initial amplitude $A = 1.0$ and an initial frequency $\kappa = 0$. The soliton then eventually propagates and emerges with a fixed value of the amplitude $\bar{A} = 7.99$, as seen in Fig. 2. This fixed value of the amplitude is also related to the fixed intensity of the pulse.

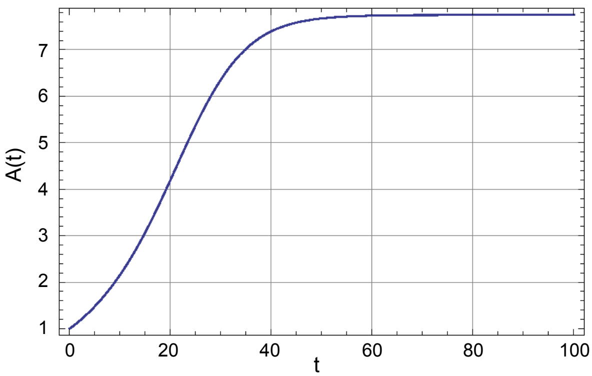

Fig. 2. Variation of the amplitude of the Gausson with perturbation terms.

The shape of this graph resembles the case of soliton cooling with Kerr law nonlinearity that was obtained earlier [1].

4. Conclusions

This paper studies the dynamics of optical solitons with log law nonlinearity in the presence of perturbation terms. There are several perturbation terms that are taken into account. The adiabatic dynamics of optical soliton parameters are obtained as well as the change in the soliton velocity. It was thus proved that the solitons will travel down the fiber with a fixed amplitude and frequency that is known as optical soliton cooling.

In the future these results will be extended. The quasi-stationary solitons will be obtained in the presence of these perturbation terms with the aid of multiple scales that will be developed. The quasi-particle theory of optical soliton interaction, with log law nonlinearity, will also be established. Those results will be reported in the future.

Acknowledgments – The research work of the second author (DM) was supported by the Serbian Ministry of Education and Science (Project III44006).

The third, fourth and fifth authors (TH, OMA and AB) would like to recognize and thankfully appreciate the support from King Saud University (KSU-VPP-117).

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Received June 2, 2011
in revised form October 31, 2011