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COMPARISON OF INCOME DISTRIBUTION FOR TWO DIFFERENT SOCIAL GROUPS

In the paper the analysis of income distribution are presented. We consider two social groups from the Tax Revenue Office from a district of Wrocław (Wroclaw-Krzyki Tax Revenue Office) in Poland. We fit Pareto distribution to the income distribution. The result of Kolmogorov-Smirnov testing is positive. In the last part of the paper we show that the difference between considered social groups is substantial.

INTRODUCTION

In the paper we propose to apply conditional distribution of incomes. Analysis of income distributions is the problem considered in many papers e.g. (Kot 1999; Luszniewicz 1982). Most of them consider all incomes in some groups and the aim is fitting the unconditional distribution. In many applications (e.g. discrimination models applied to loan problem in banks), it is enough to have conditional distribution only for some incomes greater than some sums of money (persons with less incomes are rejected). On the other hand the problem of fitting is very difficult because of outliers (small number of very big incomes). Therefore the problem of fitting conditional distribution subject to the sums of money less than some level is easier than the fitting of unconditional distribution.

In the paper we present empirical data analysis of personal incomes for two social groups in 1998. The first of them is the group of people who submit their tax declaration (PIT-30), the second is those of single parents.

We decided to examine the differences in forms of conditional distributions because of the problem with outliers. In both cases we consider the interval between 20,000 to 45,000 zlotys (approximately 5,000–11,000 EURO).

The two groups comprise people who received considerable loans (for example for cars and apartments). That is why it is very important to make a practical survey. As we show below, the attachment to a social group could be essential in a

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comparison of income distribution. Theoretical foundations of such research are presented by Dagum (1980), Shorrocks (1982) and Lehmann (1955).

In part one we present data concerning the two analysed groups. We also present hazard-rate function and conditional empirical distributions under different conditions. In our choice of conditional distributions for the subject of analysis we have been motivated by their smooth shape, as well as the fact that the good fitting is relatively easy.

In the second part we introduce the proposition to fit Pareto distribution to the analysed data. In both cases, using the standard numerical method, we obtain the optimal value of the unknown parameter. In this instance the Kolmogorov-Smirnov test gives a positive result.

In the third part we test the hypothesis about the economic equivalence of the two considered groups. As a result, we conclude that the difference is statistically substantial.

1. DATA PRESENTATION

Our data comes from the Tax Revenue Office from a district of Wroclaw (Wrocław-Krzyki Tax Revenue Office). These are incomes of 2,647 single parents (PIT-34) and incomes of 16,384 people who submit their tax declaration (PIT-30). As was mentioned above we are interested in the conditional distribution of incomes of the two considered groups. Most of bank customers have incomes on a level of between 20,000 and 45,000 zlotys. In Figures 1 and 2 we present data. In Figure 1 we present the empirical distribution of incomes for PIT-34 group.

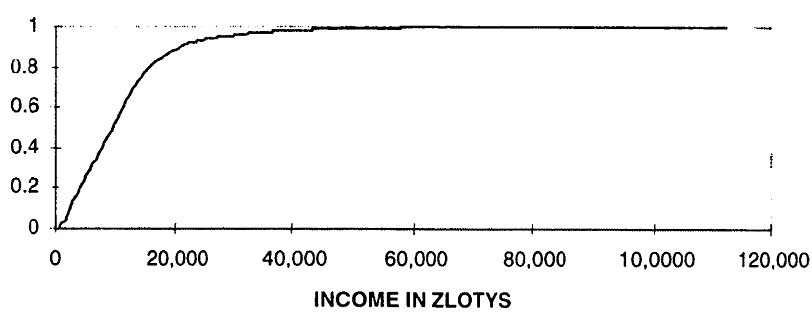

Figure 1: Empirical income distribution for PIT-34
Source: Authors' own.

In Figure 2 we present empirical distribution for PIT-30 group (cut at the level 120,000).

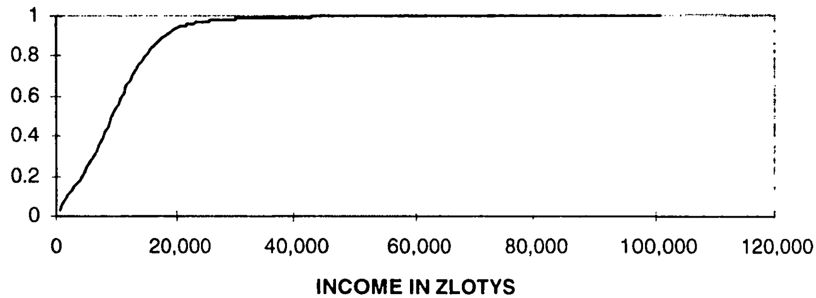

Figure 2: Empirical income distribution for PIT-30
Source: Authors' own.

In Table 1 we present basic parameters of incomes.

Table 1
Basic parameters of incomes

	PIT-30	PIT-34
Median	9134.29	9224.31
Mean	9970.18	11055.41
St. dev.	7625.63	9731.65
Q_1	5068.61	4787.29
Q_3	13389.55	14058.13
$\frac{Q_3 - Q_1}{2}$	4160.47	4635.42
$V_s = \frac{s}{\bar{x}}$	0.7648	0.88
n	16384	2647
min	0	0
max	319962.24	111925.26

Source: Authors' own.

Below we present the analysis of hazard rate (in actuarial mathematics – force of mortality) (Bowers et al. 1997) for our variables. By definition the hazard rate is given by:

$$P(x < X < x + \Delta x \,/\, X > x) = \frac{F_X(x+\Delta x) - F_X(x)}{1 - F_X(x)} \cong \frac{f_X(x)\Delta x}{1 - F_X(x)}$$

where $F_X(\cdot)$ is the distribution function of X, and $f_X(\cdot)$ is density function of X.

For the distance $\Delta x = 500$ (zlotys) we can get a hazard rate function for PIT-34 group (figure 3).

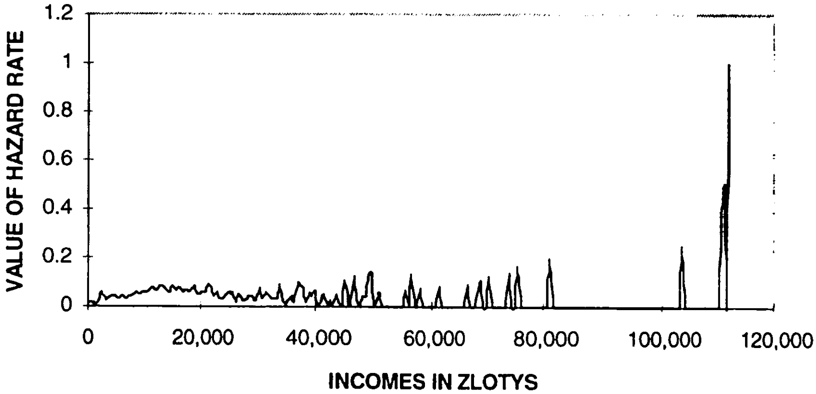

Figure 3. Empirical hazard rate for PIT-34
Source: Authors' own.

As we can see from Figure 3 a hazard rate is relatively stable only for incomes less than approximately 50,000 zlotys. For bigger ones we can see hyperbolic growth and zeros (for example between 80,000 and 100,000 zlotys).

For the interval from 20,000 to 45,000, we observe the values of hazard rate between 0 and 0.2.

2. CONDITIONAL DISTRIBUTION

In Figure 4 we present empirical conditional distributions for different conditions (for example $X > 20000$, $X > 30000$ *etc.*).

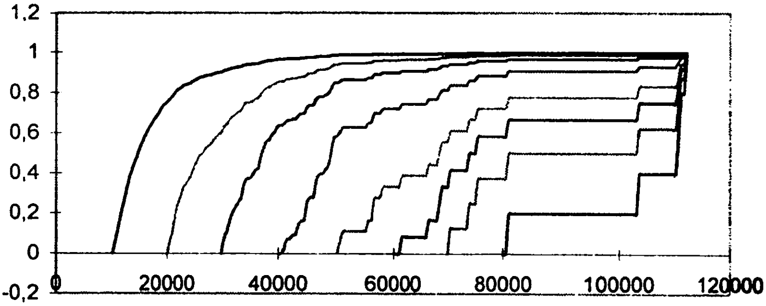

Figure 4: Conditional distribution
Source: Author's own.

In the paper we consider (as an example) the conditional distribution under condition $20000 < X < 45000$. In that case the empirical distributions are presented in Figure 5 (PIT-30) and Figure 6 (PIT-34).

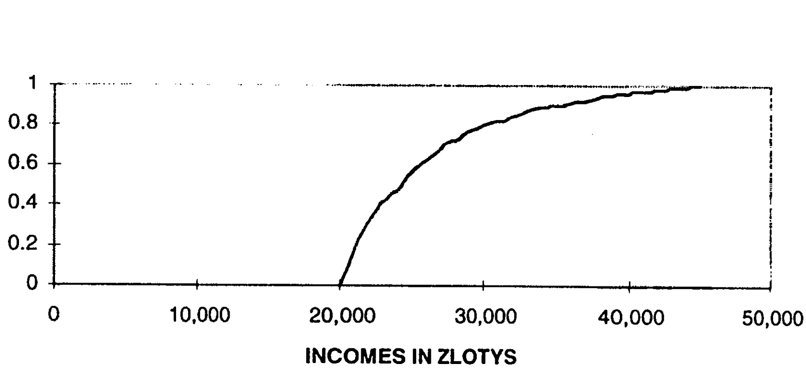

Figure 5: Conditional income distribution for PIT-30
Source: Authors' own.

4. ECONOMIC EQUIVALENCE

In the last part of the article we use the two sample Wilcoxon tests (Lehmann 1955; Serfling 1980) to verify the hypothesis:

$$H_0:\ P(X \le Y) = \frac{1}{2} \text{ versus } H_1:\ P(X \le Y) \ne \frac{1}{2}$$

where X is the income from PIT-30 group and Y is the income from PIT-34 group.

We use the asymptotic distribution of the statistics (it can be proved that it is U-statistic).

$$W = \frac{1}{n_1 n_2}\sum_{i,j} I(X_i < Y_j), i = 1,\dots,n_1, j = 1,\dots,n_2$$

The statistic W has asymptotically normal distribution with parameters :

$$E(W) = \frac{1}{2},\quad V(W) = \frac{1}{12}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)$$

under H_0:

$$W \sim N\left(\frac{1}{2}, \sqrt{\frac{1}{12}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}\right)$$

We apply this test to our conditional distribution (under condition $20000 < X < 45000$).

In our case, empirical value of W is equal :

$$W_e = \frac{134086}{236280} = 0.567$$

$$n_1 = 895, n_2 = 264, \sum_{i,j} I(X_i < Y_j) = 134{,}086$$

In our case under H_0 the statistic W has normal distribution with mean equal $\frac{1}{2}$ and standard deviation equal $\sqrt{\frac{1}{12}\left(\frac{1}{895} + \frac{1}{264}\right)} = 0.02$.

Therefore, under H_0 random variable:

$$W© = \frac{W - 0.5}{0.02}$$

has standard normal distribution.

The empirical value of W' is equal $\frac{0.567-0.5}{0.02}=3.35$.

If we take any usually applied significance level (e.g. $\alpha = 0{,}05$ or $\alpha = 0{,}01$) we should reject H_0.

CONCLUSIONS

In the paper we considered two social groups. As was mentioned above there is a significant difference between them in spite of considering conditional distribution. Therefore the presented results show that the level of income couldn't be the only criterion of classifying the person e.g. in loan problem in banks. We also showed that the distribution of incomes also depends on social groups, even in the case of equivalence of interval of incomes. So, the incomes should be researched in detail. As we showed above the application of conditional distribution, hazard rate (the measure of inequality) and Wilcoxon test for two samples could be very useful.

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