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ECONOMETRIC APPROACH OF THE ESTIMATION OF THE SINGLE MUNDELL-FLEMING EQUATIONS

1. Introduction

The theory of the exchange rate determination is intensively investigated over the last 10 years in the Slovak Republic, since the republic has turned to be independent and small open economy. During these years the exchange rate was under fixed regime, which has changed in 1998. Many papers reveal that the complexity of a small open economy is such that it is difficult to build theoretical models which are simultaneously general, comprehensible and interesting. The results of the models are usually biased because of insufficient database and non functioned economic relationships with transformed countries.

This paper is showing the theoretical background how to investigate the strategies of targeting, respectively money, the exchange rate, the nominal interest rate, the foreign uncovered interest differential and so on. Specially we will investigate the changes of the national income given changes in the exchange rate and the change of the foreign currency reserve given changes in the exchange rate.

The reasons are: the income in Slovakia during the year 2004 was rising about 5,5%, the inflow of foreign capital to the Slovakia during the last two years strongly influence the foreign reserve. In spite of this the trade balance is negative (because of the high price of oil) and the Slovak currency appreciate. For all that the first two movements are positive the central bank of Slovakia is trying to influence the Slovak currency by the various interventions. But this pressure on deprecation of the Slovak currency has only short-time effect and the value of the Slovak currency is returning to the state which satisfy the given economic trend.

Because we are involving to the research VEGA project 1/1255/04: „Analysis of Development of Economic Relationships with Abroad in Linkage to the Mone-

tary and Fiscal policy" we will describe the basic Mundell-Fleming model, which has been studied and estimated by various authors. This model could be used to answer a lot of academic questions.

2. The Model

The changes in the exchange rate use to influence the development of the basic macroeconomic indicators. The influence of exchange rate is possible to analyse with the enhanced IS-LM model which incorporates the international transactions. Such model was formulated in years 18960-1968 by Mundell and Fleming. One of the modification of the Mundell-Fleming model [Visser 2000] is following:

$$Y = Z(Y,i,T) + G + Ex(e) - \mathrm{Im}(Y,e), \tag{1}$$

$$Md(Y,i) = m(Z_{-1} + H + X), \tag{2}$$

$$X = Ex(e) - \mathrm{Im}(Y,e) + K(Y,i), \tag{3}$$

where: Y – national income,
Z – private expenditure,
i – interest rate,
T – taxes,
G – government expenditure,
Ex – receipts on the current account of the balance of payments (export),
e – rate of exchange,
Im – payments on the current account of the balance of payments (import),
Md – money demand,
m – money multiplier,
C_{-1} – volume of base money supply one period back,
H – ΔD the volume of open market transactions (purchases) = the change in the domestic credit supply D,
X – ΔV the change in the foreign exchange reserves V,
$K(Y, i)$ – net non-bank capital inflow.

The system of equations (1)-(3) defines:

- the IS curve, representing the equilibrium conditions for the real side of the economy, or investment savings (1),
- the LM curve, representing the equilibrium conditions for the monetary side of economy, or liquidity preference – money supply equilibrium (2),
- the EE curve, representing the external equilibrium conditions for the market for foreign exchange, or balance of international payments (3).

The macroeconomic indicators are following functions: the private expenditure is the function of income and interest rate; the export is a function of the exchange rate; the import is the function of national income and exchange rate; the money demand and the net non-banking capital flow respectively are the functions of na-

tional income and interest rate. All this equations is possible to rearrange to the simple econometrics equations with the random variable and to estimate their parameters structurally.

We will estimate only simple models of the equation (1) by means of OLS for consumption, export and import of quarterly data for Slovak Republic, with the assumption that the exchange rate does not change, or with static expectations for the exchange rate. The estimation is given for the short time series, starting on the first quarter 1993 till the last quarter 2003.

3. The Impact of Exchange Rate Changes on National Income

With the help of the system equations (1)-(3) it is possible to study the impact of fiscal policy (G), monetary policy (H) and exchange rate policy (e) on national income, the rate of interest and the balance of payments respectively through the account of foreign exchange reserves. In order to study the change in the variables, it is usefull to build up total differentials of the mentioned equations.

Rearranging the terms of total diferential of the system equations (1)-(3) with the dependent variables on the left side and the instrument variables on the right side, the system of equations given in matrix form is:

$$\begin{pmatrix} (1-C_Y+\mathrm{Im}_Y) & -C_i & 0 \\ Md_Y & Md_i & -m \\ (K_Y-\mathrm{Im}_Y) & K_i & -1 \end{pmatrix} \begin{pmatrix} dY \\ di \\ dX \end{pmatrix} = \begin{pmatrix} 1 & 0 & (Ex_e-\mathrm{Im}_e) \\ 0 & m & 0 \\ 0 & 0 & (\mathrm{Im}_e-Ex_e) \end{pmatrix} \begin{pmatrix} dG \\ dH \\ de \end{pmatrix} \quad (4)$$

where the subscripts denote partial derivatives. From the model it goes out, that the variables *G*, *H* and *e* are independent variables and represent the instruments of macroeconomic policy: the fiscal, monetary and exchange rate policy respectively. The macroeconomic variables, national income (*Y*), interest rate (*i*) and foreign exchange reserves (*X*) are the dependent variables.

With an eye to manipulating the system of equations and finding more exact outcomes for the results of the various policy measures, it is convenient to apply Cramer's rule. Calculate the Jacobian determinant, that is, the determinant of the matrix of the first derivatives in (4):

$$J = -\left(1-\overset{(+)}{C_Y}+\overset{(+)}{\mathrm{Im}_Y}\right)\overset{(-)}{Md_i} + \overset{(-)(+)}{C_i}\,m\left(\overset{(?)}{K_Y}-\overset{(+)}{\mathrm{Im}_Y}\right)+\overset{(+)(+)}{m\,K_i}\left(1-\overset{(+)}{C_Y}+\overset{(+)}{\mathrm{Im}_Y}\right)-\overset{(-)\ (+)}{C_i\,Md_Y}. \quad (5)$$

Generally the determinant will be positive. Determinant could be also negative if values C_Y (strong positive reaction of investment to changes in income) or K_Y (strong reaction of net capital imports to income changes) are very large. This two cases used to occure unfrequently, therefore will be neglected.

This presentation is focused on the analysis of the exchange rate policy on the national income. The system (4) is then defined for values $dG = 0$ and $dH = 0$.

The impact of exchange rate changes on national income is aimed if we replace the first column of the Jacobian matrix by the third column of the matrix from the right-hand side of formula (4). Then we calculate the determinant of this modified matrix which is

$$\begin{vmatrix} (Ex_e - \mathrm{Im}_e)de & -C_i & 0 \\ 0 & Md_i & -m \\ (\mathrm{Im}_e - Ex_e)de & K_i & -1 \end{vmatrix} =$$

$$= -Md_i(Ex_e - \mathrm{Im}_e)de + mC_i(\mathrm{Im}_e - Ex_e)de + mK_i(Ex_e - \mathrm{Im}_e)de =$$

$$= (-Md_i - mC_i + mK_i)(Ex_e - \mathrm{Im}_e)de .$$

Solution for *dY/de* then is

$$\frac{dY}{de} = \frac{\left(\overset{(-)}{-Md_i} \overset{(+)(-)}{-\,m\,C_i} \overset{(+)(+)}{+\,m\,K_i}\right)\left(\overset{(+)}{Ex_e} \overset{(-)}{-\,\mathrm{Im}_e}\right)}{J} . \qquad (6)$$

Equation (6) shows that elasticity *dY/de* will be positive. Because $J > 0$, the nominator has to be positive. It means, if the *e* will rise, national income will increase provided $Ex_e > \mathrm{Im}_e$, so difference of export and import is positive. The second term of the nominator must be also positive, because rising interest rate give rise to the net non-bank capital inflow, the consumption is decreasing $K_i \rangle C_i$ and money demand is also decreasing, so $-Md_i + m(K_i - C_i) > 0$ which imply that $m(K_i - C_i) \rangle Md_i$, since m is turnover of money and is positive.

4. The Impact of Exchange Rate Changes on Foreign Currency Reserve

Discussion about the influence of change of exchange rate on the foreign currency reserve can be also realise using the modification of the Jacobian matrix. The third column of the Jakobian matrix will be replaced by the third column of the matrix of the right-hand side formula (4). Using Cramer's rule stand the following:

$$\begin{vmatrix} 1 - C_Y + \mathrm{Im}_Y & -C_i & (Ex_e - \mathrm{Im}_e)de \\ Md_Y & Md_i & 0 \\ K_Y - \mathrm{Im}_Y & K_i & (\mathrm{Im}_e - Ex_e)de \end{vmatrix} =$$

$$= (1 - C_Y + \mathrm{Im}_Y)Md_i(\mathrm{Im}_e - Ex_e)de + Md_Y K_i(Ex_e - \mathrm{Im}_e)de -$$

$$-Md_i(K_Y - \mathrm{Im}_Y)(Ex_e - \mathrm{Im}_e)de + C_i Md_Y(\mathrm{Im}_e - Ex_e)de =$$

$$= \left[-Md_i(1 - C_Y + \mathrm{Im}_Y) + Md_Y K_i - Md_i(K_Y - \mathrm{Im}_Y) - Md_Y C_i\right](Ex_e - \mathrm{Im}_e)de.$$

Solution for the *dX/de* then is:

$$\frac{dX}{de} = \frac{\left[-\overset{(-)}{Md_i}\left(1-\overset{(+)}{C_Y}+\overset{(+)}{\mathrm{Im}_Y}\right)+\overset{(+)}{Md_Y}\,\overset{(+)}{K_i}-\overset{(-)}{Md_i}\left(\overset{(?)}{K_Y}-\overset{(+)}{\mathrm{Im}_Y}\right)-\overset{(+)}{Md_Y}\,\overset{(-)}{C_i}\right]\left(\overset{(+)}{Ex_e}-\overset{(-)}{\mathrm{Im}_e}\right)}{J}$$

$$\frac{dX}{de} = \frac{\left[Md_Y\left(K_i-C_i\right)-Md_i\left(1-C_Y+\mathrm{Im}_Y+K_Y-\mathrm{Im}_Y\right)\right]\left(Ex_e-\mathrm{Im}_e\right)}{J}$$

or

$$\frac{dX}{de} = \frac{\left[Md_Y\left(K_i-C_i\right)-Md_i\left(1-C_Y+K_Y\right)\right]\left(Ex_e-\mathrm{Im}_e\right)}{J}. \qquad (7)$$

The sign of the elasticity *dX/de* depends form the sign of the numenator, because *J* is positive.

After investigation of the numerator of the formula (7) it is possible to observe, that this value is again positive. The value of fraction *dX/de* could be negative only if K_Y is strongly negative, or if the marginal propensity to import Im_Y is improbably high.

5. Estimation of the First Equation of Mundell-Fleming model

The previous study was done only academically because the whole model.

To see how the relationships amongst the variables of the equation (1) are working we will estimate models for consumption, export and import respectively. We will use quarterly time series starting at quarter one in 1998 and ending at quarter four in 2003, giving 24 observations.

The Fig. 1 shows the development of the GDP (in constant prices 1995) together with the consumption and interest rate in Slovak Republic during the period Q1:1998 till Q4: 2003. The Fig. 2 shows how the model (8) fits the data.

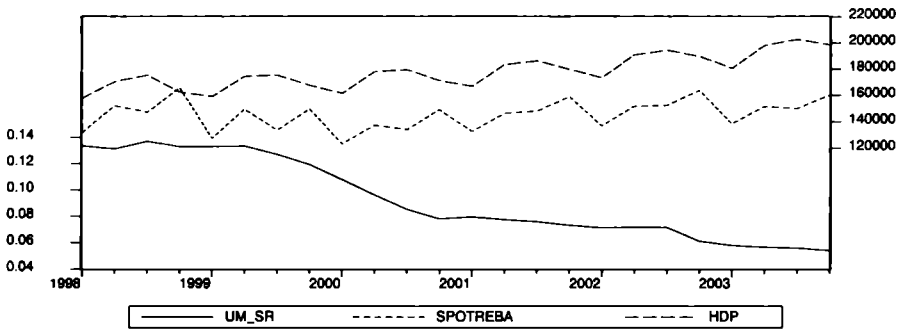

Fig. 1. Development of GDP(=Y), Consumption (=C) and Interest Rate (=i) in Slovak Republic, Q1:1998-Q4:2003

Estimation of the relationship in the logarithmic form (with the standard errors in parenthesis) is:

$$\log C_t = 0{,}980 \log Y_t + 1{,}362 i_t - 0{,}260 Q_1 - 0{,}107 Q_2 - 0{,}138 Q_3 + a_t - 0{,}895 a_{t-2} \quad (8)$$

(0,002) (0,123) (0,02) (0,02) (0,02) (0,08)

$R^2 = 0{,}92$ S.E = 0,03 D-W = 1,25 with in-sample error MAPE = 0,16 %.

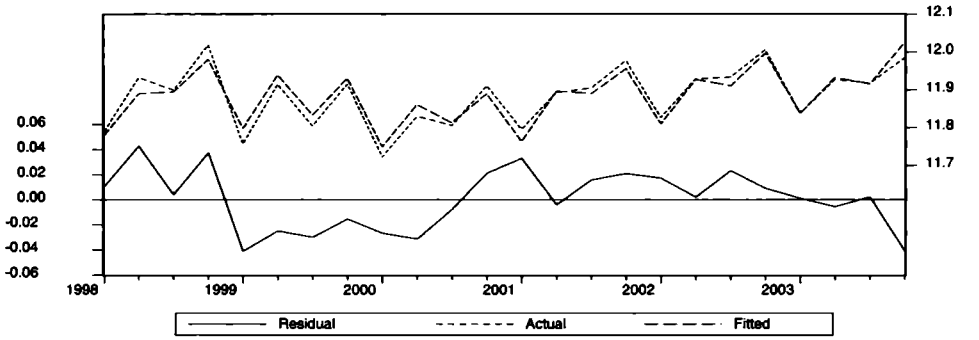

Fig. 2. Actual and predicted values of consumption under the model (8)

The Fig. 3 shows how export is related to the exchange rate EUR/SKK and Fig. 4 shows how the model (9) fits the data.

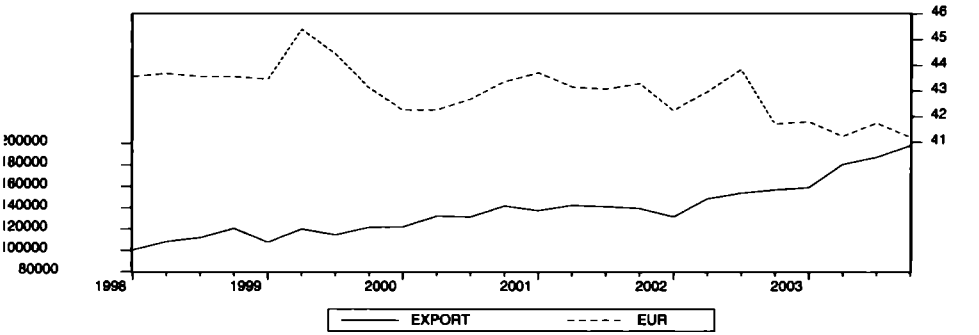

Fig. 3. Development of Export (=Ex) from Slovak Republic and EUR/SKK (=e): Q1:1998-Q4:2003

Estimation of the export in relation with the exchange rate EUR/SKK during the given period is:

$$\log Ex_t = 31{,}335 - 5{,}191 \log e_t \quad (9)$$

(4,3) (1,14)

$R^2 = 0{,}484$ S.E. = 0,13 D-W = 1,05 with in-sample error MAPE = 0,91% .

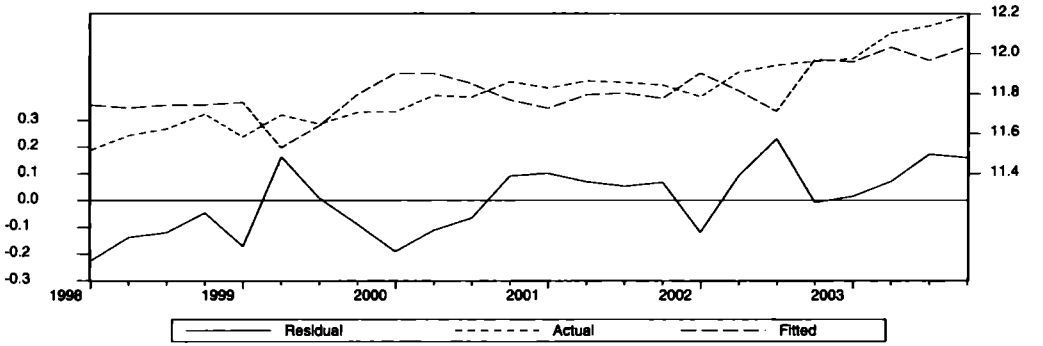

Fig. 4. Actual and predicted values of export under the model (9)

The last estimation is concerning the import in relationship with the GDP and interest rate, during the given period. Figure 5 shows the development of the GDP and Import to the slovak Republic during the given period. The Fig. 6 shows, how the data fits the model (10).

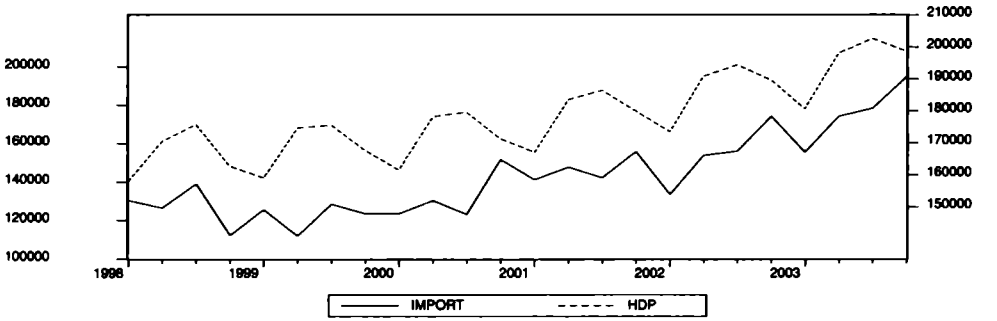

Fig. 5. Development of Import (=Im) to Slovak Republic and GDP (=Y), Q1:1998-Q4:2003

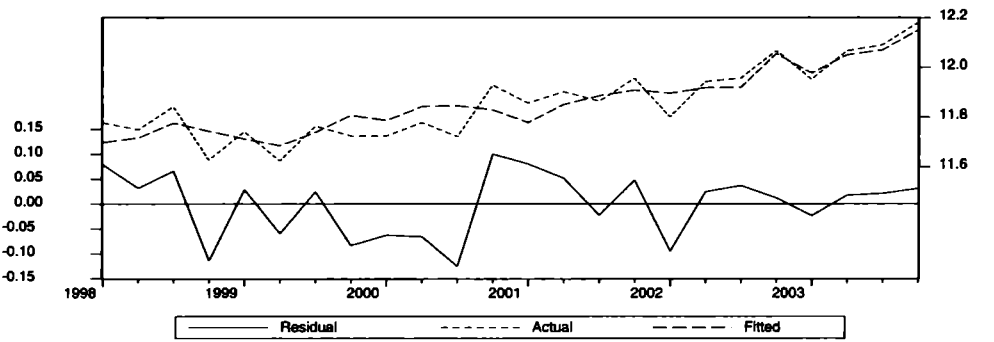

Fig. 6. Actual and predicted values of import under the model (10)

Estimation of the model is:

$$\begin{matrix} \log \mathrm{Im}_t = 1{,}538 \log Y_t & -1{,}780 \log e_t & -0{,}094 Q_2 & -0{,}085 Q_3 \\ (0{,}115) & (0{,}369) & (0{,}035) & (0{,}035) \end{matrix}, \quad (10)$$

$R^2 = 0{,}91$ S.E = 0,06 D-W =1,96, with the in-sample error MAPE = 0,46 %.

6. Conclusion

We have been concerned to examine the partial effects of the exchange rate policy within context of a simple model of a small economy which can allow for simultaneous variations in real income, product prices, asset prices and the exchange rate. We did it only theoreticaly because we do not have complete database concerning the problem at this time. At the end of the aricle, the three simple regression models for consumption, export and import respectively were estimated by means of OLS describing their development during the period Q1:1998 till Q4:2003 in the Slovak Republic and with the floating exchange rate EUR/SKK.

During the year 2004 the Slovak Republic has changed its tax policy, which attracted foreign capital to the country. We think that all these changes are a great chalange to investigate the macroeconomic situation at the country by means of econometric models.

Literature

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PODEJŚCIE EKONOMETRYCZNE W ESTYMACJI POJEDYNCZYCH RÓWNAŃ MODELU MUNDELLA-FLEMINGA

Streszczenie

Artykuł przedstawia podstawy teoretyczne rozszerzonego modelu Mundella-Fleminga. Autorki wskazują, jak na podstawie równań tego modelu można w sposób teoretyczny określić wpływ zmian kursu walutowego na dochód narodowy oraz na poziom rezerw walutowych. Następnie dokonują estymacji krzywej IS na podstawie danych ze Słowacji.