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Jumps in the Freight Rate Process in Container Shipping

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Abstract: As verified in our previous investigations (Gardoń, 2014), a weekly average net freight in container shipping may be modelled by means of the jump-diffusive process with homogeneous Poissonian jumps. So far, we have generated the relative jump size from the empirical distribution which is asymmetrical and does not seem to be one of typical distributions. Generally, upward jumps appear more often, whereas relative drops are more concentrated around their mean. In this paper, we fit to the jump data a mixture of two distributions, taking into account negative and positive jumps separately, obtaining some satisfactory results. The jump-diffusive model is mainly used for the evaluation of a derivative net premium, e.g. the European Call option for the net freight we mentioned in our previous papers (Gardoń, 2016). Without the knowledge of the underlying theoretical distribution of the relative jump size, the extremely time-consuming Monte Carlo simulations have to be conducted to this purpose. The knowledge of the theoretical jump-size distribution may lead to the analytical formula for the option premium, which will make the calculations faster and more exact.

Keywords: liner shipping, jump-diffusion, relative jump size distribution, freight rate call options

1. Introduction

Due to a popular measure of the industry concentration, namely the Herfindahl-Hirschman Index, equal to 7%, the global container shipping market is unconcentrated (see Alphaliner, 2012), thus, strongly competitive. The market share of the top 10 carriers is only about 60% and the psychology of many independent competitors affects the industry condition. In such an instance random models are a relevant tool for describing its behavior. A straight consequence of this situation is a huge price volatility which is an essential problem for the shippers and the carriers. For the sake of simplicity, the shippers may be identified as commodities owners and the carriers as vessels owners. For instance, on the main trade route from Southeast Asia to Europe the transportation prices have varied in the current millennium from below USD 2000 up to over USD 4000.

The aforementioned route from the Far East to Europe is the most important and the most competitive one in the business. As investigated by Gardoń (2014), the net freight process in the container shipping industry in the case of this route (but not only) can be modelled by a linear jump-diffusion driven by a standard Brownian motion and a homogeneous Poisson process. The most essential conditions for a proper application of the model, as a normal distribution of returns (called also ticks or relative process changes) except for jump times (see Figure 1), the independence of the returns from preceding process values guaranteeing the linearity and the exponential distribution of an *iid*-sequence of inter-jump times consistent with the homogeneity of the driving Poisson process, are fulfilled in the instance.

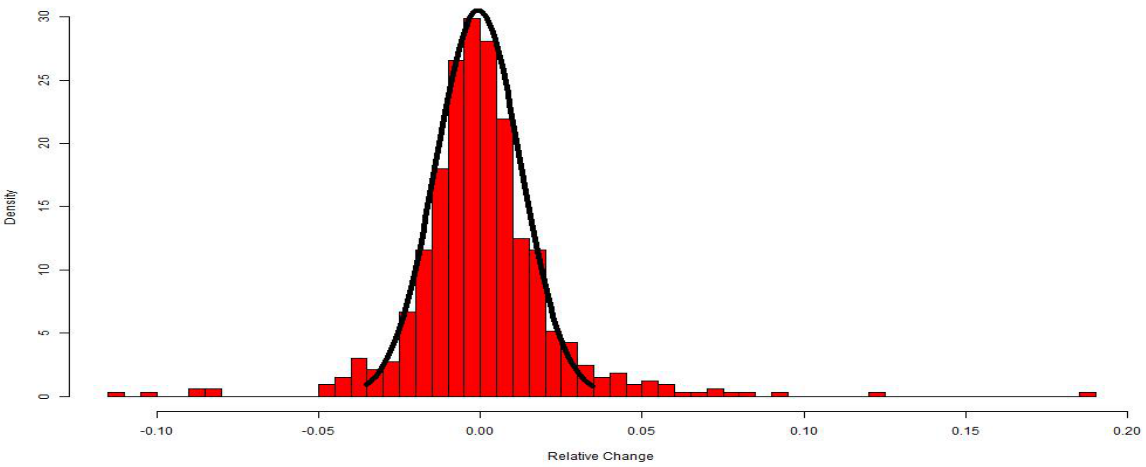

Figure 1. Empirical density of the relative average weekly net freight changes in the route from Southeast Asia to Europe versus the normal density

Source: own elaboration.

The model is defined by the following stochastic differential equation in the integral form:

$$X_t = X_{t_0} + \int_{t_0}^{t} aX_s ds + \int_{t_0^+}^{t} b\bar{X}_s \, dW_s + \int_{t_0^+}^{t} \bar{C}_s \bar{X}_s \, dN_s, \quad t > t_0, \tag{1}$$

where the modelled process X denotes the weekly average net freight (or rate, which is a transportation price consisting of basic ocean freight and different surcharges, e.g. fuel surcharge) per transported unit (*TEU* or *FFE* – the volume of a 20 or 40 feet long standard container) $\bar{X}_t = X_{t^-} = \lim_{s \nearrow t} X_s$, W is a standard Brownian motion (standardized Wiener process), N is a homogeneous Poisson process with intensity λ and both driving processes are said to be independent. Coefficients a, b and C ($\bar{C}_s$ means the left-hand side limit at s, as for X) are called the drift, the volatility and the relative jump size, respectively. Further, C is a right continuous process constant between the jump (stopping) times (Γ_i) of the driving Poisson process and its values on the consecutive interjump intervals are realizations of an independent identically distributed (*iid*) sequence of random variables ($\mathscr{C}_i$), which may be treated as independent copies of a certain pattern random variable $\mathscr{C}$ (corresponding to the jump size), independent from the both driving processes *W* and *N* as well (see Mancini, 2009). Additionally, the sequence (τ_n) of stopping times will represent times when the process is observed.

Generally, it is difficult or even impossible to find a closed analytical formula for the solution of the jump-diffusion stochastic differential equation. Therefore, usually numerical methods are used for the approximation of the solution (see e.g. Gardoń, 2004, 2006). Fortunately, the explicit formula for the solution of the linear equation (1) may be derived using the generalized Itô formula for semimartingales (see Protter, 1990):

$$X_t = X_{t_0} e^{\left(a-\frac{b^2}{2}\right)(t-t_0)+b(W_t-W_{t_0})} \prod_{i=N_{t_0}+1}^{N_t} \left(1+\bar{C}_{\Gamma_i}\right) , \quad t > t_0 \, . \tag{2}$$

The models for an underlying price process are widely used for option pricing. The knowledge of the distributions of all random parts may enable to find an analytical formula for this pricing. If they are unknown, the pricing is still possible but in a very time-consuming way based on the Monte Carlo simulations. Since the distributions of *W* and *N* are known the only issue is to find the distribution of $\mathscr{C}$. This is our motivation. In this article, we investigate the relative jump size distribution for the mentioned most crucial route from the Far East to Europe based on the real data from one of the leading carriers. Firstly, we present methods for parameter estimates. Then, looking at the empirical density histogram of the relative jump size, we choose candidates for its theoretical distribution. Further, we conduct necessary statistical goodness-of-fit tests for the candidates chosen. Finally, we discuss possible advantages and disadvantages when the theoretical distribution found were applied in comparison to the dealing with the empirical one.

2. Calibration of the Jump-Diffusive Model

The model used to be especially applied for the pricing of derivatives, e.g. options. The idea of freight options has been strongly considered in the current century (see e.g. Kou, 2002; Koekebakker, 2007; Nomikos at al., 2013), but the authors discussed usually exotic Asian options given $\mathscr{C}$ is gamma- or normally distributed. We focus on bilateral simple European Call options (see Gardoń, 2016). They are a basic example of derivatives. In our case they give a shipper the right for buying an underlying shipping service at the fixed time (expiry date) in the future for a fixed price (strike price). For this right a shipper must pay a price called the option premium.

There are several parameters necessary for the calibration of the model (2), namely the drift *a*, the volatility *b*, the Poissonian intensity λ and the relative jump size $\mathscr{C}$. Firstly, any continuous model must be discretized. A trajectory of the freight process *X* is observed at the stopping times (τ_n) and its relative changes are denoted by (Z_n):

$$Z_n = \frac{X_{\tau_n} - X_{\tau_{n-1}}}{X_{\tau_{n-1}}} = \frac{\Delta X_n}{X_{\tau_{n-1}}} = ln\frac{X_{\tau_n}}{X_{\tau_{n-1}}} , \quad n = 1, \dots, L , \tag{2}$$

where $\tau_0 = t_0$ and $\Delta X_n = X_{\tau_n} - X_{\tau_{n-1}}$ is the increment of the process *X* on the interval $(\tau_{n-1}, \tau_n]$. Further, the volatility *b* is the infinitesimal variance of the continuous part (Z_n^c) of the returns (Z_n), where $Z_n^c = a\Delta_n + b\Delta W_n$ and $\Delta_n = \tau_n - \tau_{n-1}$, that means the remained part after an extraction of jumps. Besides, this means that the standardized continuous part of the returns (Z_n^*) is a standard normally distributed *iid*-sequence:

$$Z_n^* = \frac{Z_n - a\Delta_n - \sum_{i=N_{\tau_{n-1}}+1}^{N_{\tau_n}} \bar{C}_{\Gamma_i}}{b\sqrt{\Delta_n}} = \frac{Z_n^c - a\Delta_n}{b\sqrt{\Delta_n}} \sim N(0,1), \qquad n = 1, \dots, L\,. \tag{4}$$

Thus, the volatility *b* may be estimated by means of the maximal likelihood method as the standard deviation of (Z_n^c):

$$\hat{b} = \sqrt{\frac{1}{L}\sum_{n=1}^{L}\frac{(Z_n^c)^2}{\Delta_n} - \left(\frac{1}{L}\sum_{n=1}^{L}\frac{Z_n^c}{\sqrt{\Delta_n}}\right)^2}\,, \tag{5}$$

Of course, before the estimation of the volatility b the jumps have to be recognized in the data. This follows by the so-called threshold method (see Mancini, 2009; Gardoń, 2011) with the threshold condition:

$$\frac{Z_n^2}{\hat{b}^2} > r(\Delta\tau_n), \qquad r(t) = \beta t^{1-\varepsilon} = 7.3576\mathrm{t}^{0.9}\,, \tag{6}$$

where *r* is the threshold with *ε* = 0.1 and *β* = 7.3576. If for a return the condition in (6) is valid, then such a tick is recognized as a jump. It is easy to notice that in the threshold condition for jump identification the knowledge of *b* is required, but conversely, since *b* is the volatility of the continuous part of the process *X*, then for its evaluation the exclusion of jumps is necessary, as well. As shown by Gardoń (2011), the problem may be overcome by means of an iterative procedure, where the volatility *b* and the jump returns are estimated step by step simultaneously.

It is worth mentioning here that a jump return in the data is not exactly a realization of $\mathscr{C}$. Such a return (see Equations (2), (3) and (4)) consists not only of the jump driven by a Poisson process *N*, but also includes a continuous freight process change driven by a Wiener process *W* and the drift *a*:

$$\frac{dX_t}{X_t}\Big|_{dN_t=1} = a\,dt + b\,dW_t + C_t|_{dN_t=1} \quad a.s. \qquad t > t_0.$$

In fact, realizations of $\mathscr{C}$ are represented by the difference $Z_n - Z_n^c$ on the subintervals where jumps occur, i.e. where $\Delta N_n > 0$. This leads to the following, say "continuity correction", in the jump data set:

$$(Z_n - Z_n^c)|_{\Delta N_n=1} = Z_n|_{\Delta N_n=1} - a\Delta\tau_n - b\Delta W_n \quad a.s., \qquad n = 1, \dots, L.$$

The "continuity correction" is random due to the Brownian increments ΔW_n and it must be generated artificially.

When the jumps are identified, then the Poissonian intensity *λ* can be estimated. If the process (2) is driven by the homogeneous Poisson process *N* with the intensity *λ*, then the interjump periods should create an exponentially distributed *iid*-sequence with the same parameter as the Poissonian intensity. Therefore, we chose an intensity which makes the exponential distribution fit best to the empirical interjump periods in the sense of the maximal *p*-value of the Kolmogorov--Smirnov goodness-of-fit test.

For the proper evaluation of the risk the so-called no-arbitrage assumption has to be taken into account (see Kou, 2002), which leads to the conclusion that the drift a must not be estimated from the data but set to be equal to:

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$$a = \rho - \lambda\, E\mathcal{C}, \tag{7}$$

where ρ is the LIBOR per time unit for 1 year USD contracts and E is the expectation operator. The expectation $E\mathcal{C}$ may be estimated, obviously, as the sample average from the set of relative jump values. Although, if the theoretical distribution of the relative jump size $\mathcal{C}$ were known, then the expectation could be calculated directly from the relevant formula.

3. Theoretical Distribution of the Relative Jump Size

As mentioned in the previous sections, we focus on the most interesting trade route in the container shipping, namely from Southeast Asia to Europe, headhaul (more profit-yielding) direction. We have for our disposal the data from one of the market leaders. The data set consists of 657 weekly average net freight values from the over 12 years long time period, from January 2, 2000 to August 5, 2012. This produces L = 656 weekly returns of the freight process. As already mentioned due to Gardoń (2014), the jump-diffusive model (2) fits well to the empirical data in this case. After 5 iterations of the procedure described by Gardoń (2011), 53 jump occurrences were recognized by the threshold condition (6) with the average jump size $\widehat{\mathrm{E}\mathcal{C}} = 0.0117$ and the volatility $\hat{b} = 0.013$. The empirical distribution of relative jump sizes is shown in Figure 2. On November 19, 2012 corresponding to the data set, the risk-free rate was equal to 0.86% p.a. due to the LIBOR for 1 year USD contracts, which implies ρ = 0.0165% per time unit. The time unit is a week (7 days), a common one in the entire industry. Poissonian intensity per time unit derived from the empirical distribution of the interjump periods is equal to $\hat{\lambda} = 0.1923$. Summing up, the last parameter, namely the drift corresponding to the no-arbitrage assumption, is $\hat{a} = -0.0021$ in this instance.

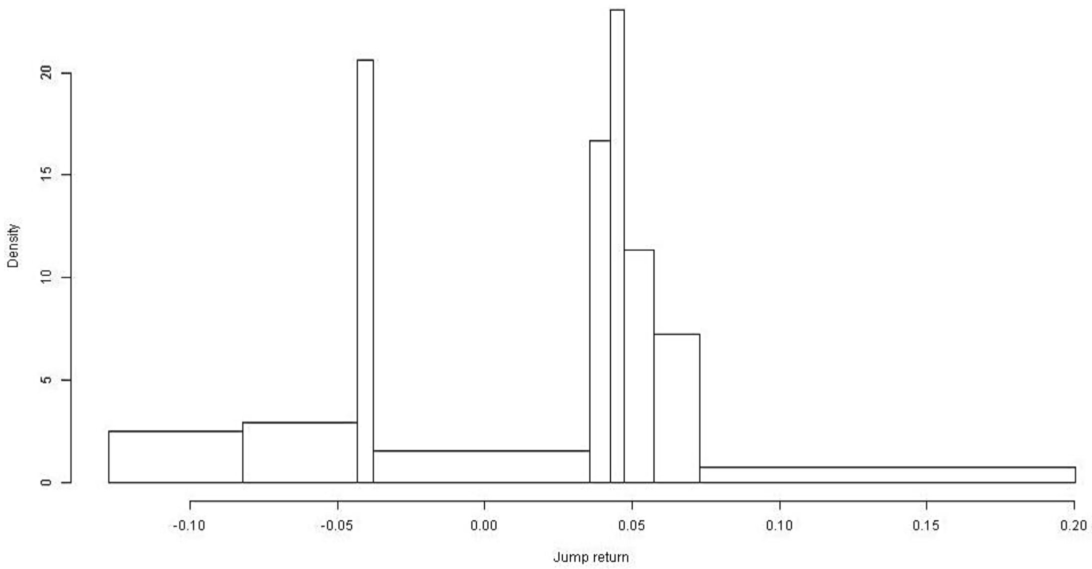

Figure 2. The empirical distribution of the relative jump size for the freight from Southeast Asia to Europe head haul direction based on 53 jumps recognized

Source: own elaboration.

The relative jump sizes may be generated directly from the empirical distribution, e.g. by means of the bootstrap method. However, another approach could be applied. Firstly, a theoretical distribution may be recognized, fitted to the empirical data and statistically tested. Then the relative jump sizes can be generated by a pseudo-random number generator from the distribution fitted. The former approach seems to be simpler and closer connected to the real data, but the latter may be more efficient. As already mentioned, the main aim of modelling the freight process is the option pricing. Without the knowledge of the underlying theoretical distribution of the relative jump size process *C*, the only possibility for the evaluation of an option net premium is the Monte Carlo simulation. It consists in multiple (hundreds thousands or even millions) calculations of the possible process trajectories which is a time consuming operation. On the contrary, if the theoretical distribution were known, a researcher could find a closed analytical formula for the net premium and, using it, calculate the net premium directly and quickly in one step. This is our motivation for the investigation of this second approach.

As visible in Figure 2, the empirical distribution of the relative jump size $\mathscr{C}$ is bimodal and asymmetrical, so already at the first sight it has been clear that no typical distribution can fit to the entire data set. The positive jumps appeared much more often; they are usually close to 0 but with large outliers up to 0.2, whereas the negative jumps are less volatile and more symmetrical.

Besides, there exists no objective limit for positive jumps and the negative ones are bound by the number 1. Hence, we split the data set into positive and negative jumps and dealt with them separately. Formally, this means that the theoretical distribution fitted must be a mixture of two typical distributions. Let us denote by $C^+ = \max(C, 0)$ a.s. – the positive relative jump size process, by $C^- = -\min(C, 0)$ a.s. – the reflected negative relative jump size process (for the sake of simplicity negative jumps are reflected to the right-hand side of the real line) and the corresponding pattern random variables by $\mathscr{C}^+$ and $\mathscr{C}^-$, respectively. Thus $C = C^+ - C^-$a.s. This leads to the mixture of distributions:

$$f_{\mathscr{C}}(u) = \alpha f_{\mathscr{C}^+}(u) + (1-\alpha)\, f_{-\mathscr{C}^-}(u)\,,\ \alpha \in (0,1)\,,$$

where $f_{\mathscr{C}}$, $f_{\mathscr{C}^+}$ and $f_{\mathscr{C}^-}$ are the densities of $\mathscr{C}$ (general relative jump size), $\mathscr{C}^+$ (positive relative jump size) and $-\mathscr{C}^-$ (negative relative jump size), respectively. Since in the data set there is about 50% more positive jumps than the negative ones (exactly 32 to 21), there has been no reason to reject the parametric hypothesis that $\alpha = 0.6$ against the alternative $\alpha \neq 0.6$, with *p*-value equal to 95.5%. Eventually, we obtained the following coefficients of the distributions mixture:

$$f_{\mathscr{C}}(u) = 0.6\, f_{\mathscr{C}^+}(u) + 0.4\, f_{-\mathscr{C}^-}(u)\,.$$

As already mentioned, there were 53 jump returns in the data set but after the division only 32 positive and 21 negative jump returns. However, the test statistic from the universal Kolmogorov--Smirnov goodness-of-fit test is only asymptotically Kolmogorovian distributed and it requires at least 50 observations for the proper evaluation of the *p*-value. Fortunately, as we mentioned in the previous section, a jump return in the data is not exactly a realization of $\mathscr{C}$ and a “continuity correction” has to be generated randomly. Obviously, there is no objection to repeat the operation many times and that results in multiple replications of every relative jump size realization. This solves the problem of insufficient number of jumps for the Kolmogorov-Smirnov goodness-of-fit test. If every correction were generated even only 10 times, the data subsets of positive and negative relative jump size realizations would enlarge, respectively, to over 300 and over 200 observations. But on the other hand, too many observations lead in practice to the rejection of any nonparametric hypothesis since the real data never fits perfectly to theoretical assumptions. Therefore, every correction was generated 50 times delivering two subsets of over 1500 positive and over 1000 negative relative jump size realizations, appropriate for the Kolmogorov-Smirnov test. As one can see in Figures 3 and 4, we found four most suitable candidates for a theoretical relative jump size distribution, taking into account the shape of the empirical density histogram of the relative jump sizes, namely:

- the log-normal distribution ~Log$N(\mu,\sigma)$;
- the gamma distribution ~$\Gamma(s,r)$;
- the beta distribution ~$B(a,b)$;
- the Weibull distribution ~Weib(s,r).

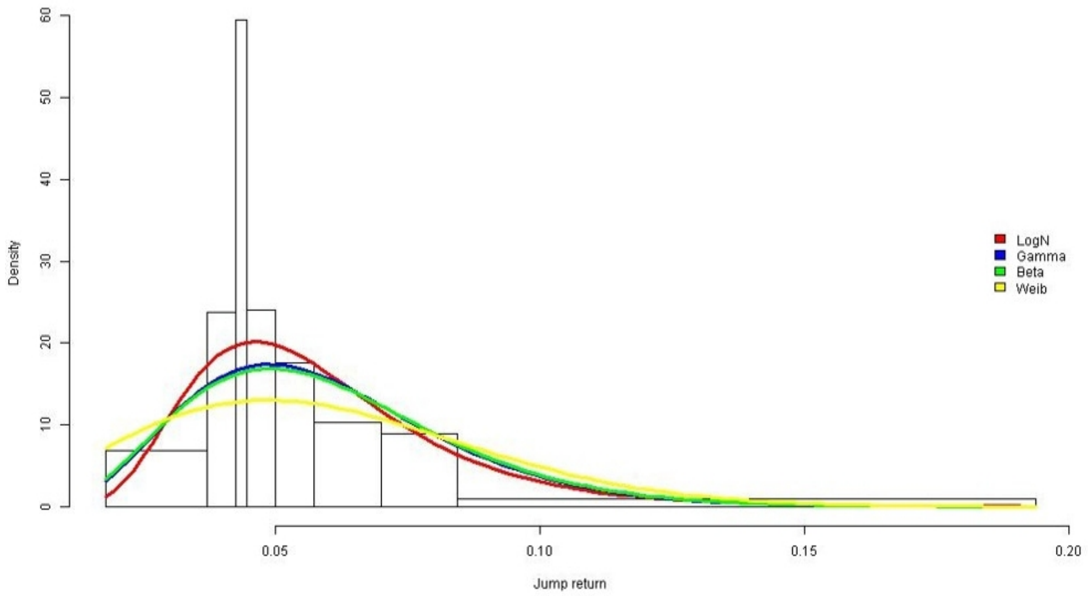

Figure 3. The empirical distribution of the positive relative jump size after the "continuity correction" vs. densities of the most appropriate typical distributions

Source: own elaboration.

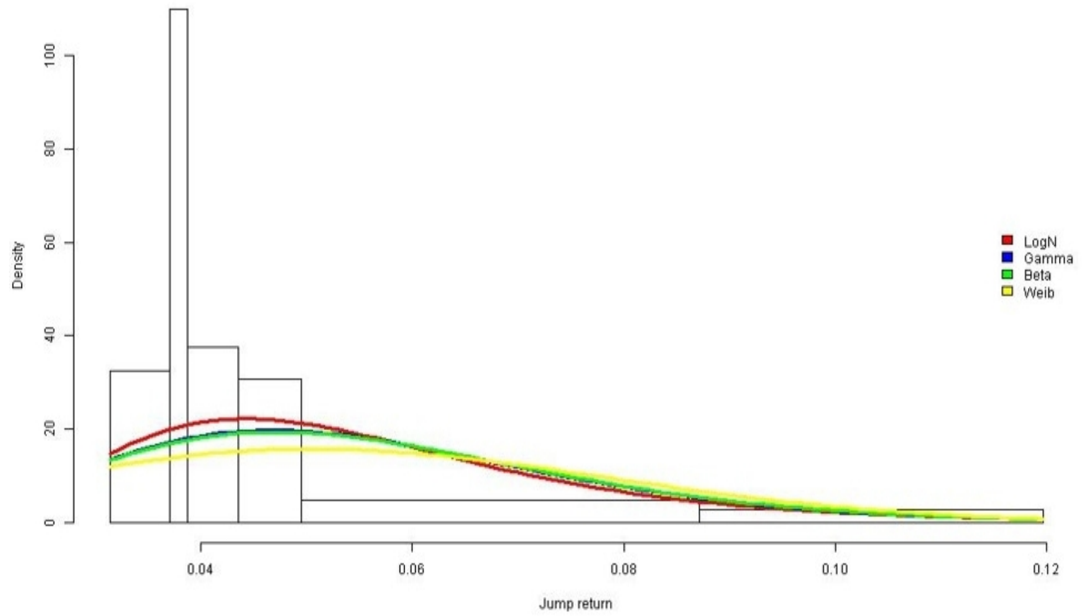

Figure 4. The empirical distribution of the negative relative jump size after the "continuity correction" vs. densities of the most appropriate typical distributions

Source: own elaboration.

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For each candidate the distribution parameters were estimated in three different manners: by means of the maximal likelihood method (ML), the moment method (Mom) and for the comparison by the Kolmogorov-Smirnov test *p*-value maximization (*p*Max), separately for the positive and the negative jumps. Then in every case the test was conducted for the hypothesis that the positive/negative relative jump size has got the relevant distribution. The results, namely the *p*-values calculated, are presented in Table 1. As one can see in the table, the really satisfactory fit yields only the log-normal distribution and only for negative relative jump size data because only in this instance a typical parameters estimation method, namely ML, resulted with p = 5.6%.

Table 1. The results of the Kolmogorov-Smirnov consistency test for the positive and negative relative jump size data subsets after 50 replications of “continuity correction” for every jump return

Method	Distribution											
	~*LogN*(*μ*, *σ*)			~*Γ*(*s*, *r*)			~*B*(*a*, *b*)			~*Weib*(*s*, *r*)		
	μ	*σ*	*p*-value	*s*	*r*	*p*-value	*a*	*b*	*p*-value	*s*	*r*	*p*-value
Positive jumps												
*p*Max	-2.9	0.41	1.7E-1	5.4	92	7.5E-2	5.2	83	5.1E-2	2.5	0.065	3.9E-3
ML	-2.9	0.49	6.6E-5	4.3	71	8.6E-8	4.0	61	5.4E-9	2.0	0.070	0
Mom	-2.9	0.25	0	3.5	56	5.1E-9	3.2	49	0	3.1	21.00	0
Negative jumps												
pMax	-3.1	0.55	9.8E-2	3.5	68	1.2E-2	3.3	61	8.8E-3	2.0	0.057	3.5E-4
ML	-3.1	0.55	5-6E-2	3.7	71	1.0E-4	3.6	64	3.2E-5	2.1	0.060	5.9E-9
Mom	-3.1	0.24	0	3.7	70	1.3E-4	3.4	62	6.0E-5	2.1	19.00	0

Source: own elaboration.

This result might be even improved to the maximal *p*-value level 9.8%. Additionally, in three next cases: log-normal- gamma- and beta-fits for the positive jumps, the theoretical distributions could be satisfactorily adjusted to the data with *p*-values 17%, 7.5% and 5.1%, respectively, but the best parameter estimates (maximizing the test *p*-value) were not met by means of the typical estimation methods we applied, either by the moment method or the ML-method, which failed the test with *p*-values much below 0.01% . It is still worth noticing that in the case of gamma- and beta-fits for the negative jumps maximal possible *p*-values were close to the accepted 5% level, namely 1.2% and 0.88%, respectively.

Another interesting observation is that the moment method delivered completely inaccurate estimates in every instance, even though there was a possibility for a proper adjustment of the distribution.

Summing up, the best candidate for the theoretical distribution of the relative jump size in the freight process connected to the trade route from Southeast Asia to Europe is the mixture of 2 log-normal distributions, namely $\sim$Log$N(-2.9,0.41)$ and $\sim$Log$N(-3.1,0.55)$, where the latter is the distribution for the absolute value of the negative jumps (so maybe more precisely it should be denoted as “minus Log*N*”), with weights 0.6 and 0.4, respectively. In Figures 5 and 6, the best log-normal-fits are drown together with the empirical distribution histogram for a graphical comparison. The reader may be stunned by this graph because despite the accepted level of *p*-value the empirical and the fitted distributions differ virtually one from another. In the further research, we would like to investigate how much this difference affects the net freight option premium.

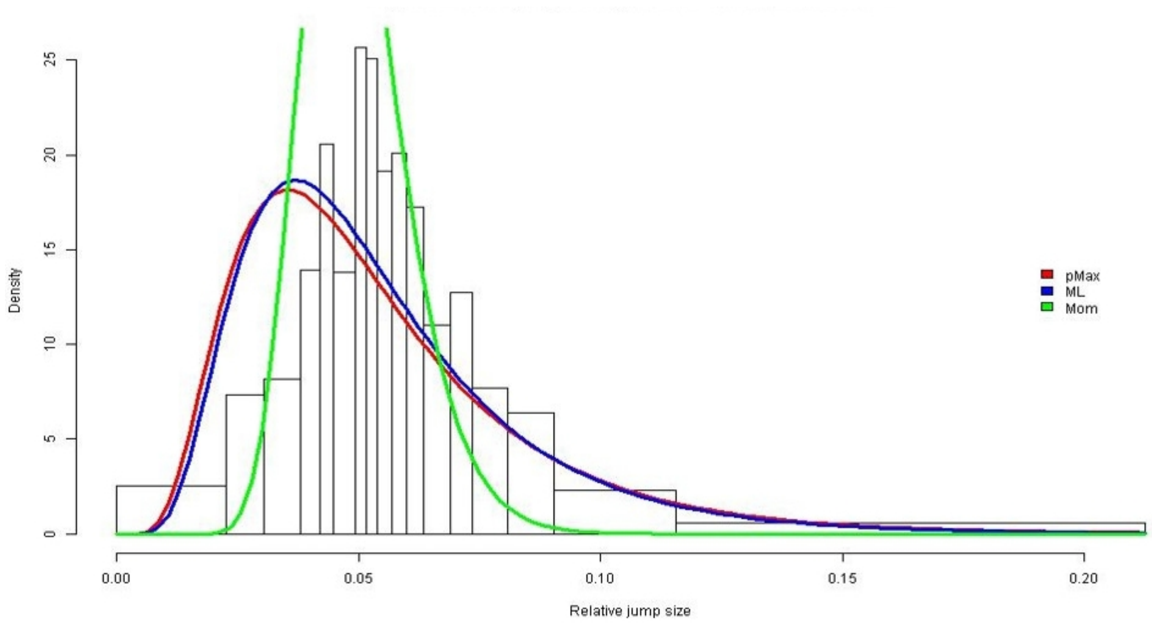

Figure 5. The log-normal fit for the positive relative jump size using three different methods for the parameters evaluation

Source: own elaboration.

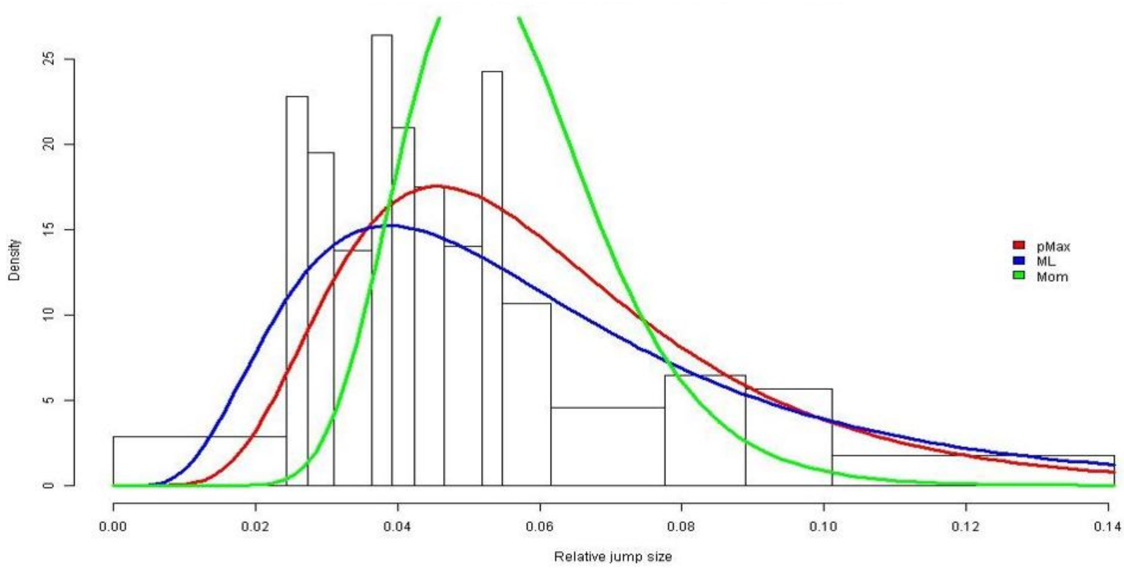

Figure 6. The log-normal fit for the negative relative jump size using three different methods for the parameters evaluation

Source: own elaboration.

A mixture of log-normal distributions seems to be worth the first consideration for other freight processes. Another reasonable candidate could be also a mixture of gamma distributions. In the Kolmogorov-Smirnov consistency test, it yielded *p*-values close to the accepted level. And as one can see in Figures 7 and 8, they differ visually from the empirical distribution in a similar way as the log-normal-fit. On the other hand, the beta distribution turned out rather inappropriate for the modeling of the relative jumps size. A completely inaccurate fit was obtained by the Weibull distribution.

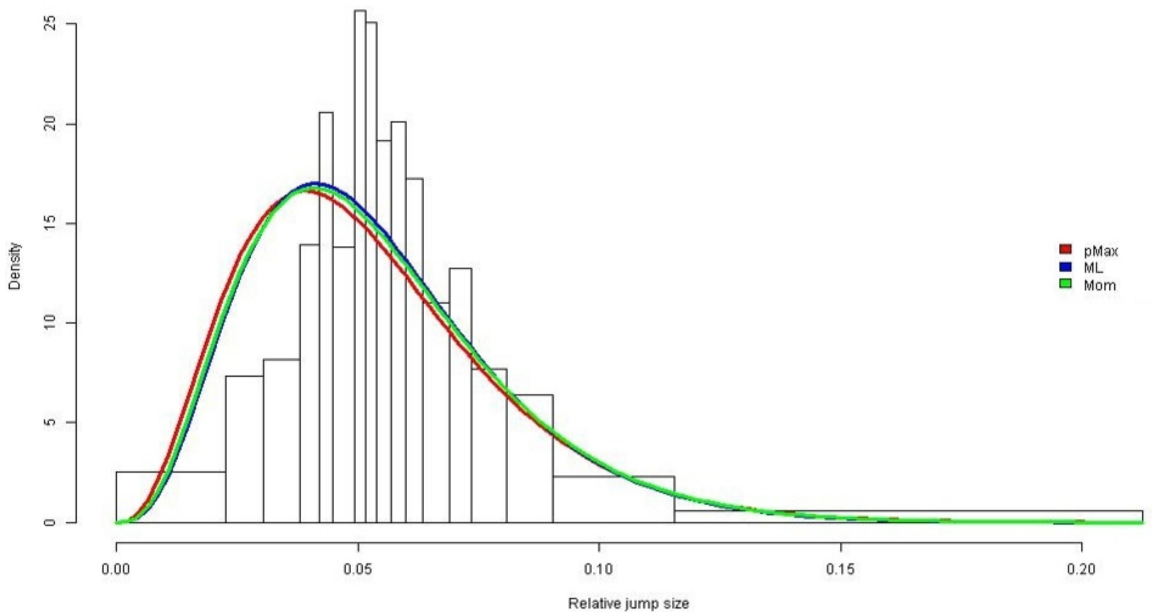

Figure 7. The gamma fit for the positive relative jump size using three different methods for the parameters evaluation

Source: own elaboration.

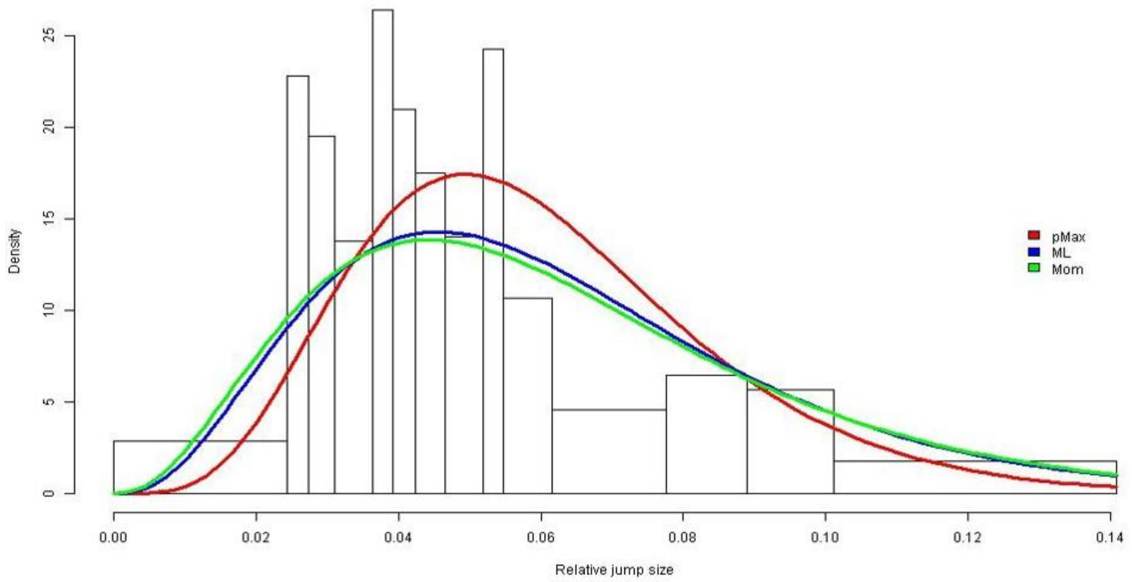

Figure 8. The gamma fit for the negative relative jump size using three different methods for the parameters evaluation

Source: own elaboration.

4. Conclusion

The knowledge of a theoretical distribution of the relative jump size from the net freight process in container shipping could let us derive the close analytical formula for the European Call option net premium. This makes the calculations faster without the time-consuming Monte Carlo simulations. As it has been investigated, the positive and negative jumps should be considered separately and the best candidate for a distribution mentioned is the mixture of log-normal distributions with weights 0.6 and 0.4, respectively. Another possibility which could be taken in the consideration is a similar mixture of gamma distributions.

However, several issues have been spotted. The first is a proper parameter estimation. Even the efficient ML-method yields results far from the adjustment which the *p*-value of the Kolmogorov--Smirnov test has the greatest value for. This implies that the parameters of the distribution have to be estimated in a quite inefficient way by means of the *p*-value maximalization. The second is that even the density of the distribution fitting the best to the data differs graphically significantly from the empirical density. The impact of this difference on the difference in evaluation of the derivative net premium should be investigated, which could be a topic for a further research. And eventually, the advantage of the theoretical distribution may be efficiently used only when the analytical formula for a net derivative premium will be found. This is a really hard task and without it the fitting may not be that advantageous because in such an instance a derivative net premium evaluation follows anyway by means of the Monte Carlo simulations. To sum up, for now, without the analytical formula for the net premium, the usage of the empirical distribution seems more confident and more appropriate.

Acknowledgements

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Skoki w procesie frachtu w kontenerowym transporcie dalekomorskim

Streszczenie: Jak zostało zweryfikowane we wcześniejszych badaniach (Gardoń, 2014), średniotygodniowy fracht w kontenerowym transporcie dalekomorskim może być modelowany za pomocą procesu skokowo-dyfuzyjnego z jednorodnymi poissonowskimi skokami. Dotychczas względna wielkość skoku była generowana z rozkładu empirycznego, który jest asymetryczny i nie przypomina żadnego z typowych rozkładów. Ogólnie skoki wzrostowe pojawiają się częściej, natomiast spadkowe są mocniej skoncentrowane wokół swojej średniej. W tym artykule do danych dotyczących skoków dopasowuje się mieszankę dwóch rozkładów, z których jeden opisuje oddzielnie skoki wzrostowe, a drugi spadkowe, co daje obiecujące wyniki. Model skokowo-dyfuzyjny jest wykorzystywany głównie do obliczania premii netto derywatyw, takich jak np. europejskie opcje kupna na fracht, których dotyczą wcześniejsze artykuły (Gardoń, 2016). Bez znajomości teoretycznego rozkładu względnej wielkości skoku podstawowego procesu do obliczania premii netto za opcje trzeba wykorzystywać czasochłonne symulacje Monte Carlo. Znajomość tego rozkładu może pozwolić na uzyskanie analitycznej, jawnej formuły na premię netto, co zwiększyłoby szybkość i dokładność obliczeń.

Słowa kluczowe: kontenerowy transport dalekomorski, dyfuzja skokowa, rozkład względnej wielkości skoku, opcje kupna na fracht