Analysis of uniform illumination system with imperfect Lambertian LEDs

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This paper offers a novel algorithm to better design LEDs arrays for illumination systems. First, the illuminance distribution of LED arrays is studied through theoretical analysis. Second, we present the algorithm criterion and steps. And finally, we use computer to simulate and verify this method. The results show that the illumination uniformity is significantly affected by the spacing of each light in the LED array. The analysis method presented here in this thesis can be usefully applied in LED in the illumination engineering.

Keywords: light emitting diode, uniform illumination, imperfect Lambertian distribution.

1. Introduction

With the rapid development of solid state lighting technologies, the light-emitting diodes (LEDs) are designed to generate 10–120 lumens per LED with an efficiency that surpasses incandescent and fluorescent lamps. So the LED will play a key role in the future lighting and widely replace incandescent and fluorescent lamps in indoor illumination system [1–3]. Though a high-power LED produces up to 120 lumens per device, a single LED still cannot provide sufficient illumination for indoor illumination system where an illumination of 300–1500 lux is needed. Several LEDs must be mounted on the panels to obtain practical luminous power and get a uniform illumination [4, 5].

To get uniform illumination, the optical modeling of the illumination system must be required. Reference [6] demonstrates how the light source model is characterized by geometry, luminous properties and intensity distribution. As is known, the intensity distribution type of LED includes Lambertian-type, batwing, and side emitting [7]. In other words, there are some LEDs with imperfect Lambertian source in the real world applications. In the literature [3–5, 8, 9] we find designs of illumination systems using

the Lambertian model or a cosine function of the viewing angle, so they can yield definite maximal condition of uniform illumination, and get definite analytical expressions.

Since those light-emitting models cannot be depicted with precision expression, such as the LUXEON K2 green, blue, red and white LED [10], when we use LED array for lighting, how to design the LED lighting system is the main purpose of this paper. We analyze the features of the luminous intensity distribution of LEDs in order to reach uniform illumination and we present an algorithm to arrange the LEDs in array. We take great efforts to improve the lighting efficiency and reduce the cost of encapsulation and of secondary optical design, and this design is more practical and easy-to-assemble.

The paper is arranged as follows. The LED illumination principle is presented in Section 2. In Section 3, we present the algorithm criterion and the algorithm steps. The simulation results are given in Section 4. Finally, in Section 5 we draw our conclusions.

2. LED illumination principle

2.1. Assumption

With LED lighting, if the light of the LED has not been assigned, the light on the target is just a round spot which produces glare to human eyes, therefore, uniform illumination plays an important role in lighting system designs. Uniform illumination depends on the following factors: the LED intensity distribution, the distance between the observer and the radiation surface, the incident angle, the reflectivity and the shape of target [4]. Here we only analyze the illumination of a flat target. The LED lighting can be catalogued into near-, mid-, and far-field according to the distance between source and target [11, 12]. When the LED illumination range is about 5 times larger than its maximum size, we think that the range belongs to far-field illumination areas, therefore the LED light source here is assumed as a point source. This paper always takes far-field illumination as its research object, and the LED arrays are placed in the same plane.

2.2. Practical model of LED

The ideal LED optical model is a perfect Lambertian, which means its intensity is proportional to the cosine of the viewing angle, namely $I(\theta) = I_0\cos(\theta)$, I_0 (lm/sr) is the intensity of the LED axis, θ is the viewing angle, $I(\theta)$ (lm/sr) is the intensity at the viewing angle. Articles [3–5] used accurately expressed models, thus the luminance of the target can be expressed as $E(r, \theta) = E_0(r)\cos^m(\theta)$, where θ is the viewing angle, the order m is related to $\theta_{1/2}$, given by $m = -\ln(2)/\ln(\theta_{1/2})$, the m depends on the relative position of the LED emitting region from the curvature center of the spherical encapsulant. $E(r, \theta)$ (lm/m^2) is the illuminance of the target plane, $E_0(r)$ is the illuminance (lm/m^2) on the axis at distance r from the LED.
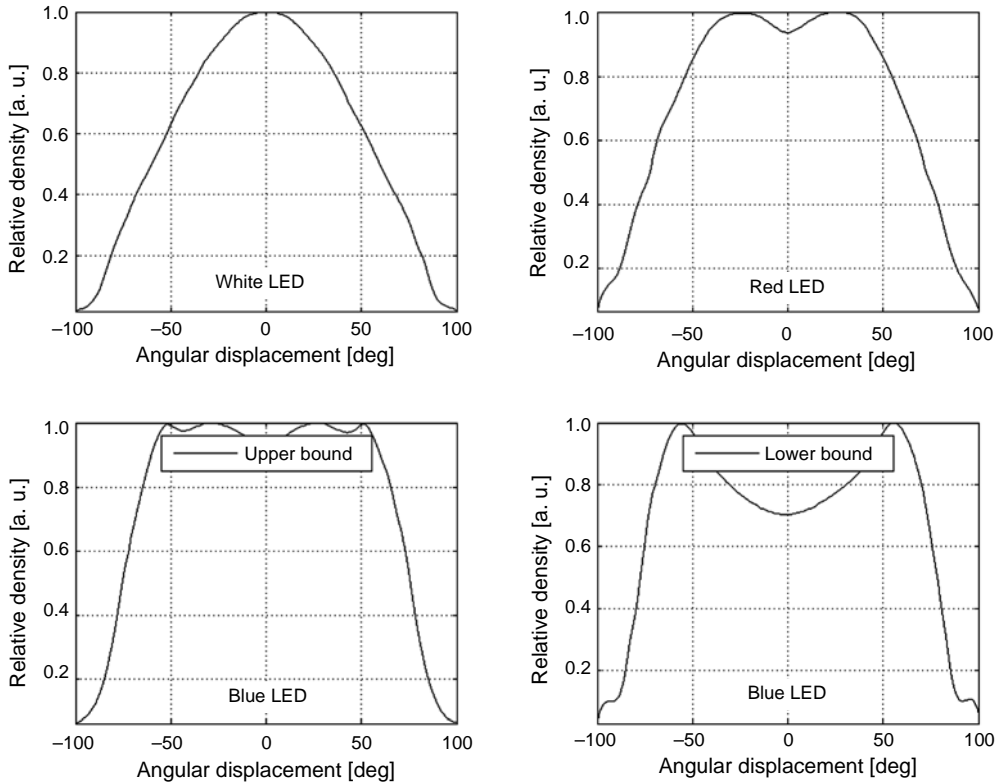
Fig. 1. Spatial luminous pattern for LUXEON K2 LED [10].

In some special scenarios, LED luminous intensity distribution $I(\theta)$ cannot be described by the above-mentioned formula. Such a kind of LED is LUMILED of LUXEON K2 series shown in Fig. 1 [10]. Here, the figures, from left to right, from top to bottom are respectively called model A, model B, model C and model D. We can know that only the angular distribution of model A is approximately depicted by the equation $I(\theta) = I_0\cos(\theta)$, and the others cannot. Consequently, it is difficult to design an LED array to get uniform illumination. When we design the layout of LED array, either we use its datasheet provided by the manufacturer, or we measure the luminous intensity distribution model by ourselves. Finally, we calculate optimal displacement parameters in the LED array.

2.3. Analysis of the illumination of the target

To describe the process of this design method more clearly, for example, we establish the Cartesian coordinates shown in Fig. 2. The LED is set in coordinate origin O, and the optical axis of the LED is perpendicular to plane *A*. Point *P* in plane *A* has coordinates *x*, *y* and *z*. In order to show the amount of light at any point *P*, take a small area d*A* around point *P*, and assume its luminous flux is dΦ (it has solid angle dω), so
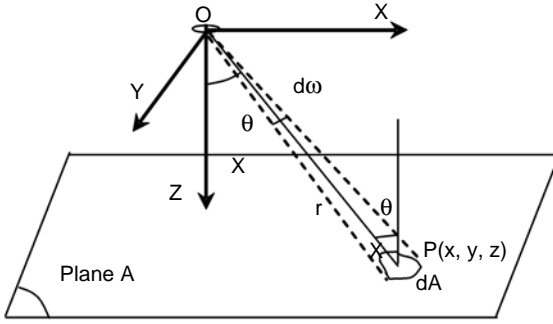
Fig. 2. Schematic of a single LED illumination.

the luminous intensity is $I_r = d\Phi/d\omega$. The relationship of the solid angle, the illuminant area and the illuminant distance can be expressed by $d\omega = (dA/r^2)\cos(\theta)$, dA is the tiny area, r is the distance from the light source to illuminant surface, θ stands for the angle of incidence. We evaluate the luminous flux by $d\Phi = (I_r/r^2)\cos(\theta)\,dA$, so the illuminance in area A is $E(r, \theta) = d\Phi/dA = I_r\cos(\theta)/r^2$. Then we translate $E(r, \theta)$ into Cartesian coordinates (x, y, z). The illuminance at point $P(x, y, z)$ can then be given by

$$E(x, y, z) = \frac{zI_r}{\left(x^2 + y^2 + z^2\right)^{3/2}} \tag{1}$$

For our scenario, the LED array is fixed on the ceiling plane XOY (ceiling plane), if the i-th LED is located in $(x_i, y_i, 0)$, the illuminance of the i-th LED is given by

$$E_i(x, y, z) = \frac{zI_r}{\left[(x - x_i)^2 + (y - y_i)^2 + z^2\right]^{3/2}} \tag{2}$$

The total illuminance E is given by the sum of the illuminance for N LEDs

$$E = \sum_{i=1}^{N} E_i \tag{3}$$

$$E_i(x, y, z) = \sum_{i=1}^{N} \frac{zI_r}{\left[(x - x_i)^2 + (y - y_i)^2 + z^2\right]^{3/2}} \tag{4}$$

I_r is the function of θ, the relation between θ and x, y, z, x_i, y_i can be expressed by

$$\theta = \mathrm{acos}\left(\frac{zI_r}{\sqrt{(x - x_i)^2 + (y - y_i)^2 + z^2}}\right) \tag{5}$$

2.4. Layout of LED array

We adopt LUXEON K2 LED for designing lighting system. The room size is 5.0 m × 5.0 m × 3.0 m, and the distance between the ceiling and the target plane is 2.15 m. LEDs fixed in the ceiling of the room are arranged as shown in Fig. 3. We take three types of configurations, that is, the linear LED array, the square LED array, and the circular LED array, as in [4, 5]. Because the linear array is too simple and impractical, we will not consider this case. Furthermore, the LED's luminous intensity is isotropic, consequently LED-to-LED has the same spacing. The specific arrangement method is given by Fig. 3. In the configuration of the square LED array, there is the same LED-to-LED separation *d*, and the LEDs are placed along axis *x* and axis *y*. In the configuration of the circular LED array, the LEDs are put in the circumference with only one LED in the center of the circle, and the radius is *r*. Finally, we discuss how many LEDs should be installed in the ceiling as in Fig. 3. Generally, illuminance of lights is standardized by ISO (International Organization for Standardization). In this standard illuminance of 300–1500 lux is required for a work office, but the typical luminous power of LUXEON K2 LED (model A) is only 130 lm from the datasheet. Therefore, 64 LEDs can provide sufficient illuminance in this system. If we use other type LED (models B, C, D), we require 81 LEDs to form the array.

3. Algorithm

3.1. Analysis of the LED separation of LED arrays

Before exploring the LED lighting array, we first study illuminance distribution of two LEDs, and then extend the method to the LED array. In order to obtain a more satisfactory uniform lighting, we should consider the separation between LEDs. In this case, the illuminance is given by the sum of the illuminance for two LEDs as follows

$$E(x, y, z) = \frac{zI_r(x_1, y_1, 0)}{\left[(x - x_1)^2 + (y - y_1)^2 + z^2\right]^{3/2}} + \frac{zI_r(x_2, y_2, 0)}{\left[(x - x_2)^2 + (y - y_2)^2 + z^2\right]^{3/2}} \tag{6}$$
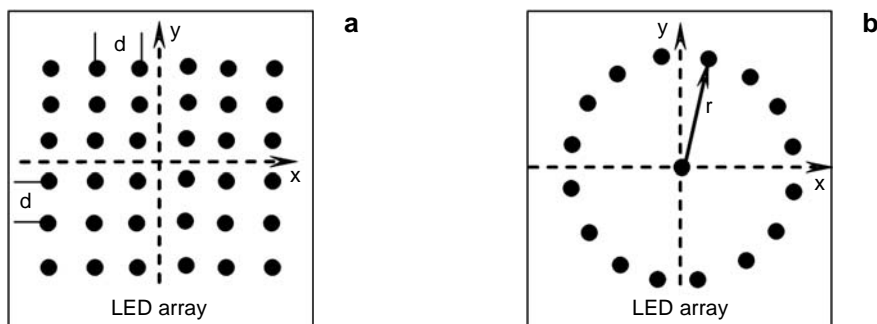

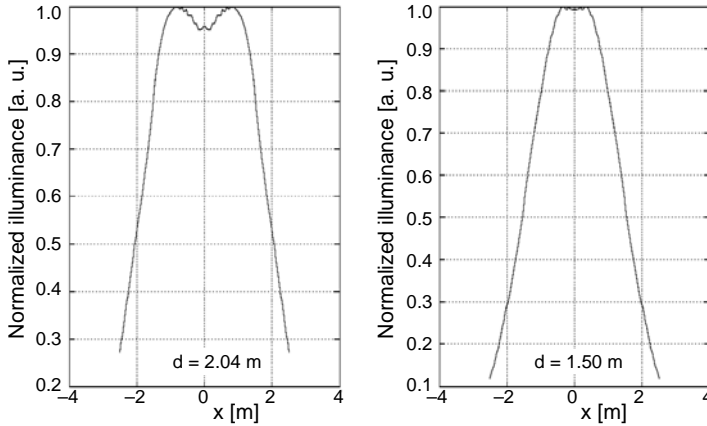Fig. 3. Configuration of LED array: square (**a**), circular (**b**).

Fig. 4. Simulation of illumination with d = 2.04 m and d = 1.50 m.

The distance between two LEDs is $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, we can adjust d to get a maximally flat illumination in central region based on Sparrow's criterion and references [4, 5]. To explain it clearly, let us take Fig. 4 for example. When the separation of the LEDs d = 2.04 m, the illumination uniformity of the central region is inadequate. If the d continues to increase, the uniformity of illumination becomes even worse. When d = 1.50 m, the illumination of the central region has the optimum illumination uniformity. We reduce d so that we can reach the optimum uniformity of illumination near the central region. Therefore, we can follow certain rules and algorithms to find the optimum parameter to achieve optimal lighting.

3.2. Algorithm criterion

Now, we are going to discuss the criteria of the algorithm. We know that LED spatial light intensity distribution is isotropic as shown in Fig. 1, and the sum of illuminance of the target is a convex function, hence there must exist an optimal lighting space. So, the LED array we design comprises the square LED array and the circular LED array. First, we determine the maximum separation d_{max} of the square LED array and the maximum radius r_{max} of the circular LED array according to the room size and the number of LEDs. Second, we calculate the illuminance of the target, and divide the room into 256×256 equal-spaced grids, which we define here as the grid element. We calculate the grid center illumination, and use it to represent the average illumination of the element.

Algorithm criteria 1:

$$\left| \frac{E_1 - E_2}{\overline{E}} \right| \le 5\% \tag{7}$$

Here, $\overline{E} = (E_1 + E_2)/2$, E_1, E_2 are the illuminances of two neighboring grid elements. If E_1 and E_2 meet Eq. (7), we deem that the two grid elements are uniform illumination.

Algorithm criteria 2 – the area surrounded by adjacent grid elements whose illuminance satisfies criteria 1 is the largest.

3.3. Algorithm steps

Under criteria 3.2, the algorithm steps are as follows.

Step 1. Get data of the LED light intensity (angular displacement) from either the datasheet provided by manufacturers or the data measured by instrument.

Step 2. Because the data obtained in step 1 is limited, we utilize spline interpolation to determine its spatial distribution.

Step 3. Discrete the reflecting surface into 256×256 equal interval grids along the x, y axes.

Step 4. To initialize the LED array parameter r or d, let $r = r_0 - \lambda r_0$, $d = d_0 - \lambda d_0$, $-1 \le \lambda \le 1$, $r_0 = r_{max}$, and $d_0 = d_{max}$. The initial values r_0 and d_0 are related to the room size and the number of LEDs.

Step 5. Evaluate the illuminance of every grid based on Eqs. (2) and (3). If the illuminance of the neighboring grid element satisfies Eq. (7), count the total area as uniform illuminance.

Step 6. Comparison of the size of the uniform illumination area. If the area is not the largest, then we change the parameter λ, and repeat step 4, step 5 and step 6, until the area is the largest. If the uniform illumination area does not increase again, then exit.

4. Simulation result

4.1. LED square array

Now, we take the square array of LEDs for example. In this case, we transform Eq. (4) into (8). The total illuminance for $N \times N$ LED array is given by

$$E(x, y, z) = \sum_{j=1}^{N} \sum_{i=1}^{N} zI_r \left[(x - x_{ij})^2 + (y - y_{ij})^2 + z^2 \right]^{-3/2} \tag{8}$$

Here, I_r is the function of x, y, z, x_{ij}, y_{ij}. N is not considered here odd nor even number. The LED array is formed by 8×8. The maximal conditions are difficult to find out, so we resort to a numerical solution. First, we start with $d = d_{max} = 0.714$ m, by restricting the room size and the number of LEDs. Then we use the algorithm mentioned in Section 3 to search the spacing of the LEDs, finally find the optimal
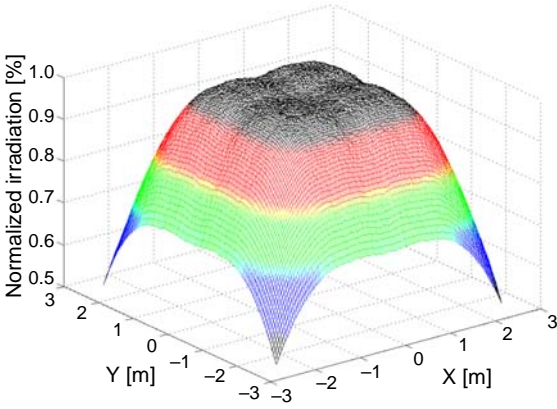
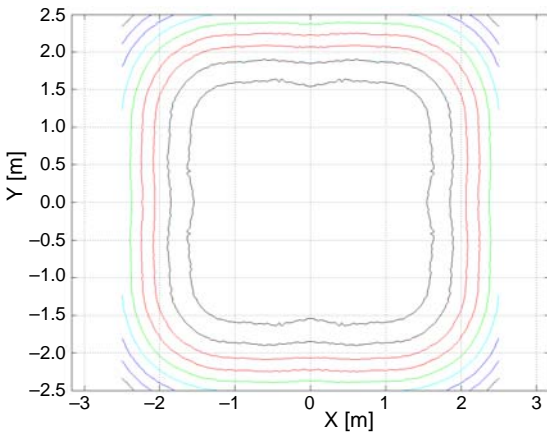Fig. 5. LED square array illuminance of the target.

Fig. 6. LED square array contour map of illuminance.
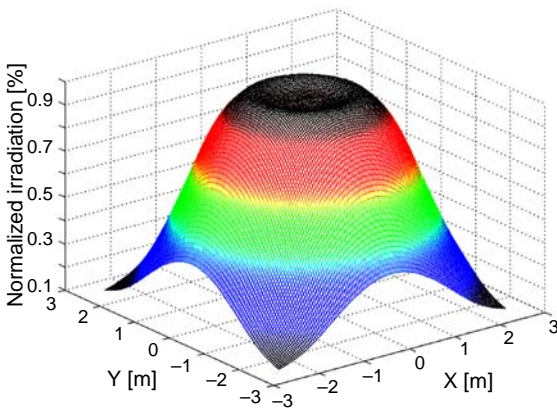

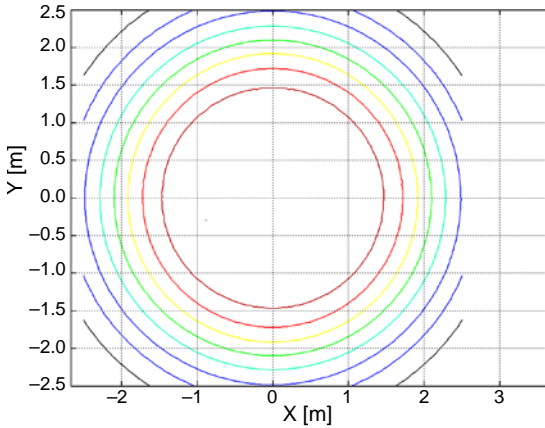Fig. 7. LED circular array illuminance of target.

Fig. 8. LED circular array contour map of illuminance.

parameters $d_{opt} = 0.704$ m. Figure 5 shows the 8×8 normalized illuminance distribution map. Figure 6 shows the illumination contour map of the optimal parameter d_{opt}, and its uniform illumination area is the largest.

4.2. LED circular array

Next, we will discuss the case of the circular array which is a popular configuration of LED lamps in practical application. We install one in the center, and the other 63 in the circle. The illumination is as follows

$$E(x, y, z) = \sum_{i=1}^{N-1} zI_r \left[\left(x - r\cos\left(\frac{2\pi i}{N-1}\right) \right)^2 + \left(y - r\cos\left(\frac{2\pi i}{N-1}\right) \right)^2 + z^2 \right]^{-3/2} +$$

$$+ zI_r \left(x^2 + y^2 + z^2 \right)^{-3/2} \tag{9}$$

The radius is initialized with $r = 2.5$ m, and is changed step by step based on the algorithm. When the radius is $r = 1.454$ m, the largest area of uniform illumination appears. The illumination is shown in Fig. 7, and the illumination contour map – in Fig. 8.

4.3. Simulation results of other models

The simulation results of other models are not listed one by one as in Sections 4.1 and 4.2. We list all the optimal simulation parameters in the Table.

5. Conclusions

In this paper, we analyze the far-field LED illumination arrays, and propose an optimal algorithm to design a LED array using the imperfect Lambertian LED model.

T a b l e. Simulation of optimal parameters.

Type of model	Square array		Circular array	
	The number of LEDs	d [m]	The number of LEDs	r [m]
Model A	64	0.704	64	1.454
Model B	81	0.615	81	1.719
Model C	81	0.448	81	1.328
Model D	81	0.332	81	1.406

The simulation results show that the algorithm structure is more reasonable, and the result is better. Compared with previously published studies, the methods in this study focus on resolving LED array lamps whose optical intensity models are not perfect Lambertian. In summary, this method is not only applicable to Lambertian, but also applicable to the imperfect Lambertian. It is the very characteristic of this algorithm.

The algorithm for a research design presented here is accurate, but it has the shortcoming of being time-consuming and having large computations. The next step should reduce the computation and computing time. Beside these, there are some interesting LED arrays which need more complex calculations which have not been considered here, such as the multi-layer concentric structure, in which the arrays are not in the same plane. Although this method has its limitations, it is still an improved and superior design method.

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