

**Krystian Pera**

Katowice University of Economics

**Piotr Saluga**

Mineral and Energy Economy Research Institute of the Polish Academy of Sciences,  
Cracow

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## ESTIMATION OF PRICE VOLATILITY AND THE CONVENIENCE YIELD – A STUDY BASED ON ZINC AND LEAD EXAMPLE

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**Summary:** The volatility of prices is one of the most important parameters in the technical analysis of assets on the financial and commodity markets as well as the major indicator of interest for the investors. For mineral commodities, apart from price volatility, it is the convenience yield which plays a significant role. The two parameters are the key-variables used in forecasting commodity prices and real option pricing models. The paper makes an attempt to estimate historical volatility of prices and the convenience yield for zinc and lead relying upon data for years 2001-2008.

**Key words:** commodity futures markets, zinc-and-lead industry, zinc and lead prices, price volatility, convenience yield.

### 1. Introduction

The constantly increasing development of electronic trading observed in recent years affected the development of commodity price quotations. The available data relate usually to commodities with standardized parameters. This in turn – independently of the increasing database – enables the construction and calculation of parameters considered when valuing derivatives; the presence of price time series as well as rates of return is the necessary condition for these calculations. The present study makes an attempt to demonstrate a method for estimating both volatility and convenience yield parameters which influence the forward / futures and option values. The calculations in this paper refer to zinc and lead quotations for the years 2001-2008.

The choice of metals is not random. Both are quoted in world's commodity exchanges, thus there is an adequate database for these metals and, in addition, the market of commodity derivatives includes both elements in standard specifications. These specifications are – in the case of zinc – *Special High Grade (SHG) Zinc (Grade Classification Z1)*, with zinc minimum purity 99.995%, and in the case of

lead – *Standard Lead*, with lead minimum purity 99.97%. A standard order includes 25 metric tons (with  $\pm 2\%$  tolerance). For calculation purposes quotations from the London Metal Exchange have been considered which, along with Tokyo and Chicago exchanges, are considered the most important commodity exchange in the world.

The future price paths may be obtained by modeling basic market parameters: supply, demand and inventories. Unfortunately, the major part of (financial) institutions uses the structural analysis only as additional information, usually forecasting prices upon:

1. the hypothesis that there is no way to predict future prices – their changes occur in accordance with the Brownian motion; hence, the only source of information about their future level are the up-to-date quotations (Markov process);
2. assumption that price levels are in principle predictable but their accurate forecast is conditioned by a proper analysis of available data.

Analysts and participants in commodity markets have in mind two parameters they consider of exceptional importance: price volatility and the convenience yield. They are of fundamental importance both for forecasting prices and real options valuation [Davis 1998; Saługa 2009]. Volatility is a measure of prices tendency to change. It is an essential indicator of risk. Since futures prices are important, the convenience yield is a pure source of information for predicting future commodity prices. It is characterized by a higher level of predictability than the indicator of price volatility. Knetsch [2006] writes, that *the convenience yield is not only a theoretical construct but also a phenomenon of empirical relevance*. This parameter is an adjustment to the cost of carry in pricing formula for futures prices on the markets with special (trading) constraints. It is also determined as a difference between the cost of carrying and the futures price.

It is important to stress a close relation between the convenience yield value and volatility of underlying instrument.

This paper, relying on historical data, features two indicators: volatility and the convenience yield. It is worth mentioning that any sophisticated investigation of volatility parameter is not the matter of the paper – its purpose is rather showing a possible range of zinc and lead prices volatility and convenience yield which gives some valuable information for mineral analysts.

The article is structured as follows: section one defines volatility and presents its well known computing method relying on historical data. Section two features transactions on the commodity markets and shows the relationship between spot and futures prices and the formulation and calculation formula of the convenience yield. Section three describes data used in computations and the results obtained.

## 2. Volatility of commodity prices in spot transactions

Volatility of an underlying asset is a crucial parameter in option valuation models. It is also an important factor in technical analysis. It was Merton [1973] who first

introduced the concept of volatility as being one of the factors the option price depends on. Volatility refers to the tendency of prices to change unexpectedly, usually as a response to new information or changes in demand for the investment; it is then a measure of quantitative instability or stochastic character of price changes determined usually as dispersion of a set of data points from its average in a period of time.

Hence, price volatility is a statistical measurement of risk usually defined as a standard deviation of their changes over a specific period of time with an assumption of continuous capitalization.

The volatility of an asset depends on the following:

- liquidity (a measure of the ease with which a security can be bought and sold on a financial market) – the lower liquidity the greater volatility,
- free-float (an estimate of the proportion of shares that are not held by large owners and that are not stock with sales restrictions) – the lower free-float the greater volatility.

Volatility may be defined upon:

- 1) past changes in financial instrument price (historical volatility),
- 2) market prices of call options on equity futures (implied volatility).

The historical (realized) volatility of prices is calculated upon past quotations. When calculating we can use both the settlement price of a trading day as well as weekly, monthly, quarterly or annual data. To obtain the most satisfactory data it is necessary to consider:

- 1) the possibly long period of market quotations,
- 2) adequate frequency of market quotations.

An indicator received on the basis of historic observations is in fact the realized volatility – it measures how volatile an asset was in the past. It is important to bear in mind that the future volatility may significantly differ from the one in the past, however, the estimation of the latter lets roughly estimate the range of possible changes.

The implied volatility is derived from options quotations on assets being subject to interest. It is considered to be the actual volatility of the underlying [Hull 1993]. It is assumed that investors making their own option valuations (with the application of different levels of underlying asset volatility) and submitting market orders on a stock exchange become the source of information on the actual volatility of the underlying.

As mentioned previously, historical volatility of prices is calculated as a standard deviation of their fluctuations from the mean over a given period with the assumption of continuous capitalization. The computing procedure with application close-to-close prices is represented as follows:

1. calculation of one-period logarithmic rates of return according to the formula:

$$r_t = \ln \left( \frac{P_t}{P_{t-1}} \right) \quad (1)$$

where:  $t = 1, 2, \dots, N$  – indicator of the period (number of observations =  $N + 1$ ),  
 $P_t$  – commodity price at the end of a period;

2. calculation of the mean from the observation:

$$\bar{r} = \frac{1}{N} \sum_{t=1}^N r_t \quad (2)$$

where:  $N$  – number of periods;

3. average period calculation of a standard deviation, according to the formula:

$$\sigma = \sqrt{\frac{1}{N} \sum_{t=1}^N (r_t - \bar{r})^2} \cdot \sqrt{N}. \quad (3)$$

Volatility estimation is the broad field of advanced studies – e.g. one of the latest interesting publications is the book by Doman & Doman [2009] which, *inter alia*, concentrates on the methodical complications of volatility investigation. In the last decade some of these problems have been overcome thanks to various computational methods. One of them is GARCH (Generalized Autoregressive Conditional Heteroskedasticity) which nowadays is one of the most promising tools of volatility estimation [Fiszeder 2009].

### 3. Relationship between spot and futures prices – the convenience yield

If a commodity is traded in the futures markets there is an opportunity to provide price dynamics. The difference between spot and futures prices is called *the basis*. There are two theories that explain the basis:

- 1) the normal backwardation theory,
- 2) the theory of storage.

The normal backwardation theory explains the basis from the point of view of hedging positions in the futures markets, whereas the theory of storage is based on an arbitrage approach which focuses on the cost of holding physical commodities.

The theory of storage is a simple tool to explain the term structure of prices. In the case of surplus stocks, the level of backwardation cannot exceed the cost of carry in other case arbitrageurs would have an incentive to gain profits. According to this theory the futures price must always be greater than the spot price because of the incurred storage costs.

In reality though, a commodity market may appear:

- 1) in backwardation – when the spot prices are higher than futures/forward prices,
- 2) in contango – when the situation is reverse.

Hence, there is an evident contrast between theory and reality that will be explained further.

When the market is in backwardation (which is the default condition) the fact of consumption being delayed in time is discounted. At this point producers and consumers are willing to minimize inventory levels in stocks. It is a typical situation for commodity trading because of scarce production in the future. A contango is present when investors expect further upward movements in prices in the case of possible events having a negative effect on supply. The market may also appear in contango in the case of surplus production when one day investors buy great amounts of a commodity at low prices in hope of selling it at a profit in the future. The greater the basis in contango the greater the willingness of producers and consumers to enlarge inventories; investors prefer buying commodities today and incur the cost of carry.

The following factors have an effect on commodities trading:

- 1) cost of storage,
- 2) capacity of stocks,
- 3) costs of transport and insurance,
- 4) level of stocks / inventories.

Mineral commodities are stored in mine dumps. There are a lot of profits related to keeping inventories in stocks which the owner of the store and commodities will gain. Among them there are such opportunities as:

- 1) smoothing production that makes the minimization of costs possible,
- 2) avoiding a situation of stockouts which would make clients' orders unrealizable,
- 3) facilitating production and sells scheduling.

The benefit flow from holding a marginal unit of a commodity from the beginning to the end of period  $t$ , net of storage and insurance costs, is called the convenience yield. It accrues to the holder of a physical commodity but not to the holder of a contract for future delivery of that commodity [Kaldor 1939; Working 1949]. This flow is of such importance that producers keep commodities in stocks even if the expected rates of return are lower than the threshold rates or even if the expected trading profits are negative.

The presence of the convenience yield explains why risk-averse investors do not sell stored commodities and do not assume a long position on the market even if there is an opportunity to arbitrage [Knetsch 2006].

First publication to notice the presence of the convenience yield dates back to 1939 [Kaldor]. Further studies in this field were carried out by, among others, Working [1949] and Brennan [1958]. Kaldor and Brennan determined the convenience yield as

a function of aggregate inventory level decreasing along with the upward movement of this level. They confirmed the presence of a convenience yield in backwardation market by the fact that keeping inventories enables economic factors to respond to the demand without incurring risk and cost of waiting for future delivery. Working noted that the amount of fixed costs of carry did not encourage producers to sell a commodity when the market was short which caused backwardation. The work of Weymar [1968], apart from confirming the thesis of a decreasing character of the convenience yield function with respect to the level of stocks, explains that stocks are retained in a backwardated market because the production, storage and transportation activities are insufficiently flexible to be able to adapt instantaneously to changes in supply and demand. Williams and Wright study [1989] describes the fact of keeping inventories as the best way of minimizing transport, processing and marketing costs. There is an interesting work of Fama and French [1988], which purpose is to verify the hypothesis on:

- greater volatility of spot prices vs. futures prices when inventory levels are low, and
- comparable volatility of these prices when inventories are high.

The convenience yield, going to the holder of commodity inventories is analogically compared to a dividend (yield) in the shares market. If there is a market of easy trading of a commodity and this commodity is well defined (i.e. its parameters are accurately determined) then we have [Pindyck 1993]:

$$P_t = \delta \sum_{i=1}^{\infty} \delta^i E_t \psi_{t+i} \quad (4)$$

- where:  $P_t$  – current price of a commodity at time/date  $t$ ,  
 $\delta$  – discount factor:  $1/(1+r)$ , where  $r$  – one-period discount rate (the expected rate of return an investor would require to hold a unit of commodity),  
 $\psi_t$  – expected future one-period (net) marginal convenience yield – the benefit flow from holding a marginal unit of a commodity from the beginning to the end of period  $t$ , net of storage and insurance costs,  
 $E$  – expectation symbol.

The price of the commodity must equal the present value of future convenience yields. The formula (4) explaining changes in commodity prices is the central equation of the present value model of rational commodity pricing, PVM.

Changes in commodity prices derive from the supply and demand structure. Given the fact that supply and demand changes cause in consequence changes in actual and expected convenience yields we may assume that price changes derive from future expected convenience yields [Pindyck 2001].

Given the fact that competitive markets equate marginal convenience yields across holders we have to note that the marginal convenience yield depends on the

level of inventories, and, in fact, is independent of the identity of the holder of those inventories. It is assumed that the convenience yield depends only on the current spot price [Moyen et al. 1996]. In the case of low stocks level the price will be high and *vice versa*.

For some mineral industries, e.g. gold mining, the convenience yield is of little importance [Pindyck 1993]. It results from the fact that the inventories level is very high when compared with the production volume. For other industries it is of exceptional importance, though: without taking into account interests and storage cost it may represent from several to a dozen or so percent per month.

For commodities being traded in futures/forward markets as well, the convenience yield may be measured directly – using (futures) prices for future delivery. This income – with the assumption of market effectiveness (no arbitrage is possible) – presents no error resulting from the relationship between spot and futures prices.

Assuming that no arbitrage is possible, the relationship between spot and futures prices may be presented as follows:

$$\psi_{t,N} = (1 + r_N)P_t - f_{t,N} \quad (5)$$

where:  $\psi_{t,N}$  –  $N$ -period (net) convenience yield (the capitalized flow of marginal convenience yield net of storage costs over the period  $t$  to  $t + N$ ),

$P_t$  – spot price in time  $t$ ,

$f_{N,t}$  – forward price for delivery at  $t + N$ ,

$r_N$  – the risk free  $N$ -period interest rate.

As to major part of mineral commodities the availability of data is far greater for futures than for forward transactions because of much greater trading of the formers. As mentioned before, futures contracts differ from forward contracts in the way they are concluded and settled. In the case of futures markets there is a central exchange which enables daily quotations, transfer of funds and transaction settlements. Hence, for the majority of commodities, differences between forward and futures contracts are scarcely perceived [French 1983].

The value of one period convenience yield (in currency units) may be obtained from the following equation [Moyen et al. 1996]:

$$CY = \frac{1}{N} \sum_{t=1}^N (1 + r) P_t - F_{t1} \quad (6)$$

where:  $r$  – one period (over the period between  $t$  and  $t + 1$ ) riskless rate of interests,

$F_{t1}$  – futures price for a contract that is purchased in period  $t$  and matures in period  $t + 1$ .

The interpretation of this equation is following: we can risklessly buy a unit of commodity on spot market and, at the same time, short a futures contract. The difference between the amounts of these transactions is strongly associated with the convenience yield, i.e. the income that accrues to the holder of the physical commodity but not to the owner of the contract.

In the case of continuous capitalization, the annual historical convenience yield (expressed in percent) is obtained from the formula [Caumon, Bower 2004]:

$$cy = -\frac{1}{T-t} \ln\left(\frac{F_{(t,T)}}{P_t}\right) + r + c \quad (7)$$

where:  $F_{(t,T)}$  – futures price of maturity  $T$  on a commodity at date  $t$ ,  
 $P_t$  – spot price at date  $t$ ,  
 $c$  – interest relative to storage cost.

#### 4. Calculation of zinc & lead historical volatility and convenience yield

The convenience yield is an interesting but methodically fairly complex financial issue. It is interesting because it combines at least three aspects and each of them is a separate and important research issue:

1. the volatility of prices of underlying assets,
2. the pricing of contracts written on these assets,
3. the valuation of real options (dividends problem).

The problem of volatility is of primary and essential value here. There were concerned the historical volatility of commodity prices, the value of contracts written on these commodities as well as the essence of the assumption of past volatility continuation in the future. The historical volatility affects not only future prices of commodities but also the value of contracts and options written on these commodities. In this way, the price of a commodity becomes the underlying asset for the contract and for the commodity option written on this good. In turn, for a real option, being a means to measure decision flexibility, the convenience yield may be understood as the equivalent of a dividend affecting the value of a financial option. Since the valuation of a financial option is based on financial option valuation methods, the adequacy of these parameters would enable the application of a larger set of option valuation methods than it was so far [Pera 2009].

The analysis of volatility in mentioned fields may be a source of vital information for an investor. It seems especially interesting in these fields to find correlations between the level of the convenience yield and the contract and real option value. The issues of convenience yield approximation has not been a matter of investigations in Polish literature yet (one of the first research papers on this subject in Poland



concerning the determination of this indicator for hard coal appeared in 2009 [Saługa, Grudziński 2009]). As far as mining industry is concerned it is important to underline that recent foreign publications concentrate especially on the convenience yield related issues in crude oil industry [e.g. Gibson, Schwartz 1990; Considine, Larson 2001; Coimbra, Esteves 2004; French 2005; Knetsch 2006]. For this reason the convenience yield for zinc and lead has been considered interesting to estimate.

#### 4.1. The procedure

The object and starting point for research was time series of zinc and lead prices in US dollars and Polish zlotys (PLNs). For these time series their historical volatility has been estimated as well as the convenience yield levels for each metal and every subsequent year.

The scope of research included years 2001-2008 and referred to zinc and lead prices according to the London Metal Exchange listings [www.lme.co.uk]. In order to estimate the basis in contracts there have been considered prices of 3-months seller contracts for metals in question quoted again on the London Metal Exchange. For the obtained prices there have been calculated consecutively:

- historical volatility of monthly rates of return,
- the basis in contracts,
- the convenience yield for every subsequent month of research period.

It was assumed in the research that the base for analysis would always be recent quotations of prices for zinc and lead for every following month. These prices, in turn, were the base to estimate  $(n - 1)$  monthly rates of return.

It was admitted that the first quotations would be metals and futures contract prices at the end of January 2001 and the last quotation analogical prices at the end of 2008. Hence, a series of 96 observations were obtained.

The logarithmic rate of return was applied for calculations according to the formula (1). Furthermore, the volatility of prices and return rates were estimated over the whole period. According to the methodic requirements of estimating volatility for each year on the base of monthly return rates volatility, the annual volatility was been determined – according to the formula (3). The estimated volatility is the first prerequisite for drawing conclusions, especially in the context of comparison of return rates volatility, calculated for prices in US dollars to return rates volatility for prices in Polish zlotys.

The notion of basis in contracts is a well known and fully described category in literature [e.g Hull 1998; Jajuga, Jajuga 2006]. It is only worth noting that sometimes there appear to be two different interpretations of the same difference being the base for calculating the basis. While usually a spot price is compared (by subtraction) to a futures price, sometimes, for calculating the basis, the spot price is subtracted from the futures price which, in the opinion of the authors, is an erratic approach. The primary and basic understanding of the basis has been assumed as the difference

between the spot price and the price of a futures contract quoted at the same time, that is:

$$\text{basis} = (P_t - F_{(t,T)}) \quad (8)$$

where:  $F_{(t,T)}$  – futures price on a commodity with time maturity  $T$  at date  $t$ ,  
 $P_t$  – spot price at time  $t$ ,

Furthermore, to calculate the monthly levels of the convenience yield the formula (7) was applied. In addition it was necessary to know riskless rates over the studied period for the American market (for the convenience yield calculated from prices in dollars) and for the Polish market (for the convenience yield calculated from prices in zlotys). For both cases it was assumed that taking into account the analysis of three-month contracts the riskless rate was understood as the interest rates on 13-week Treasury bills quoted both on the currency market in the USA and Poland. Data related to American Treasury bills were collected from the website of the Federal Reserve System of the United States [[www.federalreserve.gov](http://www.federalreserve.gov)]; as far as Poland is concerned, there are several Internet sources which allow to collect interest rates, e.g. [[www.nbp.pl](http://www.nbp.pl), [www.money.pl](http://www.money.pl)]. Another vital component for calculating the convenience yield is the cost of storage. We assumed this cost at 5%.

In order to compare the effect of currency changes on the convenience yield levels, all zinc and lead prices as well as futures contract prices for zinc and lead expressed in dollars were converted to prices in zlotys – each time according to the average exchange rates of foreign currencies of the National Bank of Poland for the last day of every following month in which metal prices in dollars were analyzed. The exchange rates were obtained from the NBP website [[www.nbp.pl](http://www.nbp.pl)]. Having in hand prices of the same commodities expressed in two different currencies enabled the comparison of prices volatility and the volatility of the convenience yield levels which, in turn, is a prerequisite for drawing conclusions about the correlation between the world's and national tendencies, for instance in investing in zinc, lead or derivatives relying on these metals.

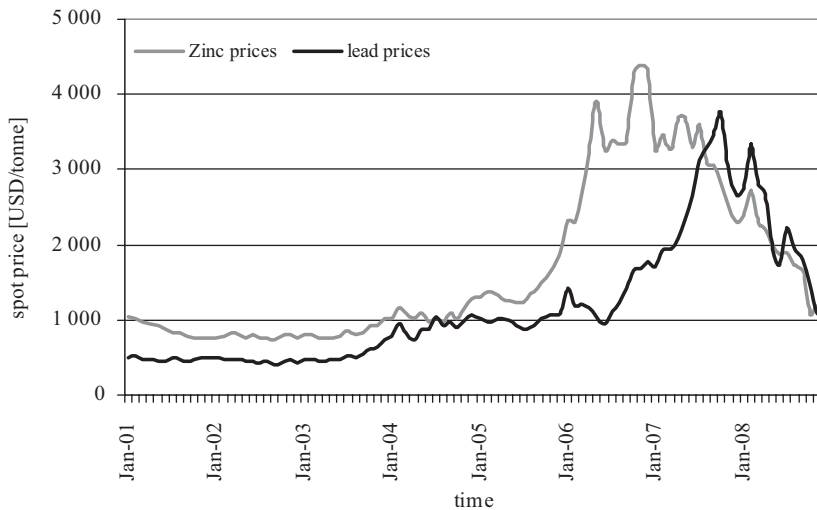
All calculations have been done in nominal values.

## 4.2. Results

The research has been done according to the methodical scheme presented above. The obtained results and conclusions have been presented below in two elementary sections:

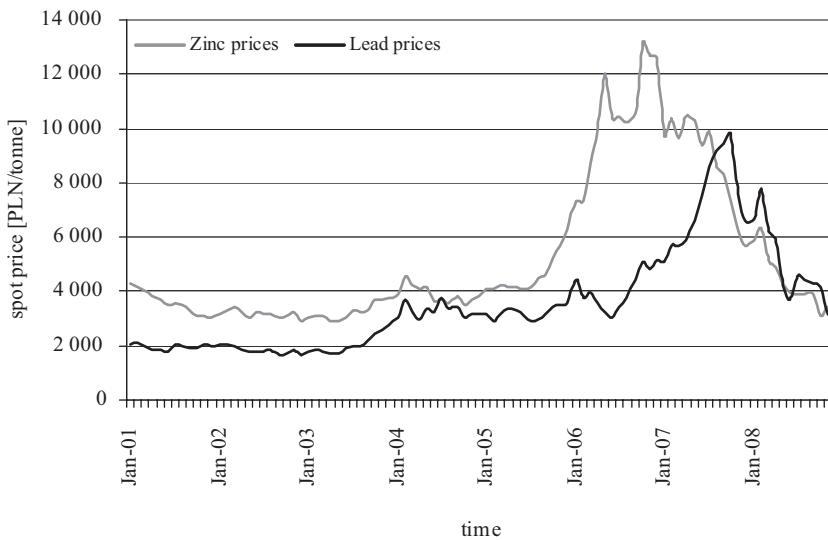
1. the analysis of prices and return rates volatility,
2. the analysis of convenience yield levels.

Price forming process for zinc and lead (in US dollars and Polish zlotys) over studied period is presented in Figures 1 and 2.



**Fig. 1.** The 2001-2008 average zinc and lead spot prices in USD/tonne

Source: own study.



**Fig. 2.** The 2001-2008 average zinc and lead spot prices in PLN/tonne

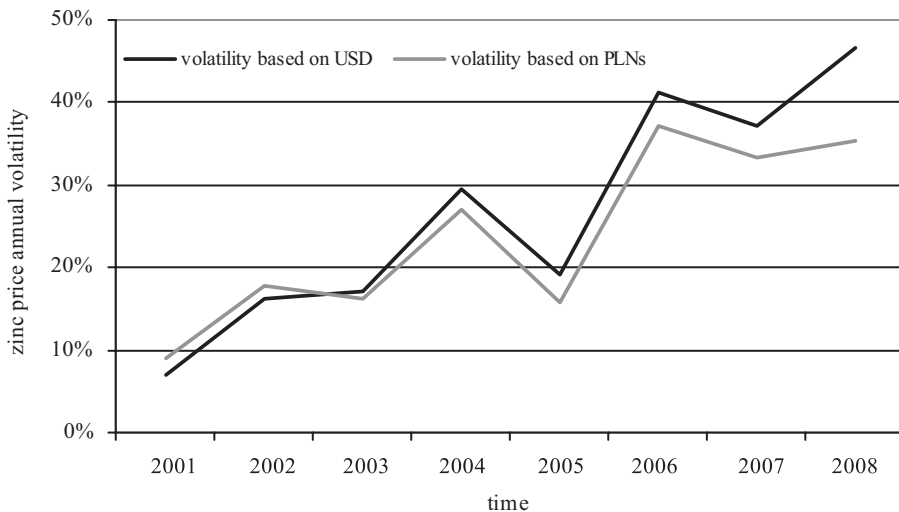
Source: own study.

The analysis of results shows that the price level of both metals increases in time – achieving the highest values in 2007 and then decreases systematically. It is interesting to note that – regardless of the chosen currency – the tendencies are

highly convergent. Relying on this we may formulate a preliminary conclusion that the US dollar exchange rate Polish currency demonstrates the stabilizing character of the relationship between currencies and investment conditions in these commodities market are – according to price’s criteria – very similar regardless of dollar or zloty currency.

In turn, the calculated average level of the basis in contracts was presented on average for zinc as minus 7.18 USD and minus 27.20 PLN, which – taking into account high nominal levels of average prices of these commodities – proves the great convergence of spot and futures prices, again regardless of dollar and zloty currencies. Analogical values for lead are of 18.71 USD and 56.48 PLN. If the average relation dollar/zloty was estimated on this base, we could get the average exchange rate of 3.79 PLN/USD from zinc prices and 3.02 from lead prices. The calculated average value of the dollar for years 2001-2008 according to the National Bank of Poland indices amounted to 3.40 PLN and was placed exactly at the average exchange rate calculated from the basis in contracts for zinc and lead as the arithmetic mean of these both values (the arithmetic mean of values 3.79 and 3.02).

The result of price levels analysis is their volatility. Figures 3 and 4 demonstrate price volatility levels for zinc and lead in US dollars and Polish zlotys.

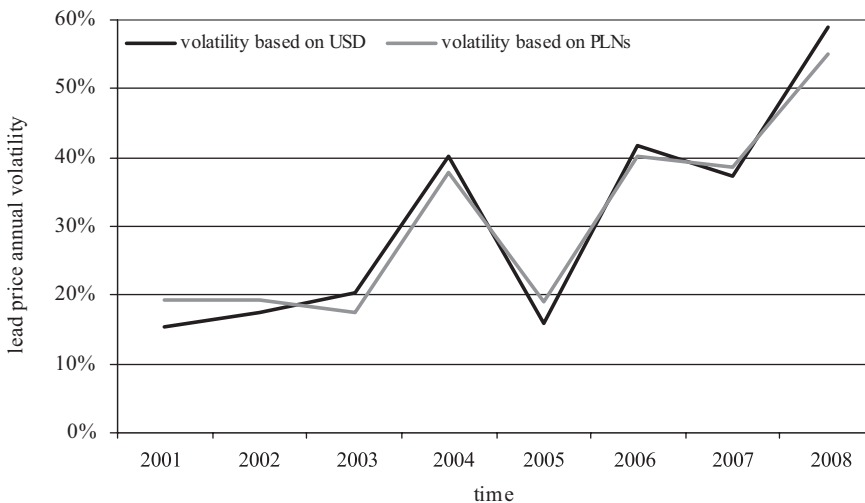


**Fig. 3.** The average annual volatility of monthly return rates for zinc

Source: own study.

As it results from the presented figures the volatility of monthly return rates for zinc and lead expressed as the standard deviation of return rates increased in time and achieved maximum values for zinc:

- 46.71% (in 2008) for prices in dollars,
  - 37.26% (in 2006) for prices in zlotys.
- For lead this relation is following:
- 59.01% (in 2008) for prices in dollars,
  - 54.95% (in 2008) for prices in zlotys.



**Fig. 4.** The average annual volatility of monthly return rates for lead

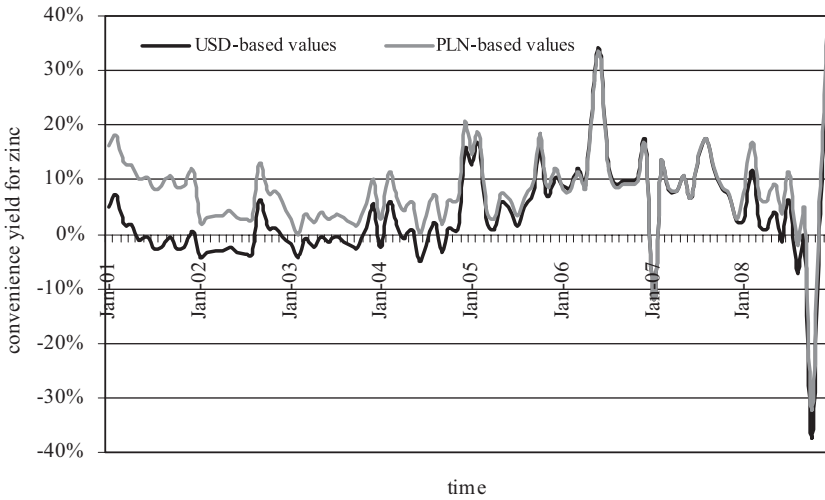
Source: own study.

It is worth noting that return rates volatility for prices in dollars is greater than the one calculated in zlotys. Consequently, we can see the increasing stabilization of Polish currency on the one hand but, at the same time, from the point of view of potential options on these commodities, taking into account that the higher volatility of the underlying asset the greater value of an option, hypothetical investing in such options quoted in dollars could generate greater profits than in the case of analogical options quoted in zlotys.

The essence of research was the approximation of convenience yield for both metals over the studied period. Figures 5 and 6 present the changes of convenience yield for zinc and lead quoted in dollars and zlotys.

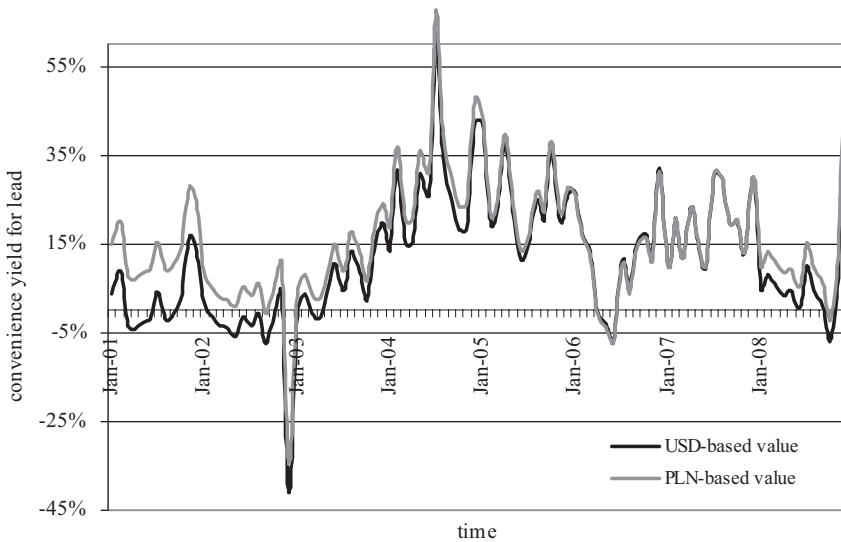
## 5. Conclusion

The convenience yield premium may but does not have to occur. Hence, the owner of a commodity and the owner of the contract on this commodity do not hold the same position. The answer to this dilemma formulated as "if it is better to own the commodity or rather to be the holder of the contract on this commodity" may be the mean value of



**Fig. 5.** The convenience yield for zinc calculated for the 2001-2008 period

Source: own study.



**Fig. 6.** The convenience yield for lead calculated for the 2001-2008 period

Source: own study.

the convenience yield. If this value is positive the owner of commodity is in the better situation, but if it is negative – the holder of a futures contract is in the better position. For studied cases the mean value of the convenience yield amounts to:

- for zinc 3.74% for prices in USD and 7.96% for prices in PLN, for lead 11.89% for prices in USD and 16.11% for prices in PLN.

In both cases and for both currencies the average level of convenience yield is positive, which means that in the studied period it was more profitable to keep the commodity than to purchase a contract on this commodity. In addition, as opposed to the specific profitability for options, the average level of the convenience yield for prices in PLN was higher than the level for prices in USD. Hence the potential owner of these commodities in the Polish market might have gained greater profits than in the case of an investor in the market where these metals were quoted in dollars. This difference, let us mention it, results exclusively from the relation of both currencies in the studied period. This convenience yield, in turn, is higher for lead and lower for zinc. If assumed that historical volatility is a good forecaster for future volatility, then the optimal investment strategy should rely on purchasing lead instead of zinc and concluding transactions on a spot market.

In addition, a high correlation between the convenience yield calculated from prices in PLN against those in USD has to be noticed. The correlation rate between the convenience yield level calculated from prices in PLN and USD for zinc amounts to 0.915, and for lead is even higher and amounts to 0.972.

As to complement – relying upon the analyzed data – we compare return rates for a hypothetical investment that consists of purchasing zinc and lead at the starting point of the studied period and then selling them at the end of the period, and alternatively the investment in dollars for the same period. Geometric and arithmetic return rates are calculated. The geometric return rate for zinc amounts to 1.71% (for prices in USD) and minus 2.31% (for prices in PLN). The geometric return rate for lead amounts to 10.33% (for prices in USD) and 5.97% (for prices in PLN). The arithmetic return rates amount to: 14.50% and minus 17.04% for zinc, and 119.56% and 59.08% for lead. If investment in dollars was an alternative for investment in commodities, in the studied period it would be the worst of the alternatives because the geometric return rate, in this case, amounts to minus 3.95%, so it has a lower level than analogical return rates for zinc and lead, and the arithmetic return rate amounts to minus 27.55% and is definitely the worst result.

The analyzed issues and conducted research seem to be a good direction in the analysis when estimating investment efficiency. The level of convenience yield – as shown – is not indifferent to investment efficiency level and should be calculated whenever we have an appropriate series of data. The authors consider combining the convenience yield related issues with those of real options valuation as particularly interesting and of educating value because this parameter is indispensable in option valuation models as the indicator of dividends. In last decades there have been developed some convenience yield assessment models that use option pricing algorithms [Heinkel et al. 1990; Gibson, Schwartz 1997; Milonas, Thomadakis 1997; Schwartz 1997; Heaney 2002; and others]. They can make a basis for further, more complex studies.

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## OKREŚLENIE HISTORYCZNEJ ZMIENNOŚCI CEN I PREMII Z TYTUŁU SKŁADOWANIA DLA CYNKU I OŁOWIU

**Streszczenie:** Jednym z najważniejszych parametrów analizy technicznej instrumentów na rynkach finansowych i towarowych oraz podstawowym wskaźnikiem interesującym inwestorów jest zmienność cen. W branży surowców mineralnych oprócz zmienności cen istotne znaczenie ma tzw. premia z tytułu składowania (convenience yield, CY), która jest ważnym parametrem modelu wartości zaktualizowanej racjonalnej wyceny towarów PVM. Obydwa parametry są kluczowymi zmiennymi wykorzystywanymi w procesie prognozy cen surowców oraz w modelach wyceny opcji rzeczowych. Niniejszy referat przedstawia próbę oszacowania historycznej zmienności cen i premii z tytułu składowania dla cynku i ołowiu na podstawie danych z 2001-2008.