ARGUMENTA OECONOMICA
No 1 (46) 2021
PL ISSN 1233-5835

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METHODOLOGICAL CONTRIBUTION TO THE DETECTION OF BACKWARD LINKAGES BETWEEN SECTORS OF THE ECONOMY

In national accounting, there are several synthetic indicators derived from the Leontief inverse, referred to as Total Material Requirement coefficients, which are commonly used in efficiency analyses and are known as input-output multipliers. Among the many indices that can be elaborated from input-output analysis, those related to the detection of key sectors are of special relevance since measuring the importance of a productive sector is especially relevant for policymakers when facing the need to make decisions to promote economic growth. There are two main alternative methods to identify key economic sectors, namely the Classical Multiplier method and the Hypothetical Extraction method (HEM), which essentially differ on the role of internal effects (the impact experienced by the sector in question). While the Classical method quantifies these internal effects, the HEM considers only the external impact. The latter method enables calculating backward linkages by isolating the column corresponding to demand-side sectors. However, such an alteration of the economic system can seem unrealistic and may give rise to doubts as to whether the results are biased, which in turn would cause incorrect public investment and false sectoral priorities. This paper offers an alternative method to detect key sectors based on a normalization of the Leontief inverse. After discussing the properties of the proposed standardization, the HEM, the Classical Multiplier method, and the one proposed here, are formally and empirically compared by using the 2010 input-output tables for Poland and Spain. The findings indicate that distinguishing and disaggregating the external effects from those that are purely internal has relevant policy implications. This disaggregation can be achieved through the proposed methodology, while avoiding the criticisms mentioned regarding the HEM, and with less effort required to calculate it.

Keywords: Leontief inverse, net multiplier, normalization
JEL Classification: Q42, C67
DOI: 10.15611/aoe.2021.1.02

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1. INTRODUCTION

The input-output (IO) methodology is usually used to assess the impact of changes in exogenous variables on a specific economy. As a consequence, whenever exogenous changes occur due to a strong single factor, and the consequences are expected to be experienced in a short time span, the term ‘impact analysis’ is used. Therefore, if the demand model is applied, the multipliers essentially depend on the information contained in the Leontief inverse. The multipliers are based on the difference between the initial effect of the exogenous changes (final demand) and the total effect of the change considered. Multiple references to these multipliers can be found in the literature, for instance in Miernyk (1967), Miernyk, Bonner, Chapman and Shellhamer (1967), Schaffer (1976), Pleeter (1980) and Hewings (1985). For a more detailed approach, see Miernyk (1976), Pibbs and Holsman (1981, 1982), Harrigan (1982), Katz and Burford (1981, 1982, 1985), Szyrmer (1992), Gim and Kim (1998), Lenzen (2001), de Mesnard (2002), Oosterhaven and Stelder (2002), Dietzenbacher (2005), Gim and Kim (2005), Liew (2005), Oosterhaven (2007), Miller and Blair (2009) and Sancho (2012). All of them present (more or less complex) alternatives to the IO model in order to disaggregate the global economic impact.

Hirschman (1958) was the first to suggest the relevance of sectoral linkages and sectoral multipliers when explaining economic growth. Following Hirschman’s approach, the authors define ‘key sectors’ as those that have either an above average backward strength (key pull sector) or an above average forward linkage index (key push sector). Various techniques exist to identify inter-sectoral relationships within an economy, in order to single out the key sectors showing higher capacity to stimulate changes in other sectors. This means finding the linkages between different sectors in order to establish a ranking which then will serve as a basis for identifying the key sectors. Initially, the linkage detection formulas were based either on direct coefficients (Chenery and Watanabe, 1958) or on entries of the Leontief inverse or Ghosh inverse (Rasmussen, 1956 and Beyers, 1976). These methods measure the relevance of the sector merely in terms of the simple averages of the technical coefficients (direct and indirect). The Hypothetical Extraction Method (HEM), in contrast, weights the relevance of a sector by way of simulating the elimination of the concrete economic connections between that sector and the remaining sectors. The output loss that would follow from this hypothetical cessation of economic activities quantifies the economic linkages of the sector and provides a measure of the degree of dependency that a given economy has upon such specific sector. This is the starting point for the propositions of net IO multipliers, which attempt to clean the entries of the inverse matrix. This

HEM is attributed to Strassert (1968), although it was previously suggested by Caevel, Degueldre and Paelinck (1965). Later, this was used by Miller (1966) as part of regional analysis, and because that idea was met with a lot of criticism, alternative methodological procedures emerged, including those developed by Meller and Marfán (1981), Cella (1984, 1986) or the one known as the partial extraction method (Dietzenbacher and Van der Linden 1997).

The wide existing academic literature is proof of the relevance of this question, but also reveals it as an 'old' problem that has failed to receive a completely satisfactory solution. This paper presents a complementary method to obtain key sectors in an economy by using a normalized Leontief inverse (by row) based on the elements of the main diagonal. This allows to reformulate the demand model in order to present a complementary/supplementary solution for the partial extraction method, and then to compare both methods. In fact, in the partial hypothetical extraction method, backward linkages are identified by isolating the column corresponding to the (demand-side) sector and by working on the modified model. However, such an alteration of the economic system seems unrealistic, thus raising doubts as to whether the results are correct, since this model does not enable reproducing the exact size of the analysed economy. In contrast, the authors' method has the following main advantages: (1) it yields a disaggregation of the global impact; (2) it quantifies the relevance of one sector in terms of its relative external contribution to market interdependencies (which could be very important in the primary sector and in sectors with a high level of aggregation of productive activities, where self-consumption is very significant) and (3) it reproduces the exact size of the economy.

In order to illustrate the viability and usefulness of this proposal, the article also presents an empirical exercise aimed at identifying the key sectors for efficiency policies in the context of the Spanish and Polish economies respectively, by using the three methodological alternatives indicated. Therefore, the combined use of the two already-existing methods and the one proposed here might enrich both the empirical results and the conclusions drawn for further policy guidance.

2. MULTIPLIER NORMALIZATION: THE CONTEXT

Disaggregating the global impact involves various transformations of the IO demand model. Hence, the corresponding formulas become complicated and may be reformulated in different ways, as shown later in this paper. This part considers various ways of disaggregating the inverse matrix, as formulated by different authors.

The demand (Leontief) model can be expressed as

$$x = (I - A)^{-1}y = Cy, \tag{1}$$

where x is the production vector, y is the vector of net final demand for imports and $(I–A)^{-1}$ is the Leontief inverse (denoted as C for convenience).

Usually the Leontief inverse is expressed as the infinite sum of powers of matrix A. More specifically, it can be written as follows:

$$(I - A)^{-1} = I + A + A^2 + \cdots + A^m + \cdots. \tag{2}$$

The general term is:

$$\alpha_{ij} = \begin{cases} 1 + a_{ij} + \sum_{k=1}^{n} a_{ik}\, a_{kj} + \sum_{k_1=1}^{n}\sum_{k_2=1}^{n} a_{ik_1}\, a_{k_1k_2} a_{k_2j} + \cdots, i = j \\ a_{ij} + \sum_{k=1}^{n} a_{ik}\, a_{kj} + \sum_{k_1=1}^{n}\sum_{k_2=1}^{n} a_{ik_1}\, a_{k_1k_2} a_{k_2j} + \cdots, i \neq j \end{cases} \tag{3}$$

It should be verified if

$$C - I = A + A^2 + A^3 + \cdots. \tag{4}$$

Indeed,

$$AC = A + A^2 + A^3 + \cdots = CA, \tag{5}$$

i.e. the product of matrices A and C is commutative.

Gim and Kim (1998) define the matrix of direct and indirect inputs:

$$\Gamma^f = C - I, \tag{6}$$

In accordance with (4) and (5), the generic element of that matrix may include various expressions as shown below:

$$\gamma_{ij}^{f} = a_{ij} + \sum_{k=1}^{n} a_{ik}\, a_{kj} + \sum_{k_1=1}^{n}\sum_{k_2=1}^{n} a_{ik_1}\, a_{k_1k_2} a_{k_2j} + \cdots = \sum_{k=1}^{n} a_{ik}\, \alpha_{kj}. \tag{7}$$

The matrix can be normalized as follows (6):

$$\Gamma^g(row) = \widehat{D}^{-1}AC = \widehat{D}^{-1}\Gamma^f, \tag{8}$$

where $\widehat{D}$ is a diagonal matrix composed of the elements of the main diagonal of matrix C. This is how the relationship between Γ^f and $\Gamma^g(row)$, is established. Note that $\Gamma^f = \widehat{D}\Gamma^g(row)$. This relationship (for the elements of main diagonal of Γ^f) was studied by Jeong (1982, 1984). Later, Gim and Kim (1998) provided a generalized version for all the elements of the matrix, based on the fact that $C(I - A) = I$. Once this property is developed, formula (8) is obtained immediately.

The generic element of $\Gamma^g(row)$ is as follows:

$$\gamma_{ij}^g(row) = \frac{1}{\alpha_{ii}}(a_{ij} + \sum_{k=1}^n a_{ik}\, a_{kj} + \sum_{k_1=1}^n \sum_{k_2=1}^n a_{ik_1}\, a_{k_1k_2} a_{k_2j} + \cdots), \quad (9)$$

or:

$$\gamma_{ij}^g(row) = \frac{1}{\alpha_{ii}} \sum_{k=1}^n a_{ik}\, \alpha_{kj}. \quad (10)$$

Other authors, such as Szyrmer (1992) and Sancho (2012), normalized matrix Γ^f by columns. Even Gim and Kim (2008) moved away from the normalization presented above by developing the following alternative formula for obtaining the direct input matrix:

$$\Gamma^g(col) = CA\widehat{D}^{-1} = \Gamma^f \widehat{D}^{-1}. \quad (11)$$

therefore:

$$\gamma_{ij}^g(col) = \frac{1}{\alpha_{jj}} \sum_{k=1}^n \alpha_{ik}\, a_{kj}. \quad (12)$$

The latter expression may be reformulated by using various substitutions related to the matrix product CA.

Matrix C may be disaggregated in various ways. Later, the possible algebraic transformations of model (1) are presented.

The following difference is derived from (1):

$$x - (I - A)x = Cy - (I - A)x, \quad (13)$$

$(I - A)x = y$, and therefore, the following is true:

$$Ax = Cy - y = (C - I)y. \quad (14)$$

Next, both terms are pre-multiplied by the inverse of matrix A:

$$A^{-1}Ax = A^{-1}(C - I)y. \quad (15)$$

However, in order to ensure that the determinant of A is not zero, it is assumed that none of the productive structures is a linear combination of any of the other ones.[1]

[1] Given the level of aggregation in the IO tables published by the main national and international statistical agencies, the possibility that the cost structure of a given sector is a linear combination of other ones is not significant. If the number of sectors is large, it can be awkward obtaining the inverse of the Leontief matrix. In fact, in the Leontief model it is essential to ensure a non-negative inverse. Indeed, an economy is said to be productive if Leontief's inverse is non-negative. To determine this fact, the necessary and sufficient condition of Hawkins-Simon is generally used. However, sometimes, when analyzing the characteristics of a matrix of Leontief, the sufficient condition of Brauer-Solow, Waugh (1950) is used.

Thus, the demand model is expressed with the following alternative formula:

$$x = A^{-1}(C - I)y = A^{-1}\Gamma^f y. \tag{16}$$

It is clear that $C = A^{-1}\Gamma^f$.

The next step analyses the role of $\Gamma^g(row)$ in the demand model. This expression is obtained by distributing (1) in a unitary matrix, hence the following expression may be obtained:

$$x = ICy = A^{-1}ACy = A^{-1}IACy = A^{-1}\widehat{D}\widehat{D}^{-1}ACy = A^{-1}\widehat{D}\Gamma^g(row)y \tag{17}$$

with:

$$\Gamma^g(row) = \begin{pmatrix} \frac{\alpha_{11}-1}{\alpha_{11}} & \frac{\alpha_{12}}{\alpha_{11}} & \cdots & \frac{\alpha_{1n}}{\alpha_{11}} \\ \frac{\alpha_{21}}{\alpha_{22}} & \frac{\alpha_{22}-1}{\alpha_{22}} & \cdots & \frac{\alpha_{2n}}{\alpha_{22}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\alpha_{n1}}{\alpha_{nn}} & \frac{\alpha_{n2}}{\alpha_{nn}} & \cdots & \frac{\alpha_{nn}-1}{\alpha_{nn}} \end{pmatrix} \tag{18}$$

Gim and Kim (2008) analysed the difference between Γ^f and $\Gamma^g(col)$. In accordance with the commonly used presentation method, it is defined as:

$$R(row) = \Gamma^f - \Gamma^g(row) \tag{19}$$

which can be formulated as:

$$R(row) = \begin{pmatrix} \frac{(\alpha_{11}-1)^2}{\alpha_{11}} & \frac{(\alpha_{11}-1)\alpha_{12}}{\alpha_{11}} & \cdots & \frac{(\alpha_{11}-1)\alpha_{1n}}{\alpha_{11}} \\ \frac{(\alpha_{22}-1)\alpha_{21}}{\alpha_{22}} & \frac{(\alpha_{22}-1)^2}{\alpha_{22}} & \cdots & \frac{(\alpha_{22}-1)\alpha_{2n}}{\alpha_{22}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{(\alpha_{nn}-1)\alpha_{n1}}{\alpha_{nn}} & \frac{(\alpha_{nn}-1)\alpha_{n2}}{\alpha_{nn}} & \cdots & \frac{(\alpha_{nn}-1)^2}{\alpha_{nn}} \end{pmatrix}. \tag{20}$$

Therefore the Leontief inverse may be disaggregated in various ways, which results – either explicitly or implicitly – in the creation of a matrix power series. The key is to look for an analytical expression that eases the decomposition of matrix C and, above all, for an appropriate economic interpretation.

In this context an important issue arises. The question is whether α_{ij} should be normalized by α_{ii} or α_{jj}. Although both methods are algebraically feasible, it should be determined which one is appropriate. Ultimately, α_{ij} measures the impact experienced in sector ***i*** because of a change in final demand for product *j*. However, that increase entails a production growth in sector *j* which, in turn, has a series of impacts on the remaining part of the economy. These impacts can be used (with adequate weights) in the decomposition of the generic element of the Leontief inverse.

Therefore the demand model can also be transformed:

$$\begin{aligned} x = Cy = y + (C - I)y = y + \left(\widehat{D}^{-1} + I - \widehat{D}^{-1}\right)(C - I)y = \\ = y + \widehat{D}^{-1}(C - I)y + [(I - \widehat{D}^{-1})(C - I)]y, \end{aligned} \quad (21)$$

which can be abbreviated as:

$$x = y + \Gamma^{g}(row)y + R(row)y. \quad (22)$$

3. DETERMINATION OF A KEY SECTOR: METHODOLOGICAL REFLECTIONS ON NORMALIZING THE INVERSE AND ITS RELATIONSHIP WITH THE HYPOTHETICAL EXTRACTION METHOD

This section explores the relationship between a specific normalization of the Leontief inverse and the formula of the partial hypothetical extraction. Therefore, the focus lies in the development of the relative net IO multipliers and then in tracing the corresponding net backward links.

The normalization procedure can also be applied directly to the Leontief inverse. This will result in the following normalization by row:

$$\bar{C}^{f}(row) = \widehat{D}^{-1}C = \begin{pmatrix} 1 & \frac{\alpha_{12}}{\alpha_{11}} & \cdots & \frac{\alpha_{1n}}{\alpha_{11}} \\ \frac{\alpha_{21}}{\alpha_{22}} & 1 & \cdots & \frac{\alpha_{2n}}{\alpha_{22}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\alpha_{n1}}{\alpha_{nn}} & \frac{\alpha_{n2}}{\alpha_{nn}} & \cdots & 1 \end{pmatrix} \quad (23)$$

and

$$\bar{C}^{g}(row) = \left(I - \widehat{D}^{-1}\right)C = \begin{pmatrix} \alpha_{11} - 1 & \frac{\alpha_{11} - 1}{\alpha_{11}}\alpha_{12} & \cdots & \frac{\alpha_{11} - 1}{\alpha_{11}}\alpha_{1n} \\ \frac{\alpha_{22} - 1}{\alpha_{22}}\alpha_{21} & \alpha_{22} - 1 & \cdots & \frac{\alpha_{22} - 1}{\alpha_{22}}\alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\alpha_{nn} - 1}{\alpha_{nn}}\alpha_{n1} & \frac{\alpha_{nn} - 1}{\alpha_{nn}}\alpha_{n2} & \cdots & \alpha_{nn} - 1 \end{pmatrix}. \quad (24)$$

Matrix (23) presents the relative values of the respective elements as a function of the multiplier of the main diagonal (this value is associated with the output *i* required directly and indirectly to deliver one additional monetary unit to *i'*s demand). These elements are greater than one, and therefore the corresponding elements of matrix (23) will be reduced.

Matrix (24) is structured similarly, except that the transformations are conditioned by the effects expressed as $(\alpha_{ii} - 1)$, $i = 1,2,..., n$. This means that the multipliers used in (23) do not take the direct impact of an additional unit of sectoral final demand into account. Thus, the Leontief inverse can be normalized by column: $\bar{C}^f(col) = C\widehat{D}^{-1}$ and $\bar{C}^g(col) = C(I - \widehat{D}^{-1})$.

Next, the demand model is transformed to visualize the role of normalization by rows. Thus, with respect to model (1), a specific modification is made, resulting in the following decomposition of the Leontief inverse:

$$\begin{aligned} x &= Cy = ICy = (\widehat{D}^{-1} + I - \widehat{D}^{-1})Cy = \\ &= [\widehat{D}^{-1}C + (I - \widehat{D}^{-1})C]y = \widehat{D}^{-1}Cy + (I - \widehat{D}^{-1})Cy \end{aligned} \tag{25}$$

In accordance with (23) and (24):

$$x = \bar{C}^f(row)y + \bar{C}^g(row)y, \tag{26}$$

so that $\bar{C}^g(row) = C - \bar{C}^f(row)$.

The first equation of system (1) can be considered as an example:

$$x_1 = \alpha_{11}y_1 + \alpha_{12}y_2 + \ldots + \alpha_{1n}y_n, \tag{27}$$

to disaggregate production by the weight of final demand and by the weight of effects driven by mutual inter-sectoral links of the self-consumption multiplier of producer sector 1 (α_{11}). Consequently, the weights can be formulated as $\frac{1}{\alpha_{11}}$ and $\frac{\alpha_{11} - 1}{\alpha_{11}}$. Therefore:

$$\begin{aligned} x_1 = \frac{1}{\alpha_{11}}x_1 + \frac{\alpha_{11} - 1}{\alpha_{11}}x_1 = \left[y_1 + \frac{\alpha_{12}}{\alpha_{11}}y_2 + \ldots + \frac{\alpha_{1n}}{\alpha_{11}}y_n \right] + \\ \left[(\alpha_{11} - 1)y_1 + \frac{\alpha_{11} - 1}{\alpha_{11}}\alpha_{12}y_2 + \ldots \frac{\alpha_{11} - 1}{\alpha_{11}}\alpha_{1n}y_n \right]. \end{aligned} \tag{28}$$

The partial extraction method consists in eliminating only the column of *j*-th sector, taking its offer into consideration, in order to explore the strength of its backward relationships on a case-by-case basis. Therefore it is necessary to split (disaggregate) the technical coefficients matrix into four sub-matrices[2]:

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \tag{29}$$

where A_{11} and A_{21} become zero matrices when the column is removed. In other words:

$$A^* = \begin{pmatrix} 0 & A_{12} \\ 0 & A_{22} \end{pmatrix} \text{ and the related inverse matrix } C^* = \begin{pmatrix} I & A_{12}C_{22} \\ 0 & C_{22} \end{pmatrix}. \tag{30}$$

It is understood that C_{22} is the inverse of $(I – A_{22})$, which is a Leontief matrix of order n–1, whereas the extraction of a single row would enable estimating the future relationships of that sector; the Ghosh model provides a highly similar approach to this issue. These two alternatives for partial extraction meet some of the criteria for normalization (by row or by column, respectively).

The multipliers which appear in the first row of matrix (23), in particular, $\frac{\alpha_{12}}{\alpha_{11}}, \dots, \frac{\alpha_{1n}}{\alpha_{11}}$, correspond to elements of the sub-matrix used by Dietzenbacher and Van der Linden in their method. Based on the above, the impact of the extraction of a column corresponding to economic activity can be easily identified. Note that as shown in (30), C_{22} can be immediately calculated based on A_{22}. That inverse matrix does not take any account of the impacts of the extracted sector. To sum up, the normalization of the Leontief inverse by row with the use of elements of the main diagonal reveals the relationships between subsequent rows as the columns are removed with the partial extraction, even if such linkages are not evident.

4. PRACTICAL USE

For practical purposes, the authors present the classification of key sectors in Poland and Spain using three alternative methods: the standard BL method based on matrix C; the normalized BL method based on $\bar{C}^f(row)$; and the hypothetical extraction (HE) based on criteria by Dietzenbacher and Van der Linden, i.e. on different matrices C^* (with an equal number of sectors). All the calculations used the national tables for the year 2010 (Central Statistical Office DATA, 2018 and INE, 2018). The Polish and Spanish IO tables are split into 77 and 65 productive sectors, respectively.

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[2] For ease of explanation, it is assumed that the first sector is eliminated.

According to formula (3), it is verified to what extent the elements of the main diagonal of the inverse of Leontief, α_{ii}, the different ones are explained by the $1 + \alpha_{ii}$ (i.e. the effect given by the final demand and the effect attributed to the self-consumption of the branch in question). Thus for Poland, a correlation coefficient of 0.985 is obtained, and for Spain, 0.984.

While the three techniques rely on similar calculation methods, they use different inverse matrices, and eventually the results are converted into relative values to facilitate the ranking of sectors and the benchmarking of methods. Similarly, forward linkages (FL) could be sought to identify the key sectors.

Based on the similarity between the normalization procedure described above and the row-based multipliers in the partial extraction method, and in accordance with a more conventional approach (Rasmusen, 1956), an attempt was made to calculate the net multipliers (based on normalized elements) and to check to what extent the results differ. In case differences are found, the underlying reasons explaining them will be investigated.

Figure 1 shows the sectors of the Polish economy (in 2010) in terms of classical backward linkages (BL) and normalized BL. The alternative, Figure 2 shows the same sectors of the Polish economy in terms of normalized BL and the hypothetical extraction-based BL. The values for the 77 sectors are specified in the attachment. Generally, the values obtained through different techniques are highly consistent with each other. The similarities are particularly noticeable between the standard BL and the normalized BL. There

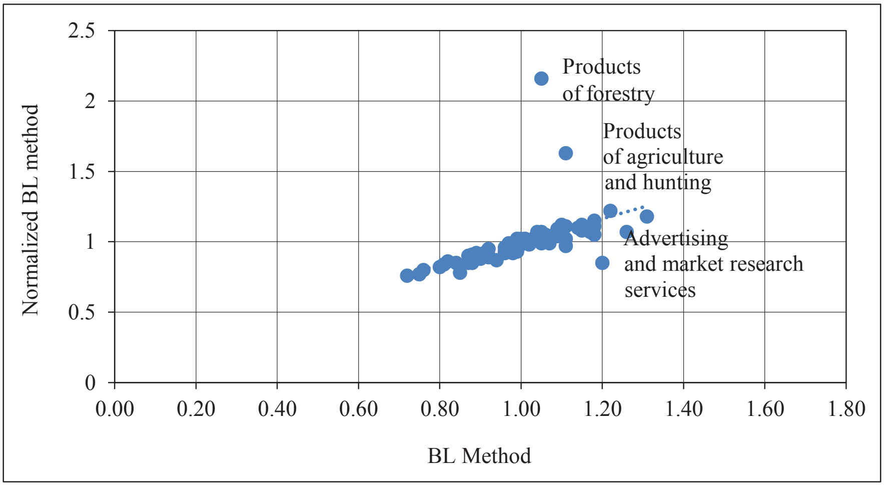

Fig. 1. Multipliers Poland I

Source: own study based on Central Statistical Office DATA (accessed on November 11, 2018).

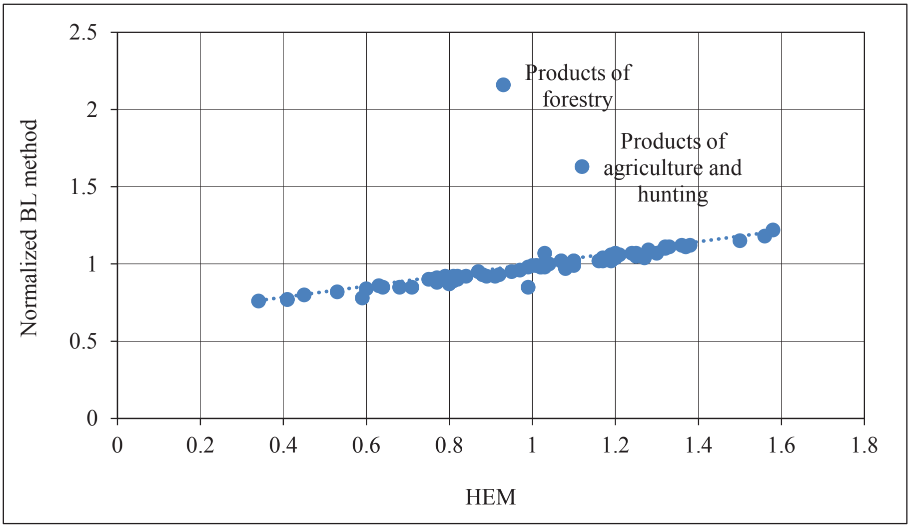

Fig. 2. Multipliers Poland II

Source: own study based on Central Statistical Office DATA (accessed on November 11, 2018).

is a group of 11 sectors with values very close to 1 (either greater or lower than 1) irrespective of the technique used.

There are some exceptions, especially regarding sector 2 (products of forestry), which was found not to have any stimulating capacity in the HEM, and sector 1 (products of agriculture and hunting). The fact that these are basic productive industries could explain why the HEMBL are not consistent; so that potential explanations are to be explored. In this context, $\alpha_{22}=1.34$ is the greatest element on the main diagonal. Therefore in accordance with the HE formula, it will considerably reduce the elements of the corresponding inverse matrix, which implies a reduction of the corresponding BL value. This is an extreme case which should be interpreted cautiously.

For sector 1, $\alpha_{11} = 1.27$ which also is one of the highest values. Most of the elements of the main diagonal of the Leontief inverse vary within a range of 1.05 to 1.15. In the normalized BL method, the values for sectors 1 and 2 distort to a certain degree the values for other sectors but their positions differ only slightly. In this approach, standardization is performed for all the sectors involved. The following key sectors are identified nearly identically by the three techniques: 62 (travel agency, tour operator and other reservation services and related services), 6 (food products), 7 (beverages), 32 (waste collection, treatment and disposal services; materials recovery services), 42 (food and beverage serving services), 39 (water and air transport services), 12 (wood and products of wood).

The results for Spain are shown in Figures 3 and 4. The values for 65 sectors are specified in Annex 2. The values obtained with the BL HE method are clearer and are precisely specified with the net multipliers used in the previous step. In other cases, the results observed for highly influential sectors are nearly fully convergent irrespective of the technique used. As in the case of Poland, the major coincidence in the relative position of multipliers is between HEM and the normalized BL method.

When looking for some differences between the results of normalized BL and BL HE, in line with the nearly perfect shift in scale between the sectors, some minor exceptions can be found in the ranking, marked by the following sectors: 19. machinery and equipment not elsewhere specified; 16. fabricated metal products except for machinery and equipment; or 15. metallurgy and metallic products. Sector 24 (electricity, gas, steam and air conditioning) has an extremely high value on the main diagonal (1.52) which explains the considerable difference in results between the standard BL and net BL methods.

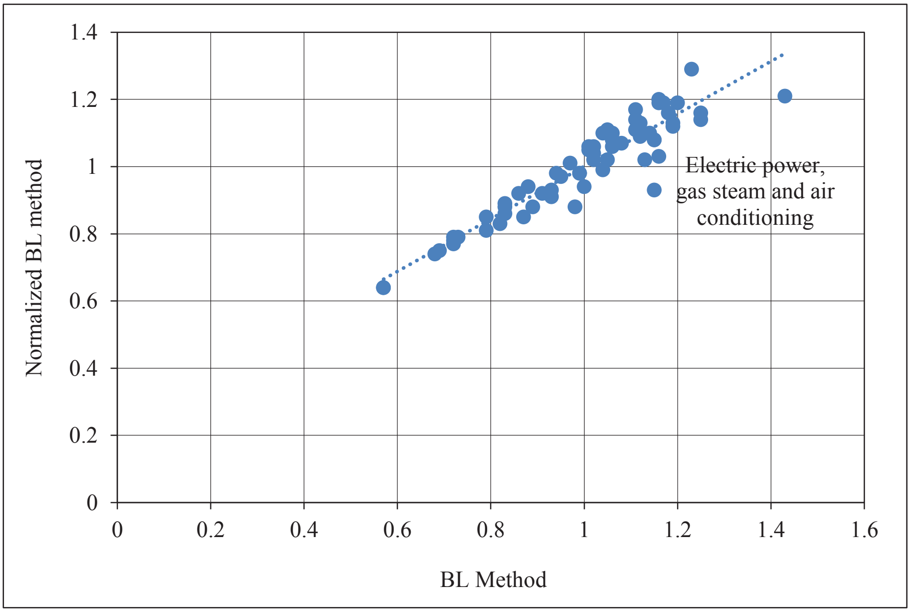

Fig. 3. Relative Multipliers Spain I

Source: own study based on INE (accessed on 11 November 2018).

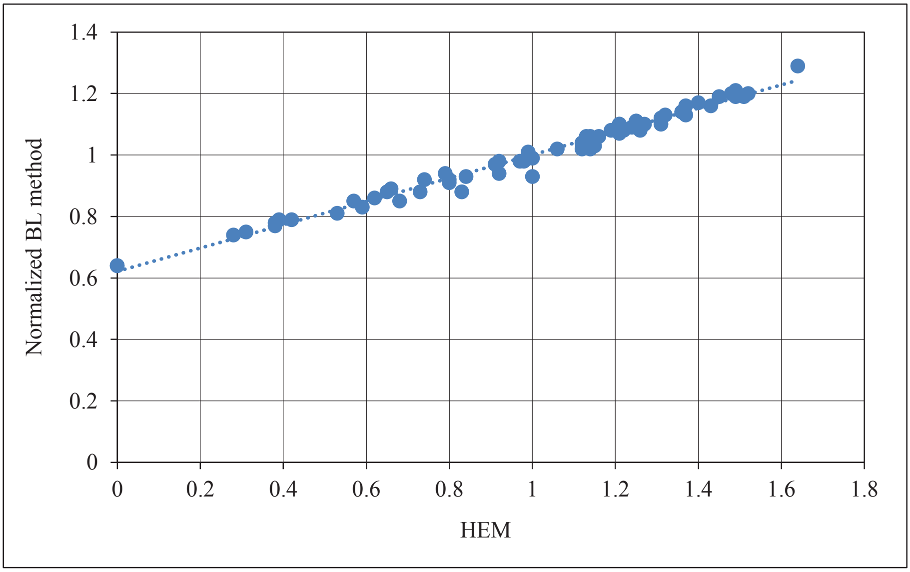

Fig. 4. Relative Multipliers Spain II

Source: own study based on INE (accessed on 11 November 2018).

Although a comparison between countries is not the purpose of this study, Poland and Spain clearly differ in the profile of their sectors. In Poland, the basic sectors are ranked at the top while in Spain, the ranks are as follows: 52. travel agency, tour operator and other reservation services and related services; 5. food products; beverages; tobacco products; 19. machinery and equipment not elsewhere specified; furniture; other products; 16. fabricated metal products except for machinery and equipment; 18. electrical equipment; 37. publishing services; and 32. maritime and inland waterway transport services.

SUMMARY AND CONCLUSIONS

One of the main applications of IO economic analysis is to identify key sectors in a given economy. The idea is that state efforts should concentrate on those sectors that might potentially maximize economy-wide impacts. There are two main approaches to identify key sectors, the classical (Rasmussen, 1957) and the HEM (Dietzenbacher and Van der Linden 1997). In recent years, multiple extensions of both approaches have been used to identify key sectors. However, despite the fact that the HEM seems to have a deeper economic foundation, it nonetheless presents important problems since the alteration of the economic system proposed by HEM can seem unrealistic and may give rise, accordingly, to doubts as to whether the results are biased, which could lead to wrong public investment decisions and false sectoral priorities. Therefore the debate, although initiated many years ago, is not yet closed.

The first main contribution of this paper was to develop a different approach, which delivers similar results to HEM, but using a complete IO model. The second main contribution was developing a formal comparison between the HEM and the proposed method. The last main contribution meant drawing a comparison among the three approaches used to identify the key sectors by using real data for Poland and Spain.

This paper offers an alternative method to identify key sectors based on a normalization of the Leontief inverse that is related to the partial hypothetical extraction method. After having discussed the properties of the proposed standardization, the HEM, the Classical Multiplier method and the alternative method proposed were formally and empirically compared by using the 2010 IO tables for Poland and Spain. The authors' findings show that distinguishing and disaggregating external effects from those that are purely internal has very relevant policy implications. It is assumed that the above conclusion favours the classification of sectors by their stimulating capacity (also extrapolated to their impulse capacity). This disaggregation can be achieved with the proposed method without incurring the criticisms mentioned in the case of the HEM, and requiring less effort to be calculated. Moreover, this approach relativizes the weight of self-consumption in the consideration of key sectors, something the HEM method penalized very severely.

As a final recommendation it is argued that studying standard isolated matrices, as in the HEM method, could lead to interpretation errors, partially because of the complexity of their elements. This approach also satisfies the accounting equilibrium and maintains it throughout all the step-by-step transformations in the guise of an appropriate system of equations, as described in this paper.

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Received: March 2019, revised: October 2019

Acknowledgement: *The paper is funded by the National Science Centre within the MINIATURA 1 research project No. NCN DEC-2017/01/X/HS4/01742, entitled "Valuation methods of biological assets in agricultural activity".*

ANNEX I

Backward linkages between Polish productive sectors in 2010

Sector	Code	Standard BL method	Normalized BL method	Partial Hypothetical Extraction
1	2	3	4	5
Products of forestry	2	1.05	2.16	0.93
Products of agriculture and hunting	1	1.11	1.63	1.12
Travel agency, tour operator and other reservation services and related services	62	1.22	1.22	1.58
Food products	6	1.31	1.18	1.56
Beverages	7	1.18	1.15	1.50
Waste collection, treatment and disposal services; materials recovery services	32	1.15	1.12	1.38
Food and beverage serving services	42	1.10	1.12	1.36
Water and air transport services	39	1.10	1.11	1.32
Wood and products of wood	12	1.18	1.11	1.37
Furniture	27	1.11	1.11	1.33
Other non-metallic mineral products	19	1.14	1.10	1.32
Printing and recording services	14	1.09	1.09	1.28
Basic metals	20	1.15	1.08	1.28
Other manufactured goods	28	1.10	1.07	1.25
Library, archive, museum services	71	1.05	1.07	1.24
Services auxiliary to financial services and insurance services	51	1.17	1.07	1.30
Programming and broadcasting services	45	1.26	1.07	1.03
Insurance services	50	1.04	1.07	1.20
Accommodation services	41	1.04	1.06	1.19
Machinery and equipment n.e.c.	24	1.05	1.06	1.21
Constructions and construction work	34	1.18	1.05	1.25
Electrical equipment	23	1.05	1.05	1.19
Management consulting services	54	1.06	1.05	1.19
Other transport equipment	26	1.09	1.04	1.20
Repair and installation services of machinery and equipment	29	1.05	1.04	1.17
Motion picture, video and television production, sound recording and music publishing	44	1.07	1.04	1.27
Architectural and engineering services; technical testing and analysis serv.	55	1.06	1.02	1.17
Motor vehicles	25	1.11	1.02	1.16
Pharmaceutical products	17	1.07	1.02	1.19
Services to buildings and landscape	64	1.03	1.02	1.10

Annex I, cont.

1	2	3	4	5
Services furnished by membership organisations	74	0.99	1.02	1.07
Fish and other fishing products	3	1.00	1.02	1.07
Land and pipeline transport services	38	1.01	1.00	1.04
Paper and paper products	13	1.05	0.99	1.08
Fabricated metal products	21	1.07	0.99	1.10
Real estate services	52	0.98	0.99	1.02
Publishing services	43	0.97	0.99	1.00
Chemicals and chemical products	16	0.99	0.99	1.01
Wholesale trade services	36	0.98	0.99	1.01
Leather and related products	11	1.00	0.98	1.02
Telecommunications services	46	0.99	0.98	1.03
Crude petroleum and natural gas; metal ores; other mining and quarrying products	5	0.98	0.98	0.99
Rubber and plastic products	18	1.02	0.98	1.02
Gambling and betting services	72	1.11	0.97	1.08
Office administrative, office support and other business support services	65	0.96	0.96	0.97
Wearing apparel	10	0.92	0.95	0.87
Warehousing; postal and courier services	40	0.99	0.95	0.95
Computer, electronic and optical products	22	0.96	0.93	0.88
Sporting services and amusement and recreation services	73	0.99	0.93	0.92
Retail trade services	37	0.89	0.92	0.81
Information services	48	0.96	0.92	0.89
Scientific research and development services	56	0.98	0.92	0.91
Coal and lignite	4	0.89	0.92	0.79
Textiles	9	0.91	0.92	0.82
Other professional, scientific and technical services	58	0.91	0.92	0.84
Tobacco products	8	0.89	0.91	0.80
Natural water; water treatment and supply services	31	0.88	0.91	0.77

1	2	3	4	5
Creative, arts and entertainment services	70	0.92	0.90	0.82
Sewerage; remediation services	33	0.88	0.90	0.77
Other personal services	76	0.87	0.90	0.75
Rental and leasing services	60	0.88	0.90	0.76
Computer programming, consultancy services	47	0.90	0.88	0.77
Security and investigation services	63	0.94	0.87	0.80
Social works services	69	0.82	0.86	0.63
Human health services	68	0.87	0.85	0.68
Advertising and market research services	57	1.20	0.85	0.99
Legal and accounting services	53	0.88	0.85	0.71
Sale and repair services of motor vehicles and motorcycles	35	0.84	0.85	0.64
Public administration services	66	0.81	0.84	0.60
Repair service of computer and personal and household goods	75	0.80	0.82	0.53
Veterinary services	59	0.76	0.80	0.45
Financial services	49	0.85	0.78	0.59
Education services	67	0.75	0.77	0.41
Coke, refined petroleum products	15	0.75	0.77	0.41
Private households with employed persons	77	0.72	0.76	0.34

Source: own study based on Central Statistical Office DATA (accessed on November 11, 2018).

ANNEX II

Backward linkages between Spanish productive sectors in 2010

Sector	Code	Standard BL method	Normalized BL method	Partial Hypothetical Extraction
1	2	3	4	5
Services of travel agencies, tour operators and other reservation services, and related services	52	1.23	1.29	1.64
Food products; drinks; manufactured tobacco	5	1.43	1.21	1.49
Machinery and equipment n.i.o.p.	19	1.16	1.20	1.52
Furniture; Other manufactured products	22	1.16	1.20	1.48
Metal products, except machinery and equipment	16	1.20	1.19	1.51
Electric equipment	18	1.17	1.19	1.49
Editing services	37	1.16	1.19	1.45
Maritime and inland waterway transport services	32	1.11	1.17	1.40
Wood and cork and products made of wood and cork, except furniture; basketware and plaiting articles	7	1.18	1.16	1.37
Metallurgical products and metal products	15	1.25	1.16	1.43
Fish and other products of fishing; aqua+'culture products; fishing support services	3	1.11	1.14	1.36
Textile products; clothing; leather goods and footwear	6	1.25	1.14	1.36
Other non-metallic mineral products	14	1.19	1.13	1.37
Air transport services	33	1.12	1.13	1.32
Wastewater collection and treatment services; waste collection, treatment and disposal services; utilization services; sanitation services and other waste management services	26	1.19	1.12	1.31
Other transport material	21	1.05	1.11	1.25
Advertising and market research services	48	1.11	1.11	1.25
Extractive Industries	4	1.06	1.10	1.27
Services provided by associations	60	1.04	1.10	1.21
Computer, electronic and optical products	17	1.05	1.10	1.21
Print and reproduction services of recorded media	9	1.12	1.09	1.24
Chemical products	11	1.15	1.08	1.26
Insurance, reinsurance and pension insurance services, except compulsory social security	42	1.06	1.08	1.19

1	2	3	4	5
Cinematographic, video and television services; sound recording and music editing; radio and television programming and broadcasting services	38	1.15	1.08	1.22
Rubber and plastic products	13	1.08	1.07	1.21
Basic pharmaceutical products and their preparations	12	1.01	1.06	1.14
Motor vehicles, trailers and semi-trailers	20	1.06	1.06	1.16
Wholesale trade and business intermediation services, except of motor vehicles, motorcycles and mopeds	29	1.02	1.06	1.13
Accommodation and food and beverages services	36	1.01	1.05	1.15
Land transport services, including pipelines	31	1.02	1.04	1.12
Construction and construction works	27	1.16	1.03	1.15
Storage services and transport auxiliaries	34	1.13	1.02	1.12
Products of agriculture, livestock and hunting, and related services	1	1.05	1.02	1.14
Sports, recreational and entertainment services	59	1.02	1.02	1.06
Repair services of computers, personal effects and household goods	61	0.97	1.01	0.99
Architectural and engineering technical services; technical testing and analysis services	46	1.04	0.99	1.00
Wholesale and retail trade services and repair services of motor vehicles and motorcycles	28	0.94	0.98	0.92
Repair and installation services of machinery and equipment	23	0.99	0.98	0.97
Other professional, scientific and technical services; veterinary services	49	0.95	0.97	0.91
Telecommunications services	39	1.00	0.94	0.92
Retail trade services, except of motor vehicles and motorcycles	30	0.88	0.94	0.79
Creation, artistic and entertainment services; library services, archives, museums and other cultural services; gambling and betting services	58	0.93	0.93	0.84
Electric power, gas, steam and air conditioning	24	1.15	0.93	1.00
Rental services	50	0.91	0.92	0.80
Social care services in residential establishments; social services without accommodation	57	0.86	0.92	0.74

Annex II, cont.

1	2	3	4	5
Services auxiliary to financial services and insurance services	43	0.93	0.91	0.80
Other personal services	62	0.83	0.89	0.66
Postal and courier services	35	0.98	0.88	0.83
Scientific research and development services	47	0.83	0.88	0.65
Programming, consulting and other services related to computer science; information services	40	0.89	0.88	0.73
Health care services	56	0.83	0.86	0.62
Public administration and defence services; compulsory social security services	54	0.79	0.85	0.57
Legal and accounting services; business headquarters services; business management consulting services	45	0.87	0.85	0.68
Security and investigation services; services for buildings and landscaping; administrative, office and other business support services	53	0.82	0.83	0.59
Financial services, except insurance and pension funds	41	0.79	0.81	0.53
Coke and petroleum refining products	10	0.72	0.79	0.39
Real Estate Services	44	0.73	0.79	0.42
Imputed rentals of the dwellings occupied by their	44a	0.72	0.78	0.38
Forestry and forestry products, and related services	2	0.72	0.77	0.38
Education services	55	0.69	0.75	0.31
Services related to employment	51	0.68	0.74	0.28
Household services as employers of domestic staff; undifferentiated goods and services produced by households for own	63	0.57	0.64	0.00
Services of extraterritorial organizations and agencies	64	0.57	0.64	0.00

Source: own study based on INE (accessed on 11 November 2018).