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INNOVATIVE ADVANTAGES RANKING. A NEW APPROACH

Assessing/ranking the innovative advantages of countries is a problem of current interest. However, the set of tools used for this purpose are very narrow and often prone to criticism. The aim of this study is to somewhat extend the arsenal of methods used to this end. For this purpose, based on a data set from the Global Innovation Index, this study develops a special multi-objective decision-making problem, the aim of which is to identify the "best countries" in the sense of their innovative advantage. Moreover, applying ranking methods (in our case the Markov-chain method and analytic hierarchy process) to this multi-objective decision-making problem, we obtain new alternative ratings/rankings of the innovative advantages of countries.

Keywords: *global innovation index, Markov chain, analytic hierarchy process, multi-objective decision making problem*

1. Introduction

In the modern economy, scientific innovation is recognised as an important component in the economic growth and competitiveness of countries, see, e.g., [1, 9, 21–23]. Correspondingly, assessing/ranking the innovative advantages of countries is crucial for policy makers. Various organisations and researchers have recently accumulated a large amount of experience in constructing different indexes/measures of the innovative advantages of countries, see, e.g., [3, 4, 8, 14].

In this study, we use data from the Global Innovation Index (GII). Note that the construction of the GII reflects the extensive experience of previous studies and the current understanding of the innovation process and its determinants. Moreover, the GII is regularly published and contains detailed data on more than 100 countries. In addition, the GII uses well-defined measurement tools and both the primary data and final

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indicators of the GII are subject to multiple external and internal tests (for details see [14]). Note also that the ranking of countries by the GII methodology (as well as by other innovation indexes) is implemented by ordering countries with the help of aggregated index scores (e.g., by the GII value, which is obtained as a weighted average of so called pillars).

In this study, we have shown that there are also other ways of ranking the innovative advantages of countries and two of them have been considered here. Based on the GII data at the pillar level, we construct a special multi-objective decision making (MODM) problem, the aim of which is to identify the "best countries" in the sense of their innovative advantage rating/ranking (hereinafter abbreviated to CIAR).

In this sense, our approach corresponds to the current trend in the development of composite indicators (see, e.g., [8]) using a multi-objective optimisation technique. However, the novelty that we are introducing here is that instead of using a traditional set of methods, we use specialized methods of ranking. Particularly, we have shown that two well-known ranking methods in the literature can be applied to the MODM problem: the Markov-chain method [5], and the analytic hierarchy process (AHP) [18–20]. Note also that two versions of the AHP will be considered here – the eigenvalue method (EV method) and the geometric mean method (GM method).

A distinctive feature of the approach we propose is that we have not used any preliminary assessments of the transition probability matrix or a pair-wise comparison matrix defined by external experts. The results obtained show that the new ratings are highly correlated with the GII and can provide quite a competitive ranking of the innovative advantages of countries.

The rest of the paper is organised as follows: In section 2, we describe our dataset (subsection 2.1), recall the basics of a MODM problem and introduce some related matrices (subsection 2.2), recall the main concepts of the Markov-chain/AHP ranking methods and introduce their connection to a MODM problem (subsection 2.3) and describe the corresponding application to the CIAR-problem (subsection 2.3). In section 3, we briefly present the results of our calculations (the results of these calculations are presented in detail in the supplement dataset [10]) and finally, section 4 gives a conclusion.

2. Materials and methods

2.1. Data

This study used the GII data for the period 2011–2015 at the pillar level as the primary dataset. For the convenience of the reader, we recall here the main features of

this dataset. The GII [14] is built on a hierarchical basis and includes the following two subindices: Innovation Input, which is a composite of five input indexes (pillars), and Innovation Output, which is a composite of two output indexes (pillars). Each pillar is divided into subpillars, each of which is built using a number of relevant individual indicators.

Table 1. Short description of the GII pillars

Pillar	Description
I11	institutions (political environment, regulatory environment, business environment)
I12	human capital and research (education, tertiary education, R&D)
I13	infrastructure (ICT, general infrastructure, ecological sustainability)
I14	market sophistication (credit, investment, trade and competition)
I15	business sophistication (knowledge workers, innovation linkages, knowledge absorption)
I21	knowledge and technology outputs (knowledge creation, knowledge impact, knowledge diffusion)
I22	creative outputs (intangible assets, creative goods and services, online creativity)

The underlying subpillars are indicated in parentheses. Pillar I21 knowledge and technology outputs, and the subpillar ecological sustainability were called scientific outputs and energy, respectively, in the 2011 GII. The subpillar online creativity was absent in the 2011 GII.

The GII is the simple average of its input and output subindices. Moreover, the subindices are the simple average of their underlying pillar scores. Each pillar score is calculated as the weighted average of its subpillar scores, and each subpillar score is calculated as the weighted average of its individual indicators. Individual indicators (their number and composition change yearly, and they totalled between 79 and 84 during the period of 2011–2015) are obtained from various sources and scaled via division by the relevant scaling factor. Individual indicators are also normalised to the [0, 100] range, with higher scores representing better outcomes. Details about the composition of individual indicators, data sources, processing techniques and methods of selecting countries can be obtained in [14]. Table 1 presents a short description of the GII pillars.

Note also that the original dataset of GII pillars has missing values for some countries in some years during the time period 2011–2015 (about 6% of country-year cases). We conducted data imputation by using the first-last accessible value and linear interpolation method. Therefore, we obtained a dataset of seven indicators for 147 countries for five years. Finally, to ensure the robustness of the estimates, it was deemed useful to average these data over time (i.e., on the interval 2011–2015) for each country. The set of data obtained by the procedure described above will be called the GII-dataset. The GII-dataset is not presented here because of its size, but it can be found elsewhere [9].

2.2. The MODM problem

Let us denote a set of alternatives by $A = \{a_1, ..., a_m\}$ a set of criteria by $C = \{c_1, ..., c_n\}$, where $c_j: A \to \mathbb{R}, j = 1, ..., n$, and consider a MODM problem as an ordered pair $< A, C >$. Without loss of generality, we may assume that the criteria are normalised so that the lower value is preferable according to each criterion. Thus, the goal of the decision making procedure can be formulated as the minimisation of all the criteria, simultaneously. Obviously, this is a very ambiguous formulation that requires clarification. Due to this reason, the concept of Pareto optimality was introduced and considered to be an appropriate concept of a solution to the MODM problem. For details see [6, 15, 17].

However, generally the set of Pareto-optimal alternatives is "large" and each one of these alternatives should be considered as mathematically equal. Hence, further factors should be considered to assist a more precise investigation of the alternatives. As a clarifying factor, we might consider some ranking method based on the set of alternatives. For this reason, we introduce specific matrices associated with a MODM problem, which will be utilised below for describing two special ranking methods.

For any natural number N, we say that the $N \times N$ matrix $S = [S_{ij}], 1 \le i, j \le N$, is a score-matrix if $S_{ij} \ge 0, S_{ii} = 0, 1 \le i, j \le N$. Following [11], for a given score-matrix $S = [S_{ij}], 1 \le i, j \le N$, we also introduce the matrix $G = [G_{ij}(S)], 1 \le i, j \le N; (G = S + S^T)$, and define the function $g_i(S) = \sum_{j=1}^{N} G_{ij}(S), 1 \le i \le N$.

Let us assume that a MODM problem $< A, C >$ (under the standard assumption that the criteria are normalised, and according to each criterion a lower value is preferable) is given and hence, the decision making goal is simultaneous minimisation of a set of values. We propose the following construction of score-matrices for the alternatives S^A. For any $a, a' \in A$, we define

$$S^A(a,a') = \sum_{c \in C} s_c^A(a,a'), \text{ where } s_c^A(a,a') = \begin{cases} 1, & c(a) < c(a') \\ 0, & c(a) \ge c(a') \end{cases} \quad \forall c \in C$$

Thus, the equality $s_c^A(a,a') = 1$ means that $c(a) < c(a')$ according to criterion $c \in C$ and the alternative a receives one point. Obviously, $m \ge S^A(a,a') \ge 0, S^A(a,a) = 0, \forall a, a' \in A$, and the matrix $S^A = \left[S^A(a,a') \right]_{a,a' \in A}$ is the score-matrix for a set of alternatives (in the sense of the definition given at the beginning of this subsection).

On the other hand, based on the score-matrix $S^A = \left[S^A(a,a') \right]_{a,a' \in A}$, we can also introduce the matrix $\varPi = \left[\varPi(a,a') \right]_{a,a' \in A}$, where

$$\varPi(a,a') = \begin{cases} S^A(a,a') + \dfrac{m - S^A(a,a') - S^A(a',a)}{2}, & a \neq a' \\ 0, & a = a' \end{cases} \quad \forall a, a' \in A,$$

2.3. Ranking methods

In this subsection, we consider two well-known ranking methods – the Markov-chain method and the analytic hierarchy process (AHP). As mentioned in the introduction, the AHP will be considered here in two versions – the classical version (the EV method) and the geometric mean method (GM method).

Note that we can interpret the matrix $\varPi = \left[\varPi(a, a') \right]_{a, a' \in A}$ as an adjacency matrix for the directed graph $\varGamma(A, C)$ associated with the MODM problem $< A, C >$. Moreover, using a well-known method, see, e.g., [5], we can transform the adjacency matrix $\varPi$ into the matrix $P = \varLambda \left(\varPi 1_m + \pi(\varPi) \right)^{-1} \varPi + \pi(\varPi) \xi^T$, where $\xi \in \text{int } \varDelta_m$ and $\pi(\varPi) = (\pi_1(\varPi), ..., \pi_m(\varPi))$ is the vector defined as follows:

$$\pi_i(\varPi) = \begin{cases} 1, & \text{if } i\text{th row of } \varPi \text{ is } 0_m \\ 0, & \text{otherwise} \end{cases}$$

The matrix P is of the same form as the transition probability matrix corresponding to the MODM problem $< A, C >$. Let us recall that a Markov chain is said to be irreducible if all the possible states communicate with each other or, in other words, exist in only one communicate class. The vector ρ, satisfying $\rho^T = \rho^T P$, is called the stationary vector of the matrix P (for details see [5]). Note that we do not use any preliminary assessments of such a transition probability matrix conducted by external experts, but define it directly from the data.

In the conclude this subsection, we note that the stationary vector $\rho = (\rho_1, ..., \rho_m)$ of the transition probability matrix P can be used as a vector rating the Markov chain's states, i.e., the entry ρ_i is defined to be the rating of the state $a_i \in A$. In other words, in the context of the MODM problem, ρ_i is the rating of the alternative $a_i \in A, i = 1, ..., m$.

The analytic hierarchy process (AHP) [17–19] has numerous applications in a variety of areas. Let us assume that m objects are given and the decision maker must

rank them based on the information obtained from pairwise (subjective and/or objective) comparison. According to the AHP, the information from pairwise comparison is used to create a matrix: $\mathcal{A} = [a_{ij}]_{i,j=1,...,m}$, which has the following properties:

$$a_{ij} > 0, \quad a_{ii} = 1, \quad a_{ij} = \frac{1}{a_{ji}}, \quad i, j = 1, ..., m$$

Such a matrix is called a positive reciprocal matrix or pairwise comparison matrix. A pairwise comparison matrix $\mathcal{A} = [a_{ij}]_{i,j=1,...,m}$ is called consistent if it satisfies the following property:

$$a_{ik} a_{kj} = a_{ij}, \quad i, j, k = 1, ..., m$$

It was shown that a pairwise comparison matrix is consistent if and only if it is of rank one. When a pairwise comparison matrix, $\mathcal{A} = [a_{ij}]_{i,j=1,...,m}$ is consistent (or almost consistent, for details see [2, 11, 16]), the rating vector (AHP Perron–Frobenius rating vector), r^{ahpPF} can be obtained as the unique solution resulting from the adaptation of the AHP-method to the MODM problem. We define the pairwise comparison matrix for the MODM problem $< A, C >$ as follows:

$$a_{ij} = \begin{cases} \exp\left(\pi_{ij} - \dfrac{m}{2}\right) & i \neq j \\ 0 & i = j \end{cases} \quad i, j = 1, ..., m$$

where $\Pi = [\pi_{tf}]_{t, j = 1, ..., m}$ is the adjacency matrix for the MODM problem $< A, C >$.

Note that here we have not used any preliminary assessments of the pairwise comparison matrix conducted by external experts, but have defined it directly from the data. Note also that pairwise comparison is carried out on the basis of simple calculations that can be conducted at little cost, even with a large number of alternatives[2].

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[2] The example considered in Section 3 shows that problems with around 150 alternatives (and with about 10 criteria) can be solved by the proposed method in a few minutes on a standard laptop (ASUS, Intel(R)Core(TM)i7-6500U CPU @ 2.5 GHz 2.59 GHz, 8 GB RAM, 64-bit operation system, ×64-based processor) without any effort to optimize the code.

2.4. Application to the CIAR problem

In particular, with regard to the CIAR problem which is considered in this study, note that we can consider a set of countries as a set of alternatives A (i.e., based on the GII-dataset $m = 147$) and the set of GII pillars as the set of criteria (i.e., based on the GII-dataset $n = 7$). In the case of the CIAR problem, all the criteria are normalised but have the "reverse orientation" (i.e., higher scores are preferable). However, using a simple transformation, it is easy to restate the CIAR problem so that lower scores will be preferable according to the criteria. This is required in order to make the ranking procedure uniform and will be used for our calculations. Thus, we obtain the standard formulation of a MODM problem and can apply the ranking methods discussed above.

3. Results

The decision-making matrix for the MODM problem under consideration is presented in [10]. In particular, the best countries in the sense of those possessing innovative advantages are represented by the set of Pareto-optimal alternatives of the MODM problem described above (the ISO alpha-3 country codes were used). These are CAN, DNK, FIN, HKG, IRL, ISR, KOR, LUX, NLD, SGP, SWE, CHE, GBR, USA. The Pareto-optimal alternatives should be considered as mathematically equal and hence the Pareto-optimal countries listed above should be considered equal in terms of their innovative advantages.

The transition probability matrix for the Markov-chain method[3] and the pairwise comparison matrix for the AHP[4] and the full rating/ranking of the countries is also presented in [10][5]. For illustrative purposes, Table 2 presents the top 30 countries (according to GII-ranking).

The correlation coefficients between the rating vectors obtained in this way are: GII-rMch: –0.96579, GII-rPFahp: 0.83641, GII-rGMahp: 0.89130, rMch-rPFahp:

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[3]All the necessary calculations related to the Markov chain were performed using the Markov chain analytic package available for the R software package (see CRAN: https://cran.r-project.org/package=markovchain). It was confirmed numerically that transition probability matrix is irreducible.

[4]Numerical calculations (using the MATLAB function eig: https://www.mathworks.com/help /matlab /ref/eig.html) indicate that for the considered pairwise comparison matrix the largest eigenvalue is given by $\lambda_{max} = 273.5918$. According to the criterion (see [2]) $\lambda_{max} < m + \alpha (1.7699m - 4.3513)$, we can accept consistency only at the $\alpha = 0.5$ level. Nevertheless, as we can see from the correlation analysis, the AHP generates a competitive ranking.

[5]Note that we use the following abbreviations: Markov-chain – rMch, AHP (Perron–Frobenius version) – rPFahp, AHP (geometric mean version) – rGMahp, and also note that the ranks take into account the direction of the rating vectors.

–0.73886, rMch-rGMahp: –0.80304, rPFahp-rGMahp: 0.98423. Note that the GII and rMch ratings are highly correlated but have opposite orientations. At the same time, the GII, rPFaph, rGMaph ratings are highly correlated and have the same orientation. Figures 1–3 represent the interrelation between the considered ratings and the GII index.
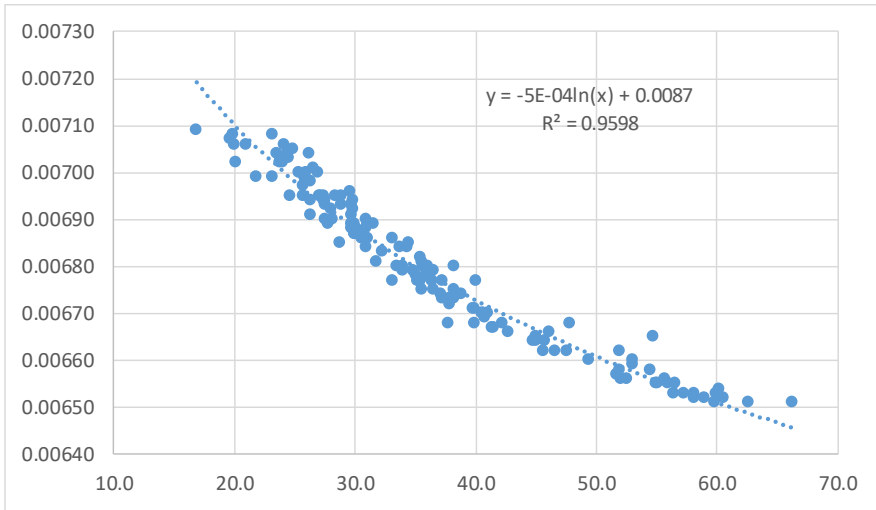

Fig. 1. rMch rating vs. GII;
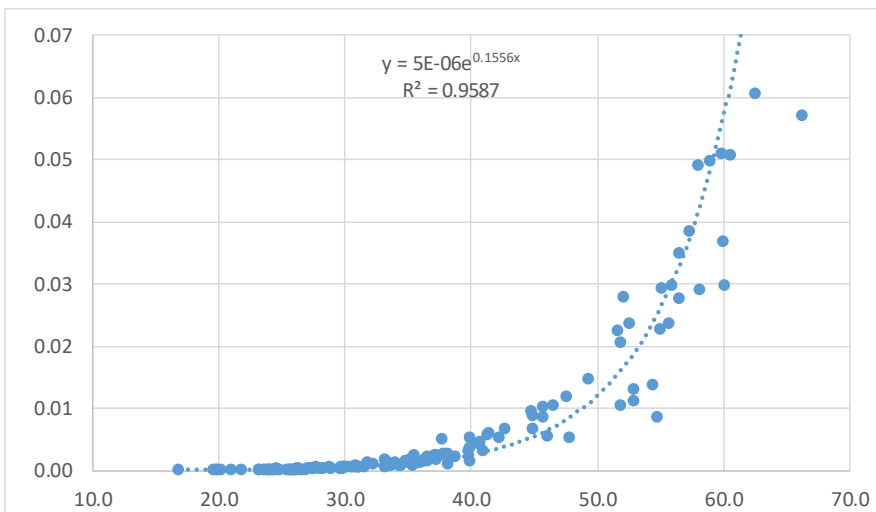vertical axis: rMch rating score, horizontal axis: GII index

Fig. 2. rPFahp rating vs. GII;
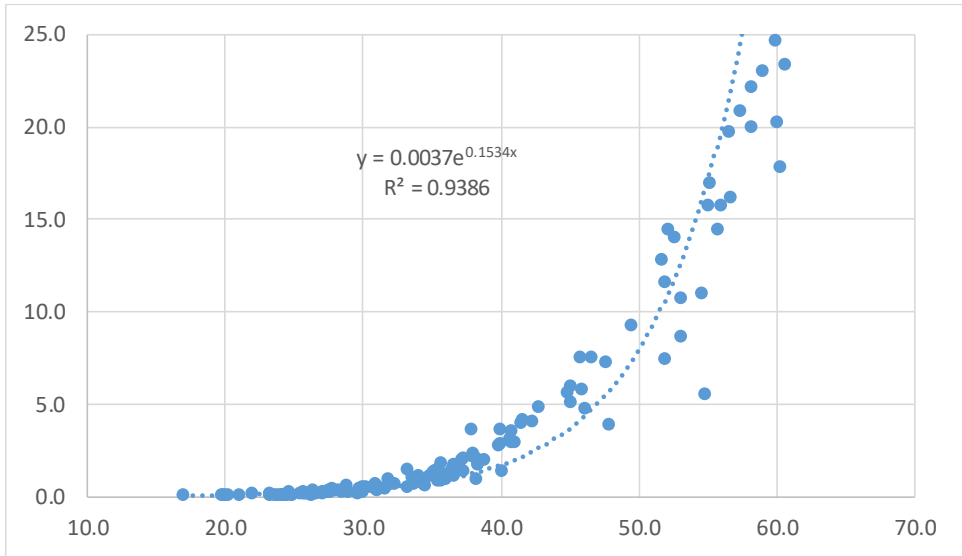vertical axis: rPFahp rating score, horizontal axis: GII index

Fig. 3. rGMahp rating vs. GII;
vertical axis: rGMahp rating score, horizontal axis: GII index

Table 2. Rankings of the top 30 GII-ranked countries
according to various methods

Country	GII	rMch	rPFahp	rGMahp	Country	GII	rMch	rPFahp	rGMahp
CHE	1	3	2	1	ISR	16	13	18	15
SWE	2	2	1	2	NOR	17	33	31	32
GBR	3	5	4	4	KOR	18	21	22	21
SGP	4	11	10	11	AUS	19	22	23	22
NLD	5	10	8	8	MLT	20	24	25	24
FIN	6	1	3	3	AUT	21	16	16	18
USA	7	6	5	5	JPN	22	18	14	17
HKG	8	8	13	9	EST	23	26	27	27
DNK	9	4	6	6	FRA	24	20	20	20
IRL	10	9	7	7	BEL	25	19	19	19
LUX	11	14	15	13	CZE	26	23	21	23
CAN	12	7	9	10	ESP	27	40	39	39
DEU	13	12	11	14	SVN	28	28	24	28
ISL	14	17	17	16	CYP	29	25	26	25
NZL	15	15	12	12	CHN	30	34	37	35

The ISO 3166-1 alpha-3 country codes are used.

As Table 2 shows, these new rankings (rMch, rPFahp and rGMahp) of countries are somewhat different from the GII ranking. For example, CHE (Switzerland) is in first position according to the GII and rMahp rankings, but occupies second position

according to the rPFahp ranking and third position according to the rMch ranking. At the same time (Figs. 1–3), the new rankings preserve the general tendencies of the GII ranking and, as we showed, are highly correlated with it. In this sense, it can be said that the new rankings are quite competitive with GII.

4. Conclusion

In this study, we have proposed new approaches to ranking countries according to their innovative advantage. Based on the data set from the Global Innovation Index at the pillar level, this study defined an MODM problem, the aim of which is to identify the "best country" in terms of innovative advantage. Applying the Markov-chain method and the AHP to solve this MODM, we obtained new ratings/rankings of the innovative advantages of countries. It must be emphasised that in this study we have not used any preliminary assessments of the transition probability matrix or of the pairwise comparison matrix made by external experts.

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