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A MULTILEVEL DECISION MAKING MODEL FOR THE SUPPLIER SELECTION PROBLEM IN A FUZZY SITUATION

Supplier selection plays a vital role in evolving an effective supply chain and the overall performance of organisations. Choosing suppliers may involve different levels arranged in a hierarchical structure. Decisions are made successively starting from the first level to the last level. Decision variables are partitioned between different levels and are called controlling factors. In the paper, we propose a multilevel supplier selection problem with uncertain or fuzzy demand and supply. Since objectives may be conflicting in nature, possible relaxations in the form of tolerances are provided by the upper level decision makers to avoid decision deadlocks. We use (linear) membership functions to fuzzily describe objective functions, as well as the controlling factors, and generate satisfactory solutions. We extend and present an approach to solving multilevel decision making problems when fuzzy constraints are employed. Different scenarios are constructed within a numerical illustration, based on the selection of controlling factors by the upper level decision makers.

Keywords: *supplier selection, supply chain, multilevel decision making, multiobjective, fuzzy optimisation*

1. Introduction

In the present scenario, suppliers play a vital role in achieving a competitive advantage based on new strategies for purchasing and manufacturing. Supplier selection, also known as sourcing decisions, involves the selection of reliable suppliers, keeping

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in view different constraints related to price, quality, time, demand and supply. Selecting the best suppliers increases customers' satisfaction, thereby improving the efficiency of the supply chain in a competitive environment. Moreover, selection of the appropriate suppliers improves quality measures by reducing the levels of rejected products, increasing the flexibility to meet the needs of the end customers, and reducing lead time at different stages. A supplier selection problem may involve multiple criteria which include both qualitative and quantitative factors. Linear programming models are widely used in formulating the supplier selection problem (VSP). Kumar et al. [11] proposed a linear model and used a fuzzy goal programming approach to deal with supplier selection problems. Their model consists of multiple objectives in which some parameters are fuzzy in nature. Kumar et al. [12] proposed a model of multiobjective integer supplier selection consisting of various parameters, which are treated as being imprecise. A linear fuzzy membership function is used to deal with the parameters that are treated as being vague.

Chen et al. [7] used a fuzzy approach to deal with the supplier selection problem where linguistic values are used to weight various factors. Their study consists of concepts of fuzzy set theory and TOPSIS is used to deal with the supplier selection problem in a supply chain. Li and Zabinsky [13] proposed a two stage stochastic model and a chance constrained model that considers discounts for bulk buying when solving the supplier selection problem. They consider uncertain supply and demand parameters and use an ε-constrained method to generate Pareto-optimal solutions to the problem. Partibhan et al. [14] presented a multicriteria approach based on strategic decisions for VSP. Their study presents an integrated approach to multicriteria decision making, using techniques such as fuzzy logic, strength weakness opportunity threat (SWOT) analysis, along with data development analysis. Aydin et al. [2] presented a new fuzzy AHP for the supplier selection problem. Their study uses a new triangular fuzzy scale containing negative fuzzy numbers and also defines fuzzy comparison decision matrices. Triangular fuzzy numbers yield the final weights of the factors without any defuzzification. Kar [9] integrated an analytical hierarchy process (AHP) and goal programming using fuzzy arithmetic to generate solutions for group decision making in a supplier selection problem. Arikan [1] proposed a novel interactive fuzzy approach to deal with the supplier selection problem with fuzzy parameters. Fuzzy parameters are described by triangular or trapezoidal membership functions, which are used to generate non-dominated solutions. Ayhan and Kilic [3] presented a two stage approach using fuzzy AHP and mixed integer linear programming for a supplier selection process that considers discounts for bulk buying. They consider the situation where different types of items are needed and none of the suppliers can provide all of these types. Büyüközkan and Göcer [6] proposed a multicriteria decision making approach to the supplier selection problem considering five suppliers, in order to determine the optimal quantities of items to be assigned to these suppliers. They consider, for the first time in the literature, fuzzy AHP and intuitive axiomatic design principles to generate solutions to their considered problem. Bakeshlou et al. [4] used a fuzzy analytical network process and fuzzy multiobjective programming to obtain the optimal quantities allocated to the available suppliers in the context of selecting a green supplier. Their multilevel decision making problem consists of different decision makers being present at different levels in a hierarchical structure. Multilevel decision making is becoming more important for contemporary decentralised organisations, where each unit seeks its own interest.

Multilevel organisations consist of interactive decision making units within a predominantly hierarchical structure. The execution of decisions is sequential, starting from the top and moving to lower levels. The decision maker at each level tries to maximise its own benefits, but is affected by the decisions of decision makers at other levels through externalities. The upper level decision maker first sets his goals and then asks each subordinate level about their solutions or optima, which are calculated in isolation. The decisions of lower level decision makers are then submitted and modified by the upper level decision maker based on the overall benefit to the organisation. Most of the models and development in multilevel programming have concerned single objectives at each level. Sakawa and Nishizaki [18], Shih et al. [19], Shih and Lee [20], Sinha and Sinha [21] proposed various methods for multilevel programming consisting of single objective functions at each level. However, to deal with more realistic concerns, multiple objectives should be introduced at each level. Baky [5] used fuzzy goal programming to develop a model for bilevel multiobjective problems and bi-level multiobjective fractional programming problems. Youness et al. [22] presented an algorithm to solve a fuzzy bi-level fractional programming problem with integer restrictions. Zheng et al. [23] proposed a model for bilevel programming problems using interactive fuzzy decision making techniques. El-Hefnawy [8] proposed a model for solving bilevel programming problems using modified particle swarm optimisation. Ke et al. [10] presented an approach to multilevel programming by integrating a genetic algorithm, neural network and simulations based on uncertainty. Pramanik et al. [17] used fuzzy goal programming to propose an approach to solving multilevel, multiobjective problems. They considered linear and linear fractional objectives, along with a set of linear constraints, and used the theory of membership functions to generate the final solutions. Osman et al. [14] developed a model for multilevel multiobjective decision making under fuzziness. The concept of fuzzy membership functions is used to generate the final solutions. However, the fuzzy approach described by Osman et al. [14] is not applicable when decision making also involves fuzzy constraints. In this paper, we extend this approach for when fuzzy objectives and constraints are present in the problem. Multilevel decision making models under fuzzy information are used to generate overall satisfactory solutions. The decision makers at each level try to optimise the outcome from their own point of view, but this outcome is affected by the decisions or solutions taken at other levels. A lower level decision maker executes policies after, and in view of, the policies or solutions adopted by upper level decision makers. These policies then constrain the feasible actions of lower level decision makers. In this study, changes in membership functions form the basis of searching for a solution. Relaxations in the form of tolerances prevent decision deadlocks in the search for a solution.

The main contributions of this paper can be summarised as follows:

- Formulating the supplier selection problem as a multilevel decision making problem in a fuzzy environment.
- Presenting an integrated approach to multilevel decision making and fuzzy programming. This approach deals with fuzzy constraints and fuzzy objectives when they are incorporated into a multilevel decision making problem involving the selection of suppliers.
- Discussing the formulations of and results for various scenarios constructed on the basis of the selection of controlling factors by upper level decision makers.
- Implementing the integrated model presented in the paper based on a numerical illustration.

The rest of the paper is organised as follows: Section 2 contains the general formulation and the procedure for solving multilevel programming problems. Section 3 contains the formulation of the supplier selection problem as a multilevel decision making problem with fuzzy constraints. Section 4 contains the implementation of this model of multilevel decision making and analyses regarding the numerical results obtained. A conclusion and perspectives are provided in Section 5.

2. Model of multilevel decision making

Consider a p-level programming problem consisting of an objective function to be minimised at each level. The general form of a p-level minimisation problem along with the set of constraints can be represented as:

$$\begin{cases} \min\limits_{x_1} F_1 = f_1(x_1, x_2, ..., x_p) & \text{(I level)} \\ \min\limits_{x_2} F_2 = f_2(x_1, x_2, ..., x_p) & \text{(II level)} \\ \vdots & \\ \min\limits_{x_p} F_p = f_2(x_1, x_2, ..., x_p) & (p\text{th level}) \\ \text{Subject to constraints:} & \\ g(x_1, x_2, ..., x_p) \quad (\le, \ge, =) \quad b & \\ x_i \ge 0, \ i = 1, 2, 3, ..., p & \end{cases} \tag{1}$$

In the above formulation (Eq. (1)), the decision variables $x_1,\ x_2,\ ...,\ x_p$ are partitioned into different levels. The decision maker at the *i*th level, $i = 1, 2, 3, ..., p,$ has control over the decision variable $x_i,\ i = 1, 2, 3, ..., p.$ The decision maker at level one is the first to execute its policies and generate its solutions. This upper level decision maker sets its goals or decisions and then asks each lower level for their optima, which are calculated individually. The decisions of the lower level decision makers are then submitted and modified by the upper level decision maker considering the overall benefit to the organisation. This process continues until a solution satisfactory to all the levels has been attained. To characterise the uncertain degrees of aspiration, the objectives are defined as fuzzy goals, using appropriately defined membership functions. Membership functions describing the decision variables under the control of different levels (also referred to as controlling factors) are also created. The membership functions for these controlling factors are created on the basis of the tolerance values provided by the decision makers at the corresponding levels.

2.1. Aspiration value of the objectives

Due to the nature of these objectives, the multiple objective functions involved in this process are often conflicting. The objective functions at individual levels are solved in isolation and these solutions are taken as the aspiration value for the associated fuzzy goal.

Let f_i^s be the aspiration value for the objective function at the *i*th level, $i = 1, 2, ..., p$. The fuzzy goals corresponding to different levels may be defined as

$$f_i^s \cong f_i(x_1, x_2, ..., x_p), i = 1, 2, ..., p \tag{2}$$

Equation (2) represents a fuzzy goal corresponding to an individual level where $\cong$ represents the fuzziness associated with different aspiration values, which can be read as essentially less than or equal to.

The upper tolerance limit, or the worst value, $u_i, i = 1, 2, 3, ..., p,$ for the objectives at each level can be determined by solving the problem represented as:

$$\begin{cases} u_i = \max\ f_i(x_1, x_2, ..., x_p), i = 1, 2, ..., p \\ \text{Subject to:} \\ g(x_1, x_2, ..., x_p)\ (\leq, \geq, =)\ b \\ x_i \geq 0, i = 1, 2, 3..., p \end{cases} \tag{3}$$

The aspiration value or the best solution for the objectives at each level can be determined by solving the following problems:

$$\begin{cases} f_i^s = \min f_i(x_1, x_2, ..., x_p), i = 1, 2, ..., p \\ \text{Subject to:} \\ g(x_1, x_2, ..., x_p)\ (\leq, \geq, =)\ b \\ x_i \geq 0, i = 1, 2, 3, ..., p \end{cases} \tag{4}$$

The membership function for the objective function at the ith level, $i = 1, 2, 3, ..., p$, may be defined as:

$$\mu\{f_i(x_1, x_2, ..., x_p)\} = \begin{cases} 1 & \text{if } f_i(x_1, x_2, ..., x_p) \leq f_i^s \\ \dfrac{u_i - f_i\left((x_1, x_2, ..., x_p\right)}{u_i - f_i^s(x_1, x_2, ..., x_p)} & \text{if } f_i^s \leq f_i(x_1, x_2, ..., x_p) \\ & \leq \mu \forall i = 1, 2, 3, ..., p \\ 0 & \text{if } f_i(x_1, x_2, ..., x_p) \geq u_i \end{cases} \tag{5}$$

2.2. Aspiration values based on the satisfaction of constraints

Multiobjective decision making problems may involve constraints with vague or imprecise information. To deal with imprecise information, fuzzy set theory is used. A fuzzy constraint may be represented as:

$$g(x_1, ..., x_p) \tilde{\leq} b \tag{6}$$

Here $\tilde{\leq}$ represents the fuzziness present in the constraints and can be read as essentially equal to. To deal with the fuzziness of constraints, the concept of membership functions is adopted in the process. Let δt be the allowable tolerance with respect to a constraint, then the membership function to deal the fuzziness of the constraint may be constructed as:

$$\mu\left\{g(x_1, x_2, ..., x_p)\right\} = \begin{cases} 1 & \text{if } g(x_1, x_2, ..., x_p) \leq b \times \delta t \\ \dfrac{\left(g(x_1, x_2, ..., x_p) - b \times \delta t\right)}{b - b \times \delta t} & \text{if } b \times \delta t \leq g(x_1, x_2, ..., x_p) \leq b \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

2.3. Evaluation of the optimum for each level

The optimal solution for the ith level may be obtained by using the membership functions (5) and (7). The optimum solution for each level can be generated by maximising the membership function associated with that level. The model used to derive the optimal solutions can be represented as:

$$
\begin{aligned}
&\max \lambda \\
&\text{Subject to:} \\
&\mu\{ f_i (x_1, x_2, ..., x_p) \geq \lambda, i = 1, 2, ..., p \\
&\mu\left\{g(x_1, x_2, ..., x_p)\right\} \geq \lambda \\
&g(x_1, x_2, ..., x_p)\ (\leq, \geq, =)\ b \\
&x_i \geq 0 \\
&i = 1, 2, 3, ..., p
\end{aligned}
\tag{8}
$$

2.4. Flexibility with respect to decision variables

Let the optimal solution for the ith level obtained according to Section 2.3 be represented as x_i^{i*}. The ith level, $i = 1, 2, ..., (p-1)$, decision maker provides some flexibility to the lower level decision makers according to his controlling factor. The tolerances Δt_i allowed with respect to the controlling factors provide a wider feasible domain to search for solutions to lower level decision makers. These tolerances are used to construct membership functions for the controlling factors and can be represented as:

$$
\mu_{x_i} = \begin{cases} \dfrac{x_i - (\, x_i^{i*} \ - \Delta t_i)}{\Delta t_i}, (x_i^{i*} - \Delta t_i) \leq x_i \leq x_i^{i*} \\ \dfrac{(x_i^{i*} + \Delta t_i) - x_i}{\Delta t_i}, x_i^{*} \leq x_i \leq (x_i^{i*} + \Delta t_i) \end{cases} \quad i = 1, 2, ..., (p-1) \tag{9}
$$

2.5. Preferences over the goals for each level

Concerning the objectives and goals of the ith level, $f_i(x_1, x_2, ..., x_p) \leq f_i^*$ is absolutely acceptable, while $f_i(x_1, x_2, ..., x_p) \geq f_i^-, i = 1, 2, p$ is absolutely unacceptable. Here, f_i^* is the optimal solution for the ith level, while f_i^- may be obtained as

$$f_i^- = f_i(x_1^{(i+1)*}, x_2^{(i+1)*}, ..., x_p^{(i+1)*}), i = 1, 2, ..., (p-1)$$

and for the pth level, the value of f_i^{j-} may be obtained as

$$f_i^- = f_i(x_1^{(i-1)*}, x_2^{(i-1)*}, ..., x_p^{(i-1)*}), i = p$$

For the goals and preferences of different levels, the membership function may be defined as:

$$\mu_{fi}\left[f\left(x_1, x_2, ..., x_p\right)\right] = \begin{cases} 1 & \text{if } f_i\left(x_1, x_2, ..., x_p\right) \geq f_i \\ \dfrac{f_i\left(x_1, x_2, ..., x_p\right) - f_i^-}{f_i^* - f_i^-} & \text{if } f_i^- \leq f_i\left(x_1, x_2, ..., x_p\right) \leq f_i^* \\ 0 & \text{if } f_i\left(x_1, x_2, ..., x_p\right) \leq f_i^- \end{cases} \tag{10}$$

$\forall i = 1, 2, ..., p$

2.6. Global evaluation for the problem

$$\begin{cases} \max \psi \\ \text{Subject to:} \\ \mu_{x_i} \geq \psi, i = 1, 2, ..., (p-1) \\ \mu_{fi}\left[f\left(x_1, x_2, ..., x_p\right)\right] \geq \psi, i = 1, 2, ..., p \\ g(x_1, x_2, ..., x_p) \leq b \\ \mu\left\{g(x_1, x_2, ..., x_p)\right\} \geq \lambda \\ 0 \leq \psi \leq 1 \\ x_i \geq 0, i = 1, 2, 3, ..., p \end{cases} \tag{11}$$

where ψ is the maximum degree of satisfaction which can be achieved by all the levels.

3. Formulation of the supplier selection problem

This section consists of the general formulation of the supplier selection problem. The following assumptions, decision variables and parameters are considered.

Assumptions

- Each item is to be purchased from only one supplier.
- Shortages are not allowed from any supplier, for any item.
- Discounts for bulk buying are not considered.
- Demand for and supply of the items is fuzzy, i.e. uncertain.
- The objectives of minimising the total cost, total number of rejected items and total number of late deliveries are fuzzy.

The supplier selection problem is a multicriterion problem and the total cost, total number of rejected items and total number of items delivered late are considered as objective functions to be minimised. The original model was proposed by Kumar et al. [11].

Decision variables

x_i – quantity ordered from supplier i, decision variable

Objective functions

Z_1 – total cost for ordering the aggregate demand
Z_2 – total number of rejected items
Z_3 – total number of items delivered late

Parameters

c_i – price of a unit ordered from a supplier i
q_i – percentage of rejected units delivered by a supplier i
l_i – percentage of late units delivered by a supplier i
D – aggregate demand for the item over a fixed planning period
P – least acceptable rating of a supplier
F – minimum value of flexibility in the supply quota that a supplier should have
B_i – budget constraint associated with each supplier
U_i – capacity of a supplier i
r_i – rating of a supplier i
s_i – quota flexibility for a supplier i

3.1. Model formulation

The Supplier selection problem for three objectives and a set of system and policy constraints may be formulated as follows:

First objective:

$$\text{minimise } Z_1 \cong \sum_{i=1}^{p} c_i x_i \tag{3.1}$$

Second objective:

$$\text{minimise } Z_2 \cong \sum_{i=1}^{p} q_i x_i \tag{3.2}$$

Third objective:

$$\text{minimise } Z_3 \cong \sum_{i=1}^{p} l_i x_i \tag{3.3}$$

Subject to

$$\sum_{i=1}^{p} x_i \cong D \tag{3.4}$$

$$x_i \tilde{\leq} U_i,\ \ i = 1, 2, ..., p \tag{3.5}$$

$$\sum_{i=1}^{p} r_i x_i \geq P \tag{3.6}$$

$$\sum_{i=1}^{p} s_i x_i \geq F \tag{3.7}$$

$$c_i x_i \leq B_i \ \ \forall i = 1, 2, ..., p \tag{3.8}$$

$$x_i \geq 0 \text{ and integer } \ \forall i = 1, 2, ..., p \tag{3.9}$$

Objective function (3.1) minimises the total cost associated with the process.

Objective function (3.2) minimises the total number of items delivered by the suppliers that are rejected.

Objective (3.3) minimises the total number of items that are delivered late by the suppliers.

Constraint (3.4) is associated with the demand for items and thus describes a restriction due to overall demand.

Constraint (3.5) represents the restrictions due to the maximum capacities of the associated suppliers.

Constraint (3.6) incorporates supplier ratings and put restrictions on the total rating of the quantities ordered.

Constraint (3.7) incorporates the minimum flexibility based on the quotas ordered from different suppliers.

Constraint (3.8) describes restrictions due to the budget for each supplier and ensures that the purchase price must not exceed the budget associated with individual suppliers.

Constraint (3.9) ensures that the quantity ordered is integer.

3.2. Model of multilevel decision making for the supplier selection problem

Multilevel structures typically exist in companies or organisations where the decision making process is hierarchical rather than central. The decision maker at the top of the hierarchy forms the top level. Following him, there is a team of decision makers forming subsequent levels. The decision makers at lower levels work under the influence of upper level decision makers. Thus the decision making starts from the first level to the last level.

I level

$$\text{minimise } Z_1 \cong \sum_{i=1}^{p} c_i x_i$$

where some of the x_i satisfy

II level

$$\text{minimise } Z_2 \cong \sum_{i=1}^{p} q_i x_i$$

where some of the x_i satisfy

III level

$$\text{minimise } Z_3 \cong \sum_{i=1}^{p} l_i x_i$$

subject to:

$$\sum_{i=1}^{p} x_i \cong D$$

$$x_i \,\tilde{\leq}\, U_i \quad i = 1, 2, ..., p$$

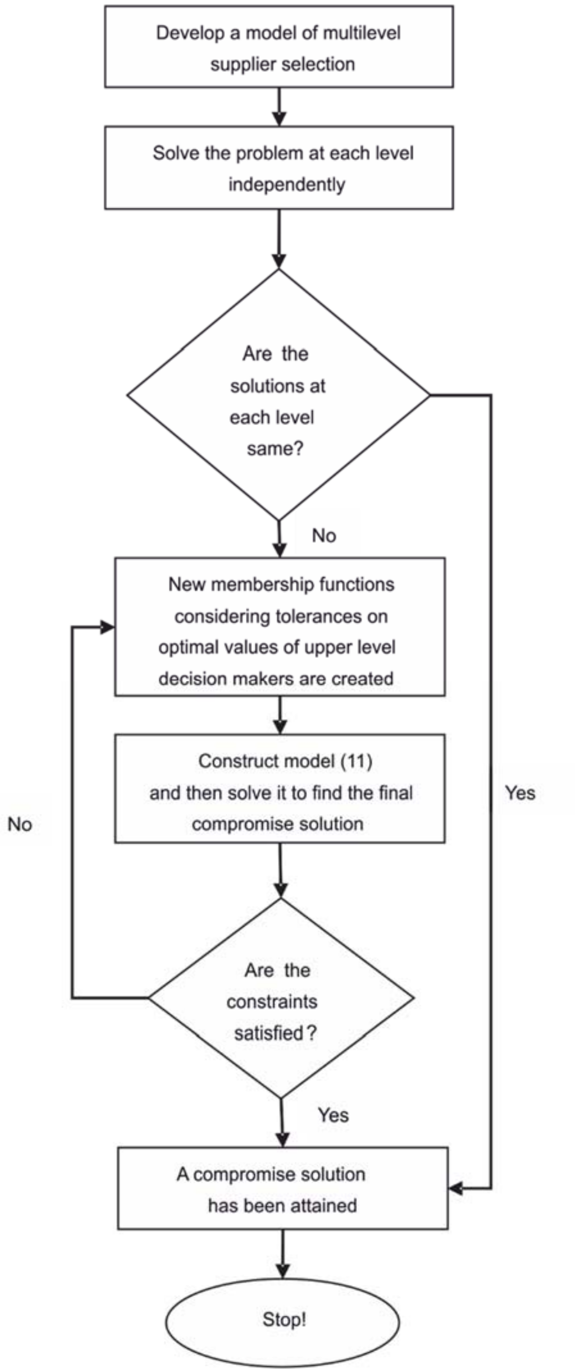

Fig. 1. Representation of the solution procedure for a multilevel decision making problem

$$\sum_{i=1}^{p} s_i x_i \geq F$$

$$\sum_{i=1}^{p} r_i x_i \geq P$$

$$c_i x_i \leq B_i, \; i = 1, 2, ..., p$$

$$x_i \geq 0, \; i = 1, 2, ..., p$$

The above formulation of the multilevel problem of selecting a supplier consists of decision makers at three levels. The decision maker at the top of the hierarchy has the sole objective of minimising cost, which ultimately results in the maximisation of profits. The decision makers at subsequent levels have objectives related to minimising the number of items that are delivered late by the suppliers and the number of such items that are rejected. The decision makers at lower levels thus work on other aspects of the process rather than cost, so as to have a better managerial environment. The decision variables are partitioned between these three levels. Hence, x_j, x_k and x_l represent the set of variables controlled by the decision makers at the first, second and third level, respectively. Here, $x_j \cup x_k \cup x_l = x_p$ and $x_j \cap x_k \cap x_l = \varphi$. The concepts of tolerance membership functions and optimisation of multiple objective functions are used to develop a technique to generate satisfactory solutions based on the theory of fuzzy sets. Here x_j, x_k and x_l are also known as controlling factors of different levels. Choosing different set of controlling factors by upper level decision makers generate different results (Fig. 1).

4. Implementation of the model and analysis of the results

To implement the proposed model, we have considered a problem of a firm which is a part of a large corporate group operating internationally.

4.1. Description of the problem

Consider a firm that manufactures auto parts. The firm buys materials from outer sources and then utilises them for the manufacture of new products. Suppose that the external purchases of the firm account for more than 76% of the total cost associated with the process. Management wants to improve the efficiency of the purchasing process and to reconsider the strategies for selecting suppliers. Optimal selection of a supplier thus improves the efficiency of the process, reduces the inventory levels and supplier base. Selection thus creates long-term trust-based relationships with potential suppliers. The task force consists of members from different departments, such as purchasing, marketing, quality control, and production, as well as research and development. Firstly, the candidate suppliers are screened and then profiled. The model proposed in this paper is then used for selecting the suppliers and amounts purchased from each of them. Twelve potential suppliers have been selected and evaluated for further process and allocation.

Table 1. Data set related to suppliers

Supplier No.	p_i($)	q_i	l_i	r_i	f_i	B_i	U_i
1	3	0.05	0.02	0.85	0.01	14000	6500
2	2	0.03	0.01	0.80	0.02	27000	16000
3	4	0	0.08	0.97	0.06	12000	4500
4	1	0.04	0.02	0.81	0.04	1900	3000
5	5	0.02	0.01	0.82	0.02	18000	4000
6	6	0.02	0.02	0.90	0.03	5000	4000
7	7	0.02	0.02	0.92	0.05	2000	2500
8	6	0.01	0.04	0.87	0.02	9000	2000
9	2	0.06	0.03	0.86	0.02	10000	6000
10	5	0	0.02	0.97	0.04	12000	2500
11	1	0.03	0.01	0.80	0.03	3000	3000
12	6	0.03	0.02	0.84	0.06	9000	2000

Table 1 presents the data set consisting of the price (in $), the percentage of rejections q_i, the percentage of items delivered late l_i, the production capacities of the various suppliers U_i, the quota flexibility f_i of various suppliers, which is represented on a scale between 0 and 1, supplier ratings r_i associated with suppliers on a scale between 0 and 1 and the maximum budgets B_i available for parts from individual suppliers. The minimum acceptable values of the total purchase value of the items supplied and the flexibility in the suppliers' quota, defined as $P = r \times D$ and $F = f \times D$, respectively, are policy decisions and depend on the demand. The aggregate demand is taken to be 34 000. If, on the scale between 0 and 1, the overall flexibility f is taken to be 0.03, then the minimum acceptable value of flexibility in the suppliers' quota is 1020. Similarly, if on the scale between 0 and 1, the minimum acceptable overall supplier rating is 0.92, then the minimum acceptable total purchase value of the supplied items is 31 280.

4.2. Creation of scenarios

The upper level decision makers have control over the set of decision variables involved in the problem. Based on the partition of the decision variables x_j and x_k, and the choice by the upper level decision makers of their controlling factors, ten scenarios are created. The upper level decision makers choose the controlling factors associated with different suppliers based on different aspects, such as highest or lowest cost, highest or lowest levels of rejected items.

Scenario 1. The upper level decision makers choose the controlling factors associated with the suppliers that are profiled as having the minimum costs considering a single product described by the data set provided in Table 1. The decision makers choose these suppliers so as to minimise the total cost of the process.

Scenario 2. In this scenario, the upper level decision makers choose the controlling factors associated with the suppliers that are profiled as having the maximum costs for a single product. The strategy according to scenario 2 is opposite to the strategy adopted in scenario 1. In this scenario, the upper level decision makers choose the suppliers associated with maximum costs, so as to have control over the allocations for these suppliers and thereby minimise the total cost of the process.

Scenario 3. This scenario is an outcome of the strategic implementation by the upper level decision makers to choose those controlling factors that are associated with higher supplier ratings.

Scenario 4. This scenario is an outcome of the strategic implementation of an approach opposite to the strategy implemented in scenario 3. In this scenario, those suppliers that are profiled as having lower supplier ratings are highlighted by the upper level decision makers. The upper level decision makers control the orders from these suppliers so as to improve the quality of the process, alongside minimising the total associated cost.

Scenario 5. The scenario is generated when the upper level decision makers select suppliers that are profiled as having large quota flexibilities. These suppliers are chosen so as to improve the efficiency of the process, alongside reducing the total cost of the process.

Scenario 6. The scenario is generated when the upper level decision makers highlight those suppliers that are profiled as having low quota flexibilities according to the data profile presented in Table 1.

Scenario 7. It results as an outcome of a strategy consisting of choosing those suppliers that are profiled as having low levels of late delivered items.

Scenario 8. It is generated when the upper level decision makers highlight suppliers from the data set in table 1 that are profiled as having high levels of late delivered items.

Scenario 9. The upper level decision makers select those suppliers from the data set in table 1 associated with low levels of rejected items in their respective supplies. This strategy improves the efficiency of the process, alongside minimising the cost of the process.

Scenario 10. The upper level decision makers try to control the quota allocations for those suppliers that are associated with higher levels of rejected items.

More scenarios may be constructed by implementing other strategies for the selection of suppliers under the control of upper level decision makers.

4.3. Results and analysis

The optimisation problems that have been solved under various scenarios using LINGO 13.0 on an AMD (A8) processor with a 1.8 GHz CPU and 8 GB of RAM. The problems faced by the individual levels are solved separately by solving the optimisation problem given by (8). The twelve decision variables associated with the suppliers are partitioned according to the three decision makers, thus each decision maker controls four decision variables. The upper level decision makers then choose the values of the controlling factors under their control. Based on this choice of controlling factors by the upper level decision makers as described in section 4.2, various scenarios are constructed. Tolerance values are then provided for the selected controlling factors to search for a solution in a wider feasible domain. Membership functions for the goals and tolerance values for the controlling factors are then created using (9) and (10). The membership functions so created are used in optimisation problem (11) to obtain the global evaluation.

Table 2. Results for all the scenarios

Variable	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
λ	0.30	0.61	0.23	0.28	0.04	0.01	0.29	0.27	0.02	0.03
Z_1	111 311	119 819	122 364	114 907	129 026	127 146	120 653	120 216	122 842	125 529
Z_2	1119	963	1045	1123	1057	1066	1121	1110	1106	1116
Z_3	787	735	799	791	831	834	789	792	834	783
X_1	4391	4453	4376	4666	4666	4666	4666	4502	4666	4552
X_2	10 834	13 106	10 690	11 115	10 716	11 862	11 098	13 500	10 721	10 840
X_3	2865	2927	2848	2113	2810	2986	2003	2856	2811	1892
X_4	1762	1900	0	1757	290	3	140	0	1900	1769
X_5	3599	3523	3532	142	3600	1483	2500	3600	1940	3598
X_6	580	2423	2348	2500	2499	2500	2000	0	2500	2373
X_7	2000	1919	1843	1146	2000	2000	1500	1865	2000	828
X_8	216	785	1348	1499	1496	960	1500	724	766	1500
X_9	4862	267	4126	4260	4988	4669	4956	4856	4997	4882
X_{10}	2774	2562	2495	2857	2838	2994	2779	2499	2836	2330
X_{11}	2883	3000	3000	2997	190	494	3000	744	194	0
X_{12}	0	0	0	1657	290	1750	1800	1656	1099	1668

Table 2 shows the final results obtained for all the scenarios. Consider scenario 1 as an illustration for analysis. According to scenario 1, the upper level decision makers select suppliers with lower costs for single piece allocation. The first level decision maker selects four suppliers with the lowest associated costs among all of the suppliers. The second level decision maker then selects the suppliers with the lowest associated costs among the remaining eight suppliers. The first level decision maker thus chooses x_2, x_4, x_9, x_{11} as his controlling factors. The second level decision maker selects x_1, x_3, x_5, x_{10} as his controlling factors. The maximum tolerance allowed by the first and second level decision makers for each of their controlling factors is 200. The maximum degree of satisfaction for all the levels achieved by the solution presented in Table 2 (the second column corresponds to the first scenario) is 0.30. The maximum number of items rejected and items delivered late for scenario 1 are 1119 and 787, respectively. Supplier 10 performs well according to several criteria such as cost, percentage of items rejected or delivered late and thus utilised 92% of his available capacity. Suppliers 5 and 11 have low levels of items rejected or delivered late in their supplies and have moderate associated costs for single piece allocation. More than 80% of their capacity was utilised by the generated solution. Supplier 2 gained 32% of the quota allocation based on total demand, because of its high capacity. Suppliers 6, 8 and 12 perform well according to several criteria such as the percentage of items rejected or delivered late in their supplies, but have high associated costs for single piece allocation and low capacities. The quota allocation for supplier12 is equal to zero, while suppliers 6 and 8 receive less than 2% of the quota allocation in the final solution. Suppliers 1, 2, 3, 4, 7 and 9 utilise more than 50% of their respective capacities.

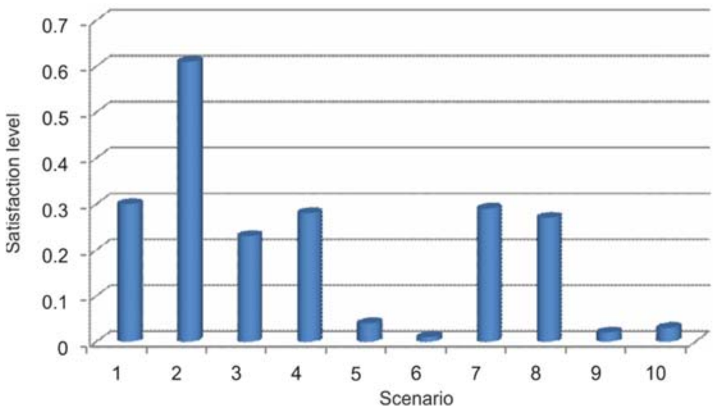

Fig. 2. Satisfaction levels for various scenarios

Figure 2 shows all of the scenarios according to the satisfaction levels obtained under the generated solutions. The maximum degree of satisfaction λ obtained under the generated solution at all of the levels is 0.6 for Scenario 2. Scenario 6 is associated with the lowest degree of satisfaction ($\lambda = 0.1$) for all the levels. The degree of satisfaction thus obtained is highest for scenario 2, when the upper decision makers highlight the suppliers that have the highest costs for single piece allocation. The degree of satisfaction is lowest under scenario 6, when the suppliers with the lowest quota flexibilities are controlled by the upper level decision makers.

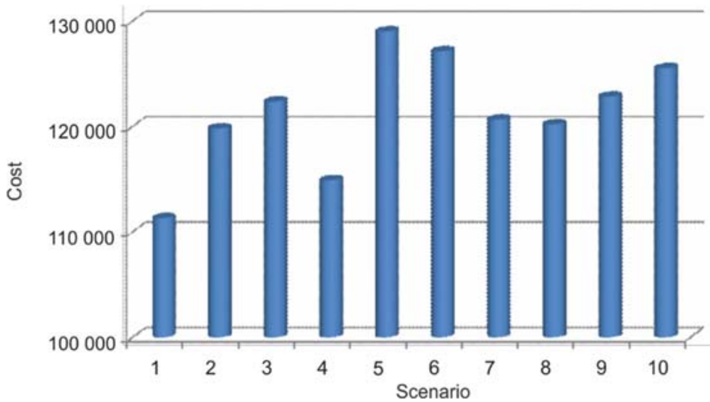

Fig. 3. Associated costs Z_1 for various scenarios [$]

Figure 3 shows associated costs for all the scenarios. Scenario 1 yields the lowest cost (111 311 $) among all of the scenarios. Under scenario 1, the upper level decision makers select suppliers that are associated with the lowest costs for single piece allocation. Scenario 5, according to which the upper level decision makers select suppliers that are associated with higher quota flexibilities, yields the largest cost.

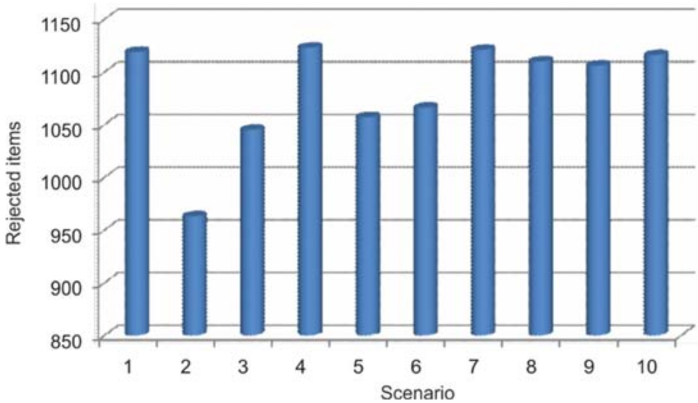

Fig. 4. Rejected items Z_2 for various scenarios

Figures 4 and 5 show the numbers of items rejected (Fig. 4) and items delivered late (Fig. 5) for all of the scenarios. The number of items provided by the suppliers that are rejected is lowest for scenario 2, when the suppliers with the highest associated costs are highlighted by the upper level decision makers. The number of rejected items is greatest under scenario 4. The number of items delivered late was least under scenario 2 and greatest under scenarios 6 and 9.

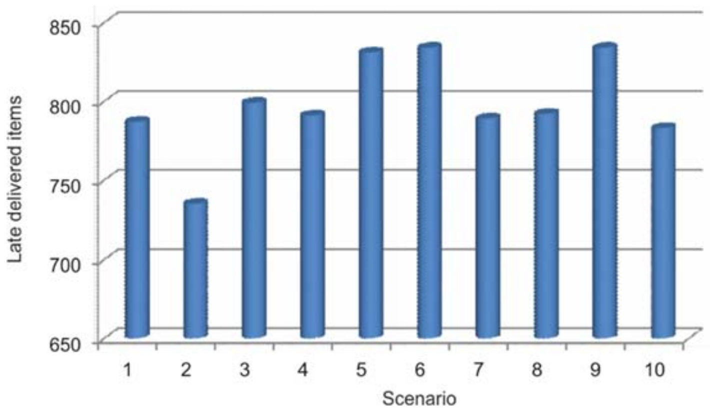

Fig. 5. Late delivered items Z_3 for various scenarios

Table 3. Quota allocations for each supplier [%]

Supplier	Quota allocations									
	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
Sup 1	12	12	12	13	13	13	12	12	13	13
Sup 2	29	35	29	30	29	33	29	37	29	30
Sup 3	8	8	8	6	8	8	5	8	8	5
Sup 4	5	5	0	5	1	0	1	0	5	5
Sup 5	10	10	10	0	10	4	7	10	5	10
Sup 6	2	7	6	7	7	7	5	0	7	7
Sup 7	5	5	5	3	5	5	4	5	5	2
Sup 8	1	2	4	4	4	3	4	2	2	4
Sup 9	13	1	11	12	14	13	13	13	14	13
Sup 10	7	7	7	8	8	8	7	7	8	6
Sup 11	8	8	8	8	0	1	8	2	1	0
Sup 12	0	0	0	4	1	5	5	4	3	5

Table 3 presents the quota allocations for the scenarios considered in terms of percentages, calculated with respect to the total allocation made in various scenarios. Considering scenario 1, the highest quota allocation is allotted to supplier 2 (29%), as the supplier gained a total allocation of 10 834 units out of the total allocation of 36 766 units. The maximum variation in the quota allocations over all the scenarios occurred for supplier 12. The quota allocations for supplier 12 are made after the allocations for the rest of the suppliers have been decided. This is because of the fact that supplier 12 is associated with higher costs, as well as a lower available budget and capacity. Supplier 2 performs well on several criteria such as cost and the number of items rejected or delivered late. In particular, supplier 2 has the maximum available supply and budget among all the suppliers. Supplier 2 thus gains a very large quota allocation in all of the scenarios. Supplier 4 has a comparatively high level of rejected items and has a low supplier rating. Moreover, the capacity and budget for supplier 4 are also low. Therefore, the quota allocation for supplier 4 is zero under some scenarios. Supplier 5 possesses a high capacity and budget. Supplier 5 therefore gained considerable quota allocations in all of the scenarios except scenario 4. Supplier 6 also gained considerable quota allocations under all of the scenarios except for scenarios 1 and 8. The quota allocation for supplier 6 ranges from zero to a highest of 7% in various scenarios. Supplier 7 performs well on criteria related to the number of items rejected or delivered late and also has a high supplier rating. Moreover, together with these characteristics, supplier 7 possesses a considerable capacity and budget to compete with other suppliers. Supplier 7 therefore gained significant quota allocations under all of the scenarios.

Supplier 8 has a low supplier rating and quota flexibility. Supplier 8 therefore received low quota allocations under all of the scenarios. Suppliers 9 and 10 received considerable quota allocations in all of the scenarios. The maximum variation in the quota allocations occurs for suppliers 11 and 12. Under some scenarios, the quota allocations for supplier 11 and 12 are equal to zero, while they are considerable in other scenarios. Supplier 12 is the only supplier for which the quota allocation equals zero in three scenarios. Supplier 12 has the highest associated cost for single piece allocation and has a low supplier rating. Supplier 2 received the highest quota allocations among all of the suppliers in all of the scenarios. Supplier 12 received the lowest quota allocations among all of the suppliers for most of the scenarios.

Supplier 1 utilised more than 62% of its budget in all of the scenarios. Supplier 1 has a low associated cost and a low rate of late delivery. Moreover, supplier 1 has a high capacity and budget. Supplier 2 utilised more than 61% of its budget in all of the scenarios. Supplier 3 also received considerable allocations in all of the scenarios. Supplier 3 utilised at least 37% of its available capacity (under scenario 10) and at most 59% (under scenario 6). Supplier 4 utilised at most 50% of its available capacity (under scenarios 2 and 9). Supplier 5 performs well based on criteria related to the rate of rejection and late delivery. The proportion of available capacity utilised by supplier 5 ranged from less than 3% (under scenario 4) to 84% (under scenario 6). Supplier 8 performs badly on several criteria related to cost and the rate of late delivery. Supplier 9 utilised more than 63% of its available capacity in all of the scenarios except for scenarios 2 and 7. Supplier 10 utilised its available capacity by more than 77% in all of the scenarios. Under scenario 7, the utilisation of supplier 10's capacity reaches 100%. Supplier 10 provides supply with zero rejections and very low levels of late delivery. Moreover, supplier 10 is one of the highest rated suppliers, along with possessing a significant capacity and budget. Hence, compared to the other suppliers over all the scenarios, supplier 10 utilises its capacity to the highest degree. Similarly, suppliers 2 and 10 utilised their budget to the maximum degree among all of the suppliers over all the scenarios.

Suppliers 4, 8, 11 and 12 utilised their budgets and capacities to a low degree in most of the scenarios.

The generated allocation patterns are definitely useful in assisting decision makers to select from potential suppliers and allocate the optimal quantities to order from these suppliers. Consideration of uncertain demand and supply provides more flexibility to the decision process and is therefore efficient in dealing with real life problems.

5. Conclusion

A model of multilevel decision making has been constructed for the supplier selection problem with fuzzy demand and supply. The decision variables are partitioned between different levels and are known as controlling factors. These controlling factors define the level of control of the upper level decision makers over the decision variables. A fuzzy programming approach to dealing with multilevel programming problems has been extended and applied to multilevel programming problems with fuzzy constraints. Using the optimal values of the controlling factors for the upper level decision makers, fuzzy theoretic membership functions have been constructed. To obtain the final solution, membership functions for the controlling factors, together with membership functions for goals/ preferences have been used. Different scenarios were constructed on the basis of the controlling factors which were under the control of the upper level decision makers. Different scenarios generate different levels of satisfaction. The results obtained can be improved by changing the values of the tolerances associated with the controlling factors adopted by the upper level decision makers.

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