OPERATIONS RESEARCH AND DECISIONS

No. 2 2017
DOI: 10.5277/ord170204

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A METHOD OF ASSIGNING A GLOBAL PREFERENCE INDEX

The issue of decision-making has been examined based on the preferences of the entire population, when the preferences of a few subpopulations varying significantly in size are known. The purpose of assigning global preferences according to the coefficients proposed here was to avoid marginalising the preferences of the smaller subpopulations. The preference coefficients for the population have been assigned using a weighted arithmetic mean, where the weights are the square roots of the sizes of the subpopulations. This is similar to the voting system known as the "Jagiellonian compromise". The statistical properties of these constants were presented in the context of decision making. These results have been illustrated by way of an example where the subpopulations exhibit significant differences, viz. students' choice of an economics university in Lower Silesia, Poland.

Keywords: *decision process, decision making, university choice*

1. Introduction

The results of studies on populations that vary significantly in size are liable to marginalise smaller subpopulations. Smaller subpopulations are therefore at risk of being marginalised. This can lead to incorrect generalisations being made with respect to the overall population. Obviously, the problem of marginalising small populations does not arise in every kind of study. Moreover, it primarily depends on the issue being studied and the assumptions of the researcher(s).

Subpopulations examined in the studies of consumer preference may vary considerably in size. It is important not to marginalise smaller populations, as their strength and market significance can contain important information, especially when the market is

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changing rapidly. Meta-regression is one of the methods used to generalise research results. This is based on secondary data, and is used, inter alia, in the medical sciences, where there is no option but to obtain various results regarding the impact of therapy [6]. Obtaining a single, global result can also be a problem in management sciences, especially in heterogeneous decision-making groups, where the diversity of views and range of experience make it difficult to achieve a result. The following methods are used in these situations:

- Multiple voting [3]. This involves multi-stage voting for the preferred options, with the aim of eliminating the least popular choice at each stage.
- The Delphi method [12], also known as the Delphi-expert method. Decisions are made on the basis of the opinion of experts in the relevant field. This opinion is then reviewed and referred back to the decision makers for further action;
- Stepladder technique [10]. This is a technique in which solutions to the problem are presented anonymously (and without any knowledge of other proposed solutions) and fed back to the team leader. Those involved in the process have no knowledge of the others' ideas to solve the problem. The advantage of this technique is that every opinion is heard – uninfluenced by any suggestions from other team members.
- Borda count [1]. A method which consists of assigning points to each solution, with the worst solution given 1 point and the others rated correspondingly higher. The option with the most accumulated points is chosen as the best.

The above mentioned decision-making techniques illustrate how heterogeneous group preferences may cause difficulties when producing a final result.

An inadequate number of demonstrated preferences may lead to over-representation of rating scores. This can be solved using a Bayesian average. In situations where a subpopulation has a strong preference, its impact on the global score will depend on the sum of the preference data from all the subpopulations, as well as the arithmetic mean of the quantities of all types of preferences. A Bayesian average can be used in ranking systems, where there is a problem with the credibility and values of ratings assigned by diverse numbers of people. The ranking may be of little use, as high assessments for a specified object, given by a few voters, are not equivalent to a lower assessment of another object given by a large number of voters. For an extensive review of Bayesian model averaging, see [4].

Considerations of inequality and the marginalisation of smaller populations are also part of the political debate. A clear problem, and a topic of discussion, is the strength of the votes of the European Union countries. In the current voting system, the Banzaf Power Index can be used to form a favourable coalition. However, this does not solve the problem of small countries not having much say in EU decision making.

One of the proposed solutions to this problem is the "Jagiellonian compromise", which weights the votes of each country according to the square root of its population. This could increase the impact of medium-sized and small countries in decision-making in the European Parliament [7]. The votes of countries with large numbers of citizens would still carry the greatest weight, but they would no longer dominate, as is the case with the current method of counting votes. The question of representation with regard to general elections has already been raised by Chamberlin and Courant [2], as well as Monroe [7]. The voting methods and procedures presented in our study bring in some alternatives to the approaches shown above. In politics, the ultimate aim of any vote is to reach a compromise and form a coalition. Our research strives at redefining these aims. Thus, as our study proves, the main goal is to analyse the preferences of given communities, i.e. decision-making processes within these communities, in terms of the interests of the general public. Our research was motivated by the empirical studies of the first author on selection criteria when choosing economics universities in Lower Silesia. Some of the study findings are demonstrated in the last chapter of the study to illustrate new applications of the global preference index.

The tools that are useful in the situations described above turned out to be less helpful in obtaining a global result in a study of the factors in choosing economics universities. The problem of obtaining a global result involved giving the study to a group of students divided into eight subpopulations that varied considerably in size.

This research, undertaken on a defined student population, separated subpopulations according to the type of institution, mode and level of study. Due to the different numbers of students in each subpopulation, there was a problem of heterogeneity in terms of quantity. It was therefore difficult to obtain a single global result. The arithmetic mean, which gives an overall score for the entire population, seemed to be the obvious solution. However, when the subpopulations vary significantly in size, the problem of small subpopulations having too great, and large populations too small, an impact on the global result arises. The final result therefore proved to be unreliable. A solution to this problem was also sought by using a weighted arithmetic mean. Each subpopulation was included in the global result in proportion to its size. In this case, however, small groups were marginalised and large groups over-represented.

This article presents a new method of constructing a global preference index. This method allows small subpopulations to retain their influence on the final result. An example of using this method, viz. in examining the factors influencing the choice of economics universities among students in Lower Silesia in Poland, is also described. The large diversity in the size of subpopulations made it necessary to construct a new index that would make it possible to obtain a global result, while maintaining the influence of smaller populations on that result.

2. Preference index

Assume existence of N subpopulations in a global population $A = A_1 \cup A_2 \cup \ldots \cup A_N$, $A_i \cap A_j = \varnothing$ for $i \neq j$, where subpopulation A_i has n_i elements, $i = 1, 2, \ldots, N$,

$n = n_1 + n_2 + \ldots + n_N$. The preference index for subpopulation A_i is $a_i = k_i / n_i$, where k_i is the number of elements (i.e., people) in the corresponding subpopulation with a given preference. Let

$$c_i = \frac{n_i}{n}$$

be the relative size of subpopulation *i*. Obviously, $c_1 + c_2 + \ldots + c_N = 1$.

Problem. How can a reasonable "global" index *a* be constructed for the population $A = A_1 \cup A_2 \cup \ \ldots \cup A_N$, i.e., one that represents all the subpopulations as a whole? Let us consider two extreme situations.

The arithmetic mean $\overline{a}$ of the preference indices a_i is determined by the formula

$$\overline{a} = \frac{a_1 + a_2 + \ldots + a_N}{N} \tag{1}$$

and the arithmetic mean $\hat{a}$ for the entire population is determined by the formula

$$\hat{a} = \frac{k_1 + k_2 + \ldots + k_N}{n} \tag{2}$$

Interpretation: Equation (1) – each subpopulation has one vote independently of their resources, Eq. (2) – each subpopulation has a voting weight proportional to its reserves. Depending on the application, both interpretations may have significant faults. Equation (1) over-represents small populations, but Eq. (2) marginalises them. To prevent these specific errors, a new global preference index needs to be introduced. The mentioned preference index may be constructed by introducing a weighted mean with particular weights as presented below. A weighted arithmetic mean is described by the formula

$$a_w = w_1 a_1 + w_2 a_2 + \ldots + w_N a_N \tag{3}$$

where $w_i \geq 0$ and $w_1 + w_2 + \ldots + w_N = 1$. Such a weighted arithmetic mean reduces to the ordinary arithmetic mean

$$a_w = \frac{a_i + a_i + \ldots + a_N}{N} = \frac{k_1 + k_2 + \ldots + k_N}{n} = \hat{a} \tag{4}$$

when $n_i = m$ and $w_i = c_i$. If $a_i = a$, then $a_w = a$. An extensive review of various kinds of means and their applications is given by Ostasiewicz and Ostasiewicz [8].

We want to select a vector of weights $w = (w_1, w_2, \dots, w_N)$ that depends on n_i, i.e., $w_i = w_i(n_i)$ and n such that the global preference index a_g, defined as the weighted mean a_w given by (3), has the properties given by Definition 1.

Let us denote

$$\bar{a}(i, j) = \frac{a_i + a_j}{2}, \qquad \hat{a}(i, j) = \frac{k_i + k_j}{n_i + n_j}$$

Definition 1. We call the number a_g a global preference index if it fulfils the following conditions:

1. If the indices in all the subpopulations are equal, $a_i = a$, then $a_g = a$.

2. If all the subpopulations have the same number of elements, $n_i = m$, i.e., $c_i = c = 1/N$, then $a_g = \bar{a}$.

3. In all remaining cases, if for any pair A_i and A_j, $i \neq j$ for which $n_i \neq n_j$ or $a_i \neq a_j$, then the following inequalities must hold:

a) if $\bar{a}(i, j) < \hat{a}(i, j)$, then $\bar{a}(i, j) < w_i a_i + w_j a_j < \hat{a}(i, j)$,

b) if $\bar{a}(i, j) > \hat{a}(i, j)$, then $\hat{a}(i, j) < w_i a_i + w_j a_j < \bar{a}(i, j)$.

For the reasons discussed in Section 1, the weights should only depend on the sizes of the subpopulations and the size of whole population. Let us assume that

$$w_i = \frac{v(n_i, n)}{\sum_{k=1}^{N} v(n_i, n)} \tag{5}$$

where $v(m, n)$ is a non-decreasing function of the variable m: $0 \le m \le n$, $v(0, n) = 0$ and $v(n, n) \le n$. Let A' be a partial population defined by

$$A' = \bigcup_{j \in \{i_1, \dots, i_M\}} A_{i_j}$$

where $\{i_1, \dots, i_M\}$ is an M-element subset of the set of indices $\{1, 2, \dots, N\}$. Let

$$n' = \sum_{j=1}^{M} n_{i_j}$$

be the number of elements in the partial population A'.

However, Definition 1 does not determine a unique global preference index. Moreover, none of the global preference indexes shown in Section 4 have as good statistical properties as the one proposed in Section 5.

We define the index d of the structure of a division of the population A into subpopulations as

$$d = \sum_{i=1}^{N} v\left(n_i, n\right) \tag{6}$$

Next, let us assume that the function $v\left(m, n\right)$ fulfils the following conditions.

For each $M \le N$ and each A', the structure coefficient d for the partial population A' achieves its maximal value if $n_i = m$ for all i and its minimal value, if $n_i = n'$ for some *i*. The case $n_i = m$ is the maximally homogeneous one and the case $n_i = n'$ is the maximally nonhomogeneous one.

For $M = 2$ and $m_1 \le m_2 \le \lfloor n/2 \rfloor$, $v\left(m_1, n'\right) \le v\left(m_2, n'\right)$.

The coefficient *d* for the whole population *A* describes the non-homogeneity of its division into subpopulations.

3. Statistical model

Let us assume that in each population A_i, the *j*th element x_{ij} exhibits a fixed preference with probability p_i independently of the other elements and $q_i = 1 - p_i$. Let X_i denote the number of elements with the given preference in the population A_i. $\hat{X}_i = X_i / n_i$ will be called the preference in subpopulation A_i. Therefore, we obtain the expectations

$$\mathrm{E}X_i = n_i p_i, \qquad \mathrm{E}\hat{X}_i = p_i$$

variances

$$\mathrm{D}^2 X_i = n_i p_i q_i, \qquad \mathrm{D}^2 \hat{X}_i = p_i q_i / n_i$$

and standard deviations

$$\sigma_i = \sqrt{n_i p_i q_i}, \qquad \hat{\sigma}_i = \sqrt{p_i q_i / n_i}$$

Let

$$Y = \sum_{i=1}^{N} w_i\left(n_i\right)\hat{X}_i \tag{7}$$

be the global preference in the population A with fixed weights $w_i\left(n_i\right)$. We assume that

$$\sum_{i=1}^{N} w_i\left(n_i\right) = 1$$

Therefore, the expectation and variance of this global preference are represented by

$$\mathrm{E}Y = \sum_{i=1}^{N} w_i\left(n_i\right) p_i \tag{8}$$

$$\mathrm{D}^2 Y = \sum_{i=1}^{N} \frac{w_i^2\left(n_i\right)}{n_i} p_i q_i \tag{9}$$

4. Square root weights

Let us define the following square-root weights

$$w_i\left(n_i\right) = \frac{\sqrt{n_i}}{\sum_{i=1}^{N}\sqrt{n_i}} \tag{10}$$

From Equations (8) and (9), we have

$$\mathrm{E}Y = \frac{\sum_{i=1}^{N}\sqrt{n_i}\, p_i}{\sum_{i=1}^{N}\sqrt{n_i}} = \frac{\sum_{i=1}^{N}\sqrt{n_i}\, p_i}{r\sqrt{Nn}} = \frac{\sqrt{n_i}\, p_i}{d} \tag{11}$$

$$\mathrm{D}^2 Y = \frac{\sum_{i=1}^{N} p_i q_i}{\sum_{i=1}^{N} \sqrt{n_i}} = \frac{1}{r\sqrt{Nn}} \sum_{i=1}^{N} p_i q_i = \frac{\sum_{i=1}^{N} p_i q_i}{d} \tag{12}$$

It follows from (12) that the variance of the variable of the global preference in the whole population does not depend on the number of elements in the individual subpopulations, but only on the total population size, n, the number of subpopulations, N, and the structure coefficient of the population division, d (or equivalently the homogeneity coefficient, r).

There are two solutions of the equation

$$\sum_{i=1}^{N} p_i q_i = pq \tag{13}$$

under the conditions $p + q = 1$, $p > 0$, $q > 0$, such that $pq \neq 1/2$. If $\mathrm{E}Y > 1/2$, then as p we accept the solution where p is greater than 1/2 and in the opposite case, a solution where p is smaller than 1/2. The parameter p may be interpreted as a global, averaged preference of an element in the population.

From (6), the structure coefficient, d, of the division of the population into subpopulations has the following form:

$$d = \sum_{i=1}^{N} \sqrt{n_i} = \sqrt{n} \sum_{i=1} \sqrt{c_i} \tag{14}$$

The coefficient d achieves its maximal value if $n_i = m$ for all i, $n = mN$. Then

$$d_{\max} = N\sqrt{m} = \sqrt{Nn}$$

The coefficient d achieves its minimal value if $n_i = n$ for exactly one i and $n_j = 0$ for the remaining j. Let us denote the non-normalised homogeneity coefficient of the division of the population into subpopulations by

$$r' = \frac{d}{\sqrt{Nn}} \tag{15}$$

Property 1. For any division of the population into subpopulations, we have

$$\frac{1}{\sqrt{N}} \leq r' \leq 1 \tag{16}$$

where

$$r' = \begin{cases} \frac{1}{\sqrt{N}} & \Leftrightarrow \; n_i = n \text{ for some } i \text{ and } n_j = 0 \text{ for } j \neq i \\ 1 & \Leftrightarrow \; n_i = m \text{ for each } i \end{cases} \tag{17}$$

The normalised homogeneity coefficient of the division of the population into subpopulations is defined by

$$r = \frac{r' - \frac{1}{\sqrt{N}}}{1 - \frac{1}{\sqrt{N}}} \tag{18}$$

Thus we obtain

Property 2. For any division of the population into subpopulations, we have

$$0 \le r \le 1 \tag{19}$$

where

$$r = \begin{cases} 0 & \Leftrightarrow \; n_i = n \text{ for some } i \text{ and } n_j = 0 \text{ for } j \neq i \\ 1 & \Leftrightarrow \; n_i = m \text{ for each } i \end{cases} \tag{20}$$

If w_i are square root weights defined by (10), then the global preference index is given by

$$a_g = \frac{a_1\sqrt{c_1} + a_2\sqrt{c_2} + \ldots + a_N\sqrt{c_N}}{\sqrt{c_1} + \sqrt{c_2} + \ldots + \sqrt{c_N}} \tag{21}$$

If $c_i = 1/N$, or equivalently $n_i = m$, then $a_g = \bar{a} = \hat{a}$, where $\bar{a}$ is the arithmetic mean of the preferences in the subpopulations defined by (1), and $\hat{a}$ is the arithmetic mean of the preference of the whole population defined by (2). The coefficient defined by (21) fulfils the conditions from Definition 1.

From the statistical model presented in Section 3, it also follows that square-root weights guarantee that the standard deviation of the global preference coefficient is independent of the size of the individual subpopulations. Square-root weights also guarantee other favourable properties. These are presented below as Properties 3 and 4.

The weights w_i can be treated as probabilities, because $w_i \geq 0$, $w_1 + w_2 + \ldots + w_N = 1$. Thus a_i are the values of a random variable Z and $\mathrm{E}Z = a_w$. Equation (21) can be reduced to the form (3) by taking

$$c_g = \sqrt{c_1} + \sqrt{c_2} + \ldots + \sqrt{c_N}$$

and

$$c_i' = \frac{\sqrt{c_i}}{c_g} \tag{22}$$

In this way, we define the random variable Z', which also assumes the values a_i with the probabilities c_i' defined by (22). Because the variance $\mathrm{D}^2 Z$ is defined by

$$\mathrm{D}^2 Z = \mathrm{E}Z^2 - \left(\mathrm{E}Z\right)^2 = c_1 a_1^2 + c_2 a_2^2 + \ldots + c_N a_N^2 - \left(c_1 a_1 + c_2 a_2 + \ldots + c_N a_N\right)^2$$

the variance $\mathrm{D}^2 Z'$ can be defined as:

$$\mathrm{D}^2 Z' = EZ'^2 - \left(EZ'\right)^2 = c_1' a_1^2 + c_2' a_2^2 + \ldots + c_N' a_N^2 - \left(c_1' a_1 + c_2' a_2 + \ldots + c_N' a_N\right)^2$$

Therefore,

$$\sigma_g = \sqrt{\mathrm{D}^2 Z'} = \sqrt{c_1' a_1^2 + c_2' a_2^2 + \ldots + c_N' a_N^2 - \left(c_1' a_1^2 + c_2' a_2^2 + \ldots + c_N' a_N^2\right)^2} \tag{23}$$

is a measure of the spread of the indexes in the individual subpopulations from the global index given by (21).

The parameter σ_g can be given a corresponding interpretation, as it indicates the diversification of the studied subpopulations according to the preference index.

Property 3. If $a_i = a$ for all *i*, then $\sigma_g = 0$.

Let us denote

$$s^2 = \frac{k_1^2 + k_2^2 + \ldots + k_N^2}{N} - \left(\overline{a}\right)^2 \tag{24}$$

and $s = \sqrt{s^2}$.

Property 4. If $n_i = m$, then $\sigma_g = s$.

Obviously, if $a_i = a$ and $n_i = m$, then s^2, defined by (24), is also equal to zero.

Example 1. The population figures (in mln) for chosen EU states (as of 1st January, 2015) and their square roots are given in Table 1. The parameters that define the non-homogeneity of these populations are calculated from Equations (14), (15) and (18):

$$d = 95.91, \quad r' = 0.8314, \quad r = 0.7912$$

Table 1. Population sizes n_i and their square roots $\sqrt{n_i}$

State	n_i	Percent of population	$\sqrt{n_i}$	Percent of $\sqrt{n_i}$ population
Germany	81.17	15.94	9.01	9.1
France	66.42	13.04	8.15	8.24
United Kingdom	64,77	12.74	8.05	8.14
Italy	60.8	11.94	7.8	7.88
Spain	46.44	9.12	6.81	6.89
Poland	38	7.46	6.16	6.23
Romania	19.86	3.9	4.46	4.5
The Netherlands	16.9	3.32	4.11	4.15
Belgium	12.26	2.41	3.5	3.54
Greece	10.81	3.29	2.12	3.32
Czech Republic	10.54	2.07	3.25	3.28
Portugal	10.37	3.22	2.04	3.25
Hungary	9.85	1.93	3.14	3.17
Sweden	9.75	1.91	3.12	3.16
Austria	8.58	1.68	2.93	2.96
Bulgaria	7.2	1.41	2.68	2.71
Denmark	5.66	1.11	2.38	2.4
Finland	5.47	1.07	2.34	2.36
Slovakia	5.42	1.06	2.33	2.35
Ireland	4.62	0.91	2.15	2.17
Croatia	4.23	0.83	2.06	2.08
Lithuania	2.92	0.57	1.71	1.73
Slovenia	2.06	0.40	1.44	1.45
Latvia	1.99	0.39	1.41	1.43
Estonia	1.31	0.26	1.14	1.16
Cyprus	0.85	0.17	0.92	0.93
Luxemburg	0.56	0.11	0.75	0.76
Malta	0.42	0.08	0.65	0.65
Total	509.34	100	98.96	100

The voting weight of each EU country depends on the number of citizens it has. There is therefore no point in formulating a global preference index. Instead, possible coalitions are considered. The weights of the votes of countries are proportional to the square root of their population size, known as the "Jagiellonian Compromise". Such weighting does not fundamentally change the methods of voting according to the rules before 11th January, 2014. However, after the ratification of the Treaty of Lisbon, the votes of countries with the smallest populations had lower weight, and the biggest countries had increased voting weight. Weighting votes according to the square root of population size rather than the population size expressly increases the importance of the voice of small and medium-sized countries. Moreover, Poland and Portugal have a greater voting weight together than Germany, even if the sum of the populations of these two countries is smaller than the population of Germany. This idea was presented in [9, 11]. An exhaustive review can be found in [13].

5. The determinants of choosing an economics university

A survey regarding the factors involved in choosing an economics university (in Lower Silesia) was conducted on a sample determined using quota sampling. The Polish education system is shared between the public and private sectors. Studies are divided into level 1 (Bachelor) and level II (Masters) in accordance with the Bologna Process[3]. Students were further subdivided according to whether their studies were full-time or part-time. Consequently, there were eight groups of students whose decision to enrol could be influenced by different determinants, and may constitute separate segments of the higher education services market. The survey sought to obtain a global result that would indicate the factors involved in choosing an economics university for students as a whole[4]. There was a problem related to calculating the values for the individual determinants corresponding to the general preferences on the market for educational services in the field of economics. Poland has had private sector higher education since 1990. There are far fewer students, especially full-time students, in the private sector than there are in the public sector. This disparity is not merely due to there being no fees for full-time studies at public universities but also because public universities are recognised "brand names" and have extensive infrastructures built up over many years. The intention in studying the factors involved in choosing an economics university was not to marginalise the factors influencing the choice of private universities, as the creation of this sector abolished public universities' monopoly on offering high-level educational

[3]The Bologna Process also defines third degree studies. This group of students, however, was not included in the study.

[4]Studies were only divided into full-time and part-time.

services, thereby bringing about competition and creating a market for educational services. Moreover, private universities have helped to popularise higher education. This makes private universities an important part of the education market, and their students are an important population in terms of the deciding factors in choosing an economics university.

There were 12 ,904 students studying for a Bachelor or Masters degree at an economics university in the 2010/2011 academic year[5]. The sample comprised 645 students, which is 5% of the defined population. The example illustrated in Table 2 presents results for full-time 1st year students only, therefore the total size of this subpopulation is 222.

Types of universities and level of study:
- Bachelor studies (1st level) public 1st Pu,
- Master studies (2nd level) public 2nd Pu,
- Bachelor studies (1st level) private 1st Pr,
- Master studies (2nd level) private 2nd Pr,

Table 2. First-year economics students divided into subpopulations

Type	i	m_i	n_i	$\sqrt{n_i}$	c_i	w_i
1st Pu	1	2129	106	10.2956	0.4775	0.3649
2nd Pu	2	1182	59	7.6811	0.2658	0.2722
1stPr	3	877	44	6.6332	0.1982	0.2351
2ndPr	4	258	13	3.6056	0.0586	0.1278
Total		4446	222	28.2156	1.0000	1.0000

Source: W. Maciejewski, *Factors influencing student's decision to study at economics universities*, PhD Thesis, Wrocław University of Economics, Wrocław 2013.

The parameters characterising the population's heterogeneity calculated from Eqs. (14), (15) and (18) are:

$$d = 28.2156, \qquad r' = 0.9469, \qquad r = 0.8937$$

Exceptional diversity can be observed among full-time students. There were 2119 undergraduate students at public universities and 877 at private universities. An even greater difference is evident among full-time MA students – 258 at private economics universities and 1182 at public universities. The only difference between the sizes of the subpopulations of part-time students can be seen at undergraduate level. The 3827

[5]Information obtained from economics universities.

first-year, part-time undergraduate students at private universities make up the largest subpopulation. This indicates the importance of private universities in the higher education system. The results of the determinants affecting the choice of economics universities were obtained separately for full-time and part-time students. This article only presents the results obtained for full-time students, the reason being that there was considerable diversity in these subpopulations and the benefits of square-root weighting were most evident here. Respondents were able to choose more than one answer in the questionnaire. Therefore, the results do not add up to 100%. Depending on the number of selected responses, each factor was assigned a certain weight[6].

Assignation of preferences:
A – higher prestige of public/private university,
B – major(s) not offered by other public/private universities,
C – higher quality of education compared to public/private university,
D – enrolment process,
E – quality of administration services,
F – other.

Table 3. Preferences for attributes when students choose a university

Type	A	B	C	D	E	F
1st Pu	0.1776	0.0220	0.2374	0.0000	0.0173	0.0283
2nd Pu	0.4067	0.0085	0.2654	0.0169	0.0000	0.0311
1st Pr	0.2053	0.2924	0.1030	0.2318	0.0727	0.0947
2nd Pr	0.0000	0.1538	0.1410	0.5000	0.1795	0.0256

Source: W. Maciejewski, *Factors influencing student's decision to study at economics universities*, PhD Thesis, Wrocław University of Economics, Wrocław 2013.

Table 3 presents, for each subpopulation, the preferences for each attribute. The intention was to present the preferences of all the students while maintaining the weights. The results obtained using the average based on square-root weights allow small subpopulations to influence the global result, but give a greater impact to large subpopulations. The data in Table 3 and Fig. 1 do not allow the results to be generalised to all students at economics universities, as the sample was not representative. The results described in Table 4 show how the choice of mean can produce different results. The results obtained using the square-root weights increase the impact of small subpopulations on the global result compared to the impact of large subpopulations.

[6]If two factors were selected, the weighting factor was 0.5. If three factors were selected, the weighting factor was 0.333. Other weights were assigned to correspond to the number of selected responses.

Table 4. Factors influencing the choice of an economics university by first-year, full-time undergraduate and masters students using different preference indexes

	A	B	C	D	E	F
$\bar{a}$	0.1974	0.1192	0.1867	0.1872	0.0674	0.0449
$\hat{a}$	0.2336	0.0797	0.2126	0.0797	0.0332	0.0420
a_g	0.2238	0.0987	0.2011	0.1230	0.0463	0.0443

Particularly interesting results were obtained when calculating the preferences for D (the enrolment process), where the use of various ratios gave the clearly distinct values of 18.72%, 7.97% and 12.3%. These results are due to the different subpopulations having significantly different preferences. Due to the use of square-root weighting, the preferences of smaller subpopulations were not marginalised, which was the intention of the authors. Figure 1 is a graphical representation of Table 3 and illustrates the differences resulting from the application of these indicators. The square-root weighted mean always gives results that are between the results obtained using the arithmetic mean and the weighted mean. In the case described in this article, these results were particularly desirable, because of the large heterogeneity of the sample.

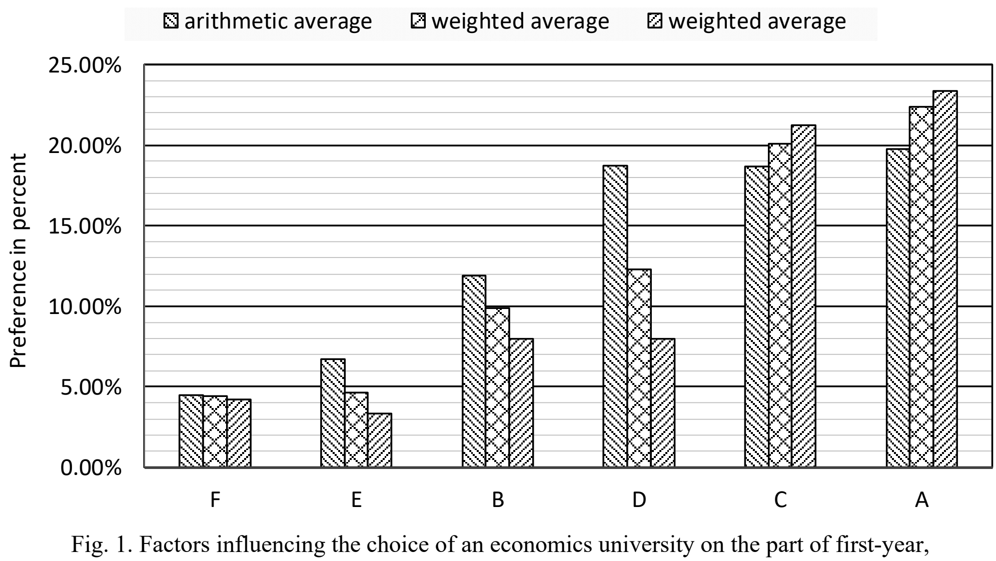

Fig. 1. Factors influencing the choice of an economics university on the part of first-year, full-time undergraduate and masters students using different preference indexes.
Source: W. Maciejewski, *Factors influencing student's decision to study at economics universities*, PhD Thesis, Wrocław University of Economics, Wrocław 2013

Table 5 presents the values of the parameter σ_g defined by Eq. (23). These values measure the spread of preferences in the individual subpopulations from the global preference indexes for each type of preference.

Table 5. Preference spread

Preference type	A	B	C	D	E	F
σ_g	0.12792	0.11649	0.06581	0.17183	0.05762	0.02797

6. Conclusions

In the case of the arithmetic mean, weighted mean, and square-root weighted mean, there are no significant differences in the rankings of factors for choosing a public or private university. However, there are large differences in the assessed importance of individual factors. Preference indexes based on the arithmetic mean and weighted means can therefore lead to different conclusions. The square-root weighted mean allows the size of the subpopulations to remain important, thus increasing the importance of small populations compared to the use of standard weighting, but still less significant than in the case of using the arithmetic mean. This parameter can be used in public opinion polls, if they are conducted on different groups, where minority communities could be marginalised. Market trends indicate that the decision-making processes of consumers are becoming increasingly multifaceted and are tending towards niche markets [5]. The same problem may apply to companies where individual departments are highly diverse and whose influence on the decisions of the management board is in proportion to the number of staff they have.

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Received 10 February 2017
Accepted 5 May 2017