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THERMAL INSTABILITY OF WALTERS' (MODEL B′) ELASTICO-VISCOUS COMPRESSIBLE FLUID IN THE PRESENCE OF HALL CURRENT

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Abstract: The thermal instability of a compressible elastico-viscous fluid is examined for viscoelastic polymeric solutions in the presence of uniform vertical magnetic field to include the Hall-current. These solutions are known as Walters' (model B′) fluids and their rheology is approximated by the Walters' (model B′) constitutive relations, proposed by WALTERS [12]. It is found that the stability criterion is independent of the effects of viscosity and viscoelasticity and is dependent on the magnitude of the magnetic field and Hall current. The magnetic field is found to stabilize a certain wave number range of the unstable configuration. The Hall current has destabilizing and stabilizing effects on the system.

NOMENCLATURE

C_p	– specific heat of fluid at constant pressure,
C_v	– specific heat of fluid at constant volume,
M	– dimensionless number accounting for Hall current,
$\vec{g}\,(0, 0, -g)$	– gravitational field,
$\vec{H}\,(0, 0, H)$	– uniform vertical magnetic field,
$\vec{h}\,(h_x, h_y, h_z)$	– perturbation in magnetic field
k	– wave number,
k_x, k_y	– wave numbers in x- and y-directions,
N_e	– electron number density,
n	– stability parameter,
ρ	– fluid density,
d	– depth of fluid layer,
p	– pressure,
R	– Rayleigh number,
Rc	– critical Rayleigh number,
T	– temperature,

t	– time coordinate,
N_Q	– Chandrasekhar number,
$\vec{v}\,(u, v, w)$	– velocity of fluid,
$x(x, y, z)$	– space coordinate,
β	– temperature gradient,
δp	– perturbation in pressure,
$\delta\rho$	– perturbation in density,
η	– electrical resistivity,
θ	– perturbation at temperature T,
μ	– fluid viscosity,
υ	– kinematic viscosity,
υ'	– kinematic viscoelasticity,
ζ	– component of vorticity,
ξ	– current density.

1. INTRODUCTION

The theoretical and experimental results of the onset of thermal instability in a fluid layer under conditions of varying hydrodynamic and hydromagnetic stability has been treated in detail by CHANDRASEKHAR [1]. GUPTA [2] studied the thermal instability of fluid in the presence of Hall currents. The Boussinesq approximation was used in all the above studies. This approximation is well justified in the case of incompressible fluids.

When the fluids are compressible, the equations governing the system become quite complicated. To simplify them, Boussinesq tried to justify the approximation for compressible fluids for density variations arising principally from thermal effects. SPIEGEL and VERONIS [3] simplified the set of equations governing the flow of compressible fluids under the following assumptions:

(a) The vertical dimension of the fluid is much less than any scale height, as defined by SPIEGEL and VERONIS [3];

(b) The motion-induced perturbations in density and pressure do not exceed, in order of magnitude, their total static variations.

Under the above approximations, SPIEGEL and VERONIS [3] showed that the equations governing convection in a compressible fluid are formally equivalent to those for an incompressible fluid if the static temperature gradient is replaced by its excess over the adiabatic one and C_v is replaced by C_p; where C_v and C_p are the specific heats at constant volume and constant pressure, respectively.

Generally, the magnetic field has a stabilizing effect on the instability, but there are a few exceptions. For example, KENT [4] has studied the effect of a horizontal magnetic field which varies in the vertical direction on the stability of parallel flows and has shown that the system is unstable under certain conditions, while in the absence of magnetic field the system is known to be stable. In stellar atmospheres and interiors, the magnetic field may be (and quite often is) variable and may altogether alter the nature of the instability. Coriolis force also plays an important role in the stability of the system.

The theory of fluids with non-linear constitutive equations, so called non-Newtonian fluids was started by REINER [5] for compressible fluids, RIVLIN [6] for incompressible materials and broadened and elaborated by ERICKSEN [7], TRUESDELL and NOLL [8] and many others. Polymer solutions and polymer melts, which provide the most common examples of non-Newtonian fluids, are usually quite viscous; it is important to note that observable non-Newtonian effects occur at fairly low Reynolds number, where inertia terms have little or no effect upon the flow of fluid. There is a vast variety of non-Newtonian fluids. The examples of such substances in the chemical laboratories and industries include polymers, synthetic lattices, protein solutions, special soap solution etc., and those encountered in daily life include asphalts, paints, pinch, starch suspensions, marine glue and certain honeys.

With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations of such fluids are desirable. FREDRICKSEN [9] has given a good review of non-Newtonian fluids whereas JOSEPH [10] has also considered the stability of viscoelastic fluids. There are many viscoelastic fluids which cannot be characterized either by Maxwell's constitutive relations or by Oldroyd's constitutive relations. One of such viscoelastic fluids is Walters' (model B′) fluid. WALTERS [11] has proposed a constitutive equation for such type of elastico-viscous fluids. Many other research workers have paid attention towards the study of Walters' (model B′) fluid. The mixture of polymethyl methacrylate and pyridine at 25 °C containing 30.5 grams of polymers per litre behaves in a very similar way as the Walters' (model B′) viscoelastic fluid proposed by WALTERS [12]. This class of fluids is used in the manufacture of parts of space cafts, aeroplane, tyres, beltconveyors, rops, cushions, seats, foams, plastics, engineering equipment, etc.

SHARMA and KANGO [13] have studied the stability of two superposed Walters' (model B′) viscoelastic fluids in the presence of suspended particles and variable magnetic field in porous medium. SHARMA and KUMAR [14] have studied the Rayleigh–Taylor instability of stratified Walters' (model B′) in the presence of suspended particles and variable horizontal magnetic field and have found that the criteria determining stability are independent of the effects of viscosity and viscoelasticity. The magnetic field stabilizes the system. The viscoelasticity of the medium has damping effects on the growth rates but has enhancing effects for certain ranges of the wave numbers. SHARMA and KUMAR [15] have studied magnetogravitational instability of a thermally conducting rotating Walters' (model B′) fluid with Hall current. SHARMA and AGGARWAL [16] have studied the effect of compressibility and suspended particles on thermal convection in a Walters' B′ elastico-viscous fluid in hydromagnetics. KUMAR and SHARMA [17] have studied the effect of suspended particles on thermal convection in a viscoelastic Walters' (model B′) fluids in hydromagnetics. KUMAR et al. [18] have studied the instability of streaming Walters' (model B′) fluid in porous medium. RANA and KUMAR [19] have studied the thermal instability of elastico-viscous rotating fluid permeating with suspended particles under variable gravity field in porous medium.

It is this class of elastico-viscous fluids we are interested in particularly to study the effect of Hall current on the thermal instability of Walters' (model B′) compressible fluid pervaded by a uniform horizontal magnetic field in addition to a constant gravity field; which is important in ground water hydrology, chemical engineering, modern technology and industries. This aspect forms the subject of the present paper.

2. FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Let T_{ij}, τ_{ij}, e_{ij}, δ_{ij}, p, v_i, x_i, μ and μ' denote, respectively, the stress tensor, shear stress tensor, rate-of-strain tensor, Kronecker delta, scalar pressure, velocity vector, position vector, viscosity and viscoelasticity. Then, Walters' (model B′) elastico-viscous fluid is described by the constitutive relations

$$\left.\begin{aligned} T_{ij} &= -p\delta_{ij} + \tau_{ij}, \\ \tau_{ij} &= 2\left(\mu - \mu' \frac{d}{dt}\right) e_{ij}, \\ e_{ij} &= \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right). \end{aligned}\right\} \tag{1}$$

Consider an infinite horizontal compressible elastico-viscous and finitely (electrically) conducting Walters' (model B′) fluid layer of depth d in which a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained. Consider Cartesian coordinates (x, y, z) with origin on the lower boundary $z = 0$ and the z-axis perpendicular to it along the vertical. A gravitational field $\vec{g}\,(0, 0, -g)$ and uniform vertical magnetic field $\vec{H}\,(0, 0, H)$ pervade the system.

Let ρ, p, ρ_m, T, α, k_T, $\kappa_T\left(=\frac{k_T}{\rho_m C_p}\right)$, g, $\upsilon\left(=\frac{\mu}{\rho_m}\right)$, $\upsilon'\left(=\frac{\mu'}{\rho_m}\right)$, $\vec{v}$, η, N_e and e denote, respectively, the density, scalar pressure, constant spatial average of density, temperature, thermal coefficient of expansion, thermal conductivity, thermal diffusivity, gravitational acceleration, kinematic viscosity, kinematic viscoelasticity, fluid velocity (zero initially), resistivity, electron number density and electron charge.

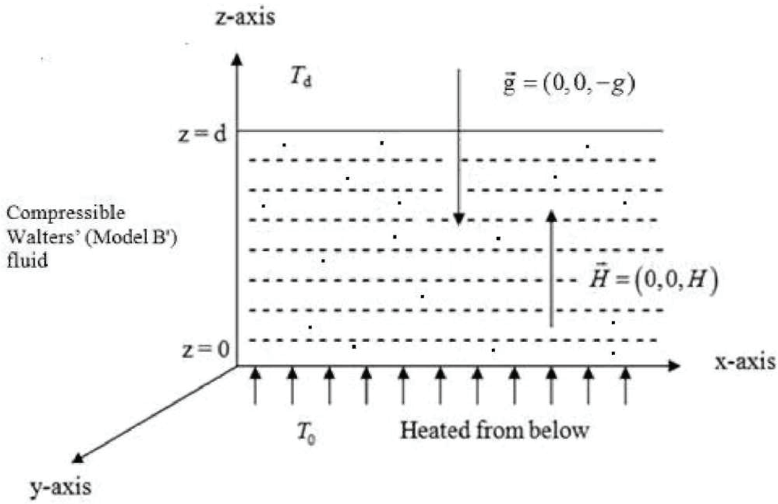

Fig. 1. Geometrical configuration

Then the equations expressing the conservation of momentum, mass, heat and the equation of state are

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{1}{\rho_m}\nabla p + \vec{g}\frac{\rho}{\rho_m} + \frac{1}{4\pi\rho_m}(\nabla \times \vec{H}) \times \vec{H} + \left(\upsilon - \upsilon' \frac{\partial}{\partial t}\right)\nabla^2 \vec{v}, \tag{2}$$

$$\frac{\partial \rho}{\partial t} + (\vec{v} \cdot \nabla)\rho + \rho(\nabla \cdot \vec{v}) = 0, \tag{3}$$

$$\frac{\partial T}{\partial t} + (\vec{v} \cdot \nabla)T = \kappa_T \nabla^2 T, \tag{4}$$

$$\rho = \rho_m[1 - \alpha(T - T_m)], \tag{5}$$

where T_m is the temperature at which $\rho = \rho_m$. The magnetic permeability has been taken to be unity.

Maxwell's equations yield

$$\frac{\partial \vec{H}}{\partial t} = \nabla \times (\vec{v} \times \vec{H}) + \eta \nabla^2 \vec{H} - \frac{1}{4\pi N_e e}\nabla \times [(\nabla \times \vec{H}) \times \vec{H}], \tag{6}$$

$$\nabla \cdot \vec{H} = 0\,. \tag{7}$$

Initially, $\vec{v} = (0, 0, 0)$, $\vec{H} = (0, 0, H)$, $T = T(z)$, $p = p(z)$, $\rho = \rho(z)$, where, following SPIEGEL and VERONIS [3], we have

$$\left.\begin{aligned}
&T(z) = -\beta z + T_0,\\
&p(z) = p_m - g\int_0^z (\rho_m + \rho_0)dz,\\
&K_m = \left(\frac{1\partial\rho}{\rho\partial p}\right)_m,\\
&\rho(z) = \rho_m[1 - \alpha_m(T - T_m) + K_m(p - p_m)],\\
&\alpha_m = -\left(\frac{1\partial\rho}{\rho\partial p}\right)_m (= \alpha, \text{say}).
\end{aligned}\right\} \tag{8}$$

SPIEGEL and VERONIS [3] expressed any state variable, say X, in the form $X = X_m + X_0(z) + X'(x, y, z, t)$, where X_m is the constant spatial distribution of X, X_0 is the variation in X in the absence of motion and $X'(x, y, z, t)$ is the perturbation in X due to the motion of the fluid. Thus p_m and ρ_m are the constant spatial distributions of p and ρ, and ρ_0 and T_0 are the density and temperature of the fluid at the lower boundary $z = 0$. The pressure and temperature have been shown to be related by relations of the form (9) (SPIEGEL and VERONIS [3]), which have been obtained from the basic equations by integration.

Let δp, $\delta\rho$, θ, $\vec{h}\,(h_x, h_y, h_z)$ and $\vec{v}\,(u, v, w)$ denote, respectively, the perturbations in pressure p, density ρ, temperature T, magnetic field $\vec{H}\,(0, 0, H)$ and velocity $(0, 0, 0)$. The change in density $\delta\rho$, caused mainly by the perturbation θ in temperature, is given by

$$\delta\rho = -\rho_m\alpha\theta. \tag{9}$$

Then the linearized hydromagnetic perturbation equations appropriate for the problem, under the SPIEGEL and VERONIS [3], are

$$\frac{\partial\vec{v}}{\partial t} = -\frac{1}{\rho_m}\nabla\delta p + \frac{1}{4\pi\rho_m}(\nabla\times\vec{H})\times\vec{H} - \vec{g}\alpha\theta + \left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)\nabla^2\vec{v}, \tag{10}$$

$$\nabla\cdot\vec{v} = 0, \tag{11}$$

$$\frac{\partial\theta}{\partial t} = \left(\beta - \frac{g}{C_p}\right)w + \kappa_T\nabla^2\theta, \tag{12}$$

$$\frac{\partial\vec{h}}{\partial t} = \nabla\times(\vec{v}\times\vec{H}) + \eta\nabla^2\vec{h} - \frac{1}{4\pi N_e e}\nabla\times[(\nabla\times\vec{h})\times\vec{H}]. \tag{13}$$

$$\nabla \cdot \vec{h} = 0, \tag{14}$$

where $\frac{g}{C_p}$ is the adiabatic gradient. Under the above mentioned approximations, as shown by SPIEGEL and VERONIS [3], the equations governing convection in a compressible fluid have been written as formally equivalent to those for an incompressible fluid, except that the static temperature gradient β is replaced by $\beta - \frac{g}{C_p}$.

3. DISPERSION RELATION

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[w, \theta, h_z, \zeta, \xi] = [W(z), \Theta(z), K(z), Z(z), X(z)]\exp(ik_x x + ik_y y + nt), \tag{15}$$

where k_x and k_y are the wave numbers along the x- and y-directions. $k = (k_x^2 + k_y^2)^{\frac{1}{2}}$ is the resultant wave number and n is, in general, a complex constant. $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ are the z-components of the vorticity and current density, respectively.

Expressing the coordinates (x, y, z) in the new unit of length d and letting $a = kd$, $\sigma = \frac{nd^2}{\upsilon}$, $p_1 = \frac{\upsilon}{\kappa_T}$, $p_2 = \frac{\upsilon}{\eta}$, $\upsilon'' = \frac{\upsilon'}{d^2}$ and $D = \frac{d}{dz}$, equations (10)–(14), with the help of (9) and (15), in non-dimensional form, become

$$[\sigma - (1 - \upsilon''\sigma)(D^2 - a^2)](D^2 - a^2)W + \frac{g\alpha d^2}{\upsilon}a^2\Theta - \frac{Hd}{4\pi\rho_m\upsilon}(D^2 - a^2)DK = 0, \tag{16}$$

$$[\sigma - (1 - \upsilon''\sigma)(D^2 - a^2)]Z = \frac{Hd}{4\pi\rho_m\upsilon}DX, \tag{17}$$

$$(D^2 - a^2 - p_2\sigma)X = -\frac{Hd}{\eta}DZ - \frac{Hd}{4\pi N_e e\eta d}(D^2 - a^2)DK, \tag{18}$$

$$(D^2 - a^2 - p_2\sigma)K = -\frac{Hd}{\eta}DW + \frac{Hd}{4\pi N_e\eta}DX, \tag{19}$$

$$(D^2 - a^2 - p_1\sigma)\Theta = -\frac{d^2}{\kappa_T}\left(\beta - \frac{g}{C_p}\right)W. \tag{20}$$

Eliminating Θ, K, X and Z between (17)–(20), we obtain

$$\left.\begin{aligned}
&[(D^2-a^2-p_2\sigma)^2\{\sigma-(1-\upsilon''\sigma)(D^2-a^2)\}+N_Q D^2(D^2-a^2-p_2\sigma)\\
&+MD^2(D^2-a^2)\{\sigma-(1-\upsilon''\sigma)(D^2-a^2)\}]\\
&\times\left[(D^2-a^2)(D^2-a^2-p_1\sigma)\{\sigma-(1-\upsilon''\sigma)(D^2-a^2)\}-Ra^2\left(\frac{G'-1}{G'}\right)\right]W\\
&+N_Q(D^2-a^2)(D^2-a^2-p_1\sigma)\\
&[\rangle\{\sigma-(1-\upsilon''\sigma)(D^2-a^2)\}(D^2-a^2-p_2\sigma)+N_Q D^2\langle D^2]W=0,
\end{aligned}\right\} \tag{21}$$

where $N_Q = \dfrac{H^2 d^2}{4\pi\rho_m \upsilon\eta}$ is the Chandrasekhar number, $R = \dfrac{g\alpha\beta d^4}{\upsilon\kappa_T}$ is the Rayleight number, $M = \left(\dfrac{H}{4\pi N_e e\eta}\right)^2$ is the non-dimensional number accounting for Hall current and $G' = \dfrac{C_p\beta}{g}$. Consider the case where both the boundaries are free and the medium adjoining the fluid is non-conducting. The appropriate boundary conditions for this case are (CHANDRASEKHAR [1]).

$$W = D^2W = 0, \quad \Theta = 0, \quad DZ = 0, \quad X = 0 \quad \text{at} \quad z = 0 \text{ and } 1 \tag{22}$$

and $\vec{h}$ are continuous.

The case of two free boundaries, although rather artificial, is the most appropriate for the stellar atmospheres (SPIEGEL [20]). Using the boundary conditions (22), one can show that all the even-order derivatives of W must vanish for z = 0 and 1, and hence the proper solution of (21) characterizing the lowest mode is

$$W = W_0 \sin \pi z\,, \tag{23}$$

where W_0 is a constant. Substituting (23) into (21) and letting $a^2 = \pi^2 b^2$, $R_1 = \dfrac{R}{\pi^4}$, $Q_1 = \dfrac{N_Q}{\pi^4}$, $\bar{\upsilon} = \pi^2\upsilon''$, and $i\sigma_1 = \dfrac{\sigma}{\pi^2}$, we obtain the dispersion relation

$$R=\left(\frac{G'}{G'-1}\right)\Bigg[\frac{1+b}{b}\{i\sigma_1+(1+b)(1-i\sigma_1\overline{\upsilon})\}(1+b+ip_1\sigma_1)$$
$$+\frac{1}{b}\{Q_1(1+b)(1+b+ip_1\sigma_1)$$
$$[\rangle i\sigma_1+(1+b)(1-i\sigma_1\overline{\upsilon})\langle\rangle 1+b+ip_2\sigma_1\langle+Q_1]\}$$
$$\times[(1+b+ip_2\sigma_1)^2\{i\sigma_1+(1+b)(1-i\sigma_1\overline{\upsilon})\}$$
$$+Q_1(1+b+ip_2\sigma_1)+M(1+b)\{i\sigma_1+(1+b)(1-i\sigma_1\overline{\upsilon})\}]^{-1}\Bigg]. \tag{24}$$

4. STATIONARY CONVECTION

When instability sets in as stationary convection, the marginal state will be characterized by $\sigma=0$. Putting $\sigma=0$, the dispersion relation (24) reduces to

$$R_1=\frac{G'}{G'-1}\left[\frac{(1+b)^3}{b}+\frac{1}{b}\{Q_1(1+b)\langle(1+b)^2+Q_1\rangle\}\times\{(1+b)^2+Q_1+M(1+b)\}^{-1}\right], \tag{25}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number b and the parameters Q_1, M and G'. For fixed Q_1 and M, let G' (accounting for the compressibility effects) also be kept fixed. Then we find that

$$\overline{R}_c=\frac{G'}{G'-1}R_c, \tag{26}$$

where $\overline{R}_c$ and R_c denote, respectively, the critical Rayleigh numbers in the presence and absence of compressibility. The effect of compressibility is thus to postpone the onset of thermal instability. Hence compressibility has a stabilizing effect. $G' > 1$ is relevant here. The cases $G' < 1$ and $G' = 1$ correspond to negative and infinite values of the critical Rayleigh numbers in the presence of compressibility, which are not relevant in the present study.

It follows from (25) that

$$\frac{dR_1}{dQ_1}==\left(\frac{G'}{1-G'}\right)\frac{1}{b}\{(1+b)\langle(1+b)^2+Q_1\rangle+Q_1(1+b)\}\times\{(1+b)^2+Q_1+M(1+b)\}^{-2}. \tag{27}$$

This shows that the magnetic field has a stabilizing effect for all wave number range for $G' < 1$ and destabilizing effect for $G' > 1$. In the presence of Hall current for $G' > 1$, the magnetic field may have a stabilizing or destabilizing effect.

Equation (25) also yields

$$\frac{dR_1}{dM}=\left(\frac{G'}{1-G'}\right)\frac{1}{b}Q_1(1+b)^2\langle(1+b)^2+Q_1\rangle\langle(1+b)^2+Q_1+M(1+b)\rangle^{-2}\,. \qquad (28)$$

It is evident from (28) that $\frac{dR_1}{dM}$ is always positive for $G' < 1$, implying thereby the stabilizing effect of Hall currents and $\frac{dR_1}{dM}$ is always negative for $G' > 1$, implying thereby the destabilizing effect of Hall currents for all wave numbers.

We thus conclude that the presence of viscoelasticity does not affect the stability or instability of the system.

5. STABILITY OF THE SYSTEM AND OSCILLATORY MODES

Multiplying (16) by W^*, the complex conjugate of W, integrating over the range of z, and making use of (17)–(20), we obtain

$$[\sigma I_1+(1-\upsilon''\sigma)I_2]+\frac{C_p\alpha\kappa_T a^2}{\upsilon(1-G')}(I_3+p_1\sigma^* I_4)+\frac{\eta d^2}{4\pi\rho_m\upsilon}(I_5+p_2\sigma I_6)$$

$$+\frac{\eta}{4\pi\rho_m\upsilon}(I_7+p_2\sigma^* I_8)+d^2[\sigma^* I_{10}+(1-\upsilon''\sigma^*)I_9]=0, \qquad (29)$$

where

$$I_1=\int_0^1(|DW|^2+a^2|W|)dz,$$

$$I_2=\int_0^1(|D^2W|^2+a^4|W|^2+2a^2|DW|^2)dz,$$

$$I_3=\int_0^1(|D\Theta|^2+a^2|\Theta|)dz,$$

$$I_4=\int_0^1|\Theta|^2\,dz,$$

$$I_5=\int_0^1(|DX|^2+a^2|X|^2)dz,$$

$$I_6=\int_0^1|X|^2\,dz,$$

$$I_7 = \int_0^1 (|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2)dz,$$

$$I_8 = \int_0^1 (|DK|^2 + a^2|K|)dz,$$

$$I_9 = \int_0^1 (|DZ|^2 + a^2|Z|)dz,$$

$$I_{10} = \int_0^1 |Z|^2\, dz, \tag{30}$$

which are all positive definite. The real and imaginary parts of (29) give

$$\sigma_r\left[I_1 - \upsilon'' I_2 + \frac{C_p \alpha \kappa_T a^2}{\upsilon(1-G')} p_1 I_4 + \frac{\eta p_2}{4\pi\rho_m \upsilon}(I_8 + d^2 I_6) + d^2(I_{10} - \upsilon'' I_9)\right]$$
$$= -\left[I_2 + \frac{C_p \alpha \kappa_T a^2}{\upsilon(1-G')} I_3 + \frac{\eta}{4\pi\rho_m \upsilon}(I_7 + d^2 I_5) + d^2 I_9\right], \tag{31}$$

and

$$\sigma_i\left[I_1 - \upsilon'' I_2 + \frac{C_p \alpha \kappa_T a^2}{\upsilon(1-G')} p_1 I_4 + \frac{\eta p_2}{4\pi\rho_m \upsilon}(d^2 I_6 - I_8) + d^2(I_{10} - \upsilon'' I_9)\right] = 0. \tag{32}$$

It follows from (31) that σ_r is negative if $G' < 1$, $I_1 - \upsilon'' I_2 > 0$ and $I_{10} - \upsilon'' I_9 > 0$. The system is therefore stable for $\frac{C_p \beta}{g} < 1$, $d^2 I_1 > \upsilon' I_2$ and $d^2 I_{10} > \upsilon' I_9$. It is evident from (32) that σ_i may be zero or non-zero. Thus the modes may be non-oscillatory or oscillatory. The oscillatory modes are introduced owing to the presence of a magnetic field (and hence Hall current). In the absence of magnetic field, the oscillatory modes are not allowed for $\frac{C_p \beta}{g} > 1$, but the presence of a magnetic field and Hall currents introduce oscillatory modes in the system.

REFERENCES

[1] CHANDRASEKHAR S., *Hydrodynamic and Hydromagnetic Stability*, Dover Publications, New York, 1981.

[2] GUPTA A.S., *Hall effects on thermal instability*, Rev. Roumaine Math. Pures Appl., 1967, 12, 665–677.

[3] SPIEGEL E.A., VERONIS G., *On the Boussinesq approximation for a compressible fluid*, Astrophys. J., 1960, 131, 442–447.

[4] KENT A., *Instability of laminar flow of a magnetofluid*, Phys. Fluids, 1966, 9, 1286–1289.

[5] REINER M., *A mathematical theory of dilatancy*, American J. Math., 1945, 67, 350–362.

[6] RIVLIN R.S., *The hydrodynamics of non-Newtonian fluids. I*, Proc. Roy. Soc., London, 1948, A193, 260–281.

[7] ERICKSEN J.L., *Characterstic surfaces of equations of motion of non-Newtonian fluids*, ZAMP, 1953, 4, 260–267.

[8] TRUESDELL C., NOLL W, *The non linear field theories of mechanics*, (Handbuch der Physik III/3), Berlin–Heidelberg–New York, Springer, 1965.

[9] FREDRICKSEN A.G., *Principles and Applications of Rheology*, Prentice–Hall Inc., New Jersey, 1964.

[10] JOSEPH D.D., *Stability of fluid motion II*, Springer-Verlag, New York, 1976.

[11] WALTERS K., *The motion of an elastico-viscous liquid contained between concentric spheres*, Quart. J. Mech. Applied Math., 1960, 13, 325–333.

[12] WALTERS K., *Non-Newtonian effects in some elastico-viscous liquids whose behaviour at small rates of shear is characterized by a general linear equation of state*, Quart. J. Mech. Applied. Math., 1962, 15, 63–76.

[13] SHARMA R.C., KANGO S.K., *Stability of two superposed Walters'* (*model B'*) *viscoelastic fluids in the presence of suspended particles and variable magnetic field in porous medium*, Appl. Mech. Engng., 1999, 4(2), 255–269.

[14] SHARMA V., KUMAR S., *Rayleigh–Taylor instability of stratified Walters'* (*model B'*) *fluid in the presence of a variable horizontal magnetic field and suspended particles*, Journal of Indian Math. Soc., 2001, 68, 209–219.

[15] SHARMA V., KUMAR S., *Magnetogravitational instability of a thermally conducting rotating Walters'* (*model B'*) *fluid with Hall current*, Proc. Nat. Acad. Sci. India, 2000, 70(A)I, 87–97.

[16] SHARMA R. C., AGGARWAL A. K., *Effect of compressibility and suspended particles on thermal convection in a Walters' B' elastico-viscous fluid in hydromagnetics*, Int. J. of Appl. Mech. Eng., 2006, 2, 391–399.

[17] KUMAR S., SHARMA V., *Effect of suspended particles on thermal convection in a viscoelastic Walters'* (*model B') fluid in hydromagnetics*, Shekher (New Series) Int. J. Maths., 2009, 1, 1–11.

[18] KUMAR S., GUPTA D., JASWAL R., *The instability of streaming Walters'* (*model B'*) *fluid in porous medium*, Int. J. Math. Sci. & Engg. Appls., 2010, 4, 355–367.

[19] RANA G.C., KUMAR S., *Thermal instability of Rivlin–Ericksen elastico-viscous rotating fluid permeating with suspended particles under variable gravity field in porous medium*, Studia Geotechnica et Mechanica, 2010, XXXII, No. 4, 39–54.

[20] VERONIS G., *Convective instability in a compressible atmosphere. I*, Astrophys. J., 1965, 141, 1068–1090.