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Experimental study of an anomalous hollow beam with orbital angular momentum

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Anomalous hollow beam has potential applications in the optical trapping and free space optical communications, *etc.* It is noted that, thus far, although a large number of studies have been carried out in this field, most of them are theoretical studies and only quite a few cases have experimental results. Here, we experimentally study the generating of anomalous hollow beam carrying orbital angular momentum, and measure its topological charge. We show that the number of dark rings in the Fourier transform of intensity patterns is equal to the topological charge. The experimental results agree well with the simulations.

Keywords: anomalous hollow beam, orbital angular momentum, focused intensity, Fourier transform.

1. Introduction

Over the past decades, vortex beams with spiral phase have attracted much attention and have found many applications in varied fields, such as free-space optical communications [1–4], optical trapping [5], quantum information processing [6, 7], optical micromanipulation [8], super-resolution microscope [9]. Until now, the research of vortex beams is still a hot topic [10–14]. A vortex beam with an azimuthal phase $\exp(iM\varphi)$ can carry an orbital angular momentum (OAM) of $M\hbar$ per photon (where M denotes the topological charge (TC), φ is the azimuthal angle about the optical axial, and $\hbar$ denotes reduced Plank's constant). In theory, many models can be used to describe the vortex beams, such as Laguerre–Gaussian beams, high-order Bessel–Gaussian beams, *etc.* [15]. In many applications of vortex beams, we need to know their TC exactly. Therefore, recently, lots of methods have been reported to determine the TC, such as analyzing interferometric pattern of the vortex beam [16, 17], or direct Fourier transform (FT) of the diffracted intensity of the vortex beam [18–20].

In 2005, WU *et al.* reported an anomalous hollow electron beam with an elliptical solid core to study the transverse instability for the first time, which provided a useful tool for studying the linear and nonlinear particle dynamics in the storage ring [21]. In 2007 and 2008, CAI *et al.* introduced two theoretical models to describe such beams, named an anomalous hollow beam (AHB) [22, 23]. Since then, due to its special properties, extensively theoretical studies in AHB have been carried out, such as propagation properties passing through turbulent atmosphere [24, 25], nonparaxial propagation [26, 27], uniaxial crystals [28], fractional Fourier transforming optical system [29], misaligned optical system [30], highly focused lenses [31] and isotropic nonlocal media [32]. More recently, ZHAO *et al.* introduced a theoretical model to describe an anomalous hollow vortex beam and studied its statistical properties [33]. However, to the best of our knowledge, the experimental generation of the AHB with OAM has not been reported yet.

In this paper, we report on an experimental study of generating AHB with OAM and measuring its TC through operating FT of the focused intensity. The results show that the number of dark rings in the FT is equal to the TC.

2. Theory

The electric field of an AHB with elliptical symmetry at $z = 0$ can be expressed as a superposition of astigmatic Gaussian modes and astigmatic doughnut modes as follows [22]:

$$E(x, y, 0) = \left(-2 + \frac{8x^2}{w_{0x}^2} + \frac{8y^2}{w_{0y}^2}\right) \exp\left(-\frac{x^2}{w_{0x}^2} - \frac{y^2}{w_{0y}^2}\right) \tag{1}$$

The electric field of an AHB with OAM at the input plane can be defined as follows:

$$E(x, y, 0) = \left(-2 + \frac{8x^2}{w_{0x}^2} + \frac{8y^2}{w_{0y}^2}\right) \exp\left(-\frac{x^2}{w_{0x}^2} - \frac{y^2}{w_{0y}^2}\right) (x + iy)^M \tag{2}$$

where w_{0x} and w_{0y} are the beam waist widths of an astigmatic Gaussian mode in the x and y directions, respectively, and M is the TC.

Within the framework of paraxial approximation, the propagation of the AHB with OAM through a paraxial *ABCD* optical system can be described with the help of the Collins integral formula [34]. The analytical expressions for an AHB carrying OAM in the output plane can be written as [33]:

$$\begin{aligned}
E(x_1, y_1, z) = {} & \frac{ik}{2\pi B}\exp(-ikz)\exp\left[-\frac{ikD}{2B}(x_1^2+y_1^2)\right] \\
& \times \exp\left(\frac{q_x^2}{p_x}+\frac{q_y^2}{p_y}\right)\frac{\pi}{\sqrt{p_x p_y}}\sum_{l=0}^{M}\frac{M!\,i^l}{l!(M-l)!} \\
& \times\left\{-2\left[n_{11}!\left(\frac{q_x}{p_y}\right)^{n_{11}}\sum_{k=0}^{E(n_{11}/2)}\frac{1}{(n_{11}-2k)!\,k!}\left(\frac{p_x}{4q_x^2}\right)^k\right]\right. \\
& \times\left[n_{12}!\left(\frac{q_y}{p_y}\right)^{n_{12}}\sum_{k=0}^{E(n_{12}/2)}\frac{1}{(n_{12}-2k)!\,k!}\left(\frac{p_y}{4q_y^2}\right)^k\right] \\
& +\frac{8}{w_{0x}^2}\left[n_{21}!\left(\frac{q_x}{p_x}\right)^{n_{21}}\sum_{k=0}^{E(n_{21}/2)}\frac{1}{(n_{21}-2k)!\,k!}\left(\frac{p_x}{4q_x^2}\right)^k\right] \\
& \times\left[n_{22}!\left(\frac{q_y}{p_y}\right)^{n_{22}}\sum_{k=0}^{E(n_{22}/2)}\frac{1}{(n_{22}-2k)!\,k!}\left(\frac{p_y}{4q_y^2}\right)^k\right] \\
& +\frac{8}{w_{0y}^2}\left[n_{31}!\left(\frac{q_x}{p_x}\right)^{n_{31}}\sum_{k=0}^{E(n_{31}/2)}\frac{1}{(n_{31}-2k)!\,k!}\left(\frac{p_x}{4q_x^2}\right)^k\right] \\
& \left.\times\left[n_{32}!\left(\frac{q_y}{p_y}\right)^{n_{32}}\sum_{k=0}^{E(n_{32}/2)}\frac{1}{(n_{32}-2k)!\,k!}\left(\frac{p_y}{4q_y^2}\right)^k\right]\right\}
\end{aligned} \tag{3}$$

where

$$p_x = \frac{1}{w_{0x}^2}+\frac{ikA}{2B} \tag{4a}$$

$$q_x = \frac{ikx_1}{2B} \tag{4b}$$

$$p_y = \frac{1}{w_{0y}^2} + \frac{ikA}{2B} \tag{4c}$$

$$q_y = \frac{iky_1}{2B} \tag{4d}$$

$$n_{11} = n_{31} = M - l \tag{4e}$$

$$n_{12} = n_{22} = l \tag{4f}$$

$$n_{21} = 2 + M - l \tag{4g}$$

$$n_{32} = 2 + l \tag{4h}$$

The transfer matrix for free space propagation along a distance z is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \tag{5}$$

Based on these analytical results, we can calculate the contour graph and the crossline of an AHB with OAM at different propagation distances in free space with $w_{0x} = w_{0y} = 2$ mm, $M = 3$, $\lambda = 632.8$ nm and $z_r = kw_{0x}^2/2$. Figure 1 shows that an AHB carrying OAM has a doughnut profile and maintains its shape during propagation, which is very different from the propagation properties of the AHB having no OAM. For AHB without OAM, the central intensity increases gradually and in the far field, the dark region disappears completely [22].

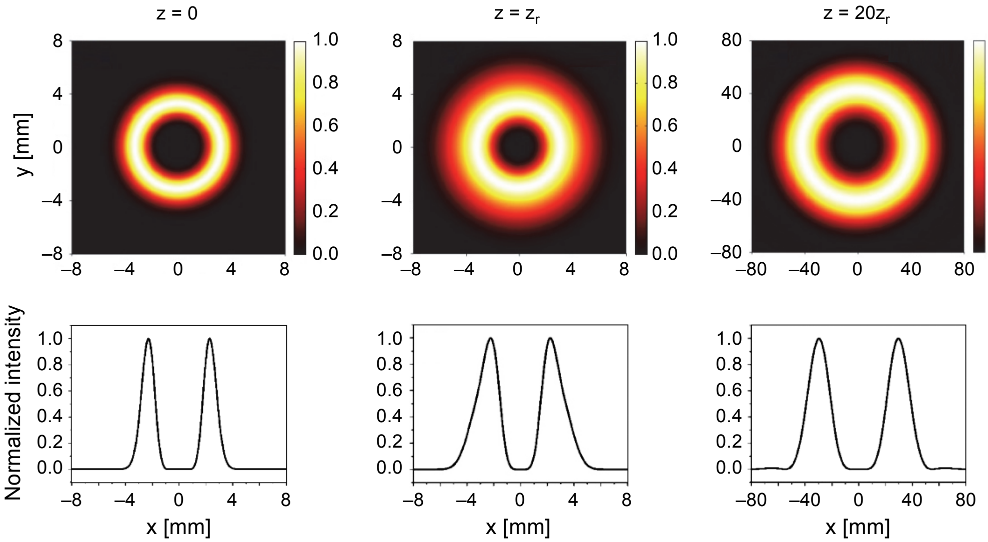

Fig. 1. Contour graph and the crossline distribution of an AHB with OAM at different propagation distances in free space with $w_{0x} = w_{0y} = 2$ mm, $M = 3$, and $\lambda = 632.8$ nm.

3. Experimental results

Now we carry out the experiment to generate an AHB with OAM and measure its topological charge. Figure 2 shows the experimental setup for generating AHB with OAM. The laser beam generated by a He-Ne laser ($\lambda = 632.8$ nm) firstly passes through a beam expander, then goes toward a spatial light modulator (SLM), which acts as a grating pattern designed by the method of computer-generated holograms and controlled by a personal computer (PC1). The first-order diffraction pattern of the beam from the SLM can be regarded as an AHB with OAM. After passing through the SLM, the generated AHB with OAM arrives at the beam profile analyzer, which is used to measure the beam intensity in the source plane ($z = 500$ mm) and focal plane. Figure 3 shows the experimental results of intensity patterns and the corresponding simulations for dif-

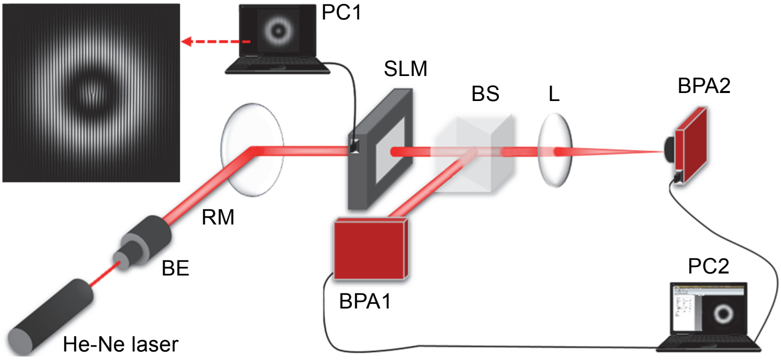

Fig. 2. Experimental setup for generating an AHB with OAM; BE – beam expander, RM – reflecting mirror, PC1, PC2 – personal computers, SLM – spatial light modulator, BS – beam splitter, L – lens, and BPA1, BPA2 – beam profile analyzers.

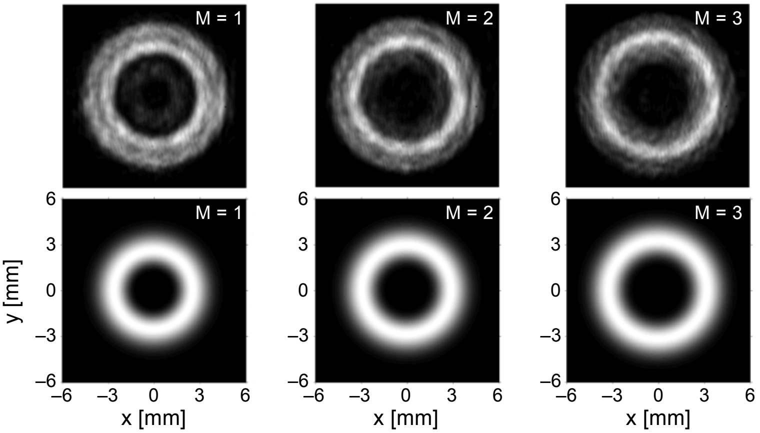

Fig. 3. Experimental results and the corresponding theoretical results of the intensity distribution of the generated AHB with OAM for different TC at the source plane ($z = 500$ mm) with widths $w_{0x} = w_{0y}$ = 2 mm, and $\lambda = 632.8$ nm.

ferent topological charges. From Fig. 3 we can see that the dark center of the AHB with OAM increases with increasing topological charge M. The experimental results agree with the simulations very well.

In order to study the focusing properties of the generated AHB with OAM, a thin lens L with focal length $f = 400$ mm is located behind the SLM in Fig. 2. After passing through the thin lens L, the generated AHB with OAM arrives at the beam profile analyzer (BPA), which is located at the focal plane and is used to measure the focal intensity. At the focal plane, the transfer matrix reads as [35]:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & f \\ -1/f & 1 \end{bmatrix} \tag{6}$$

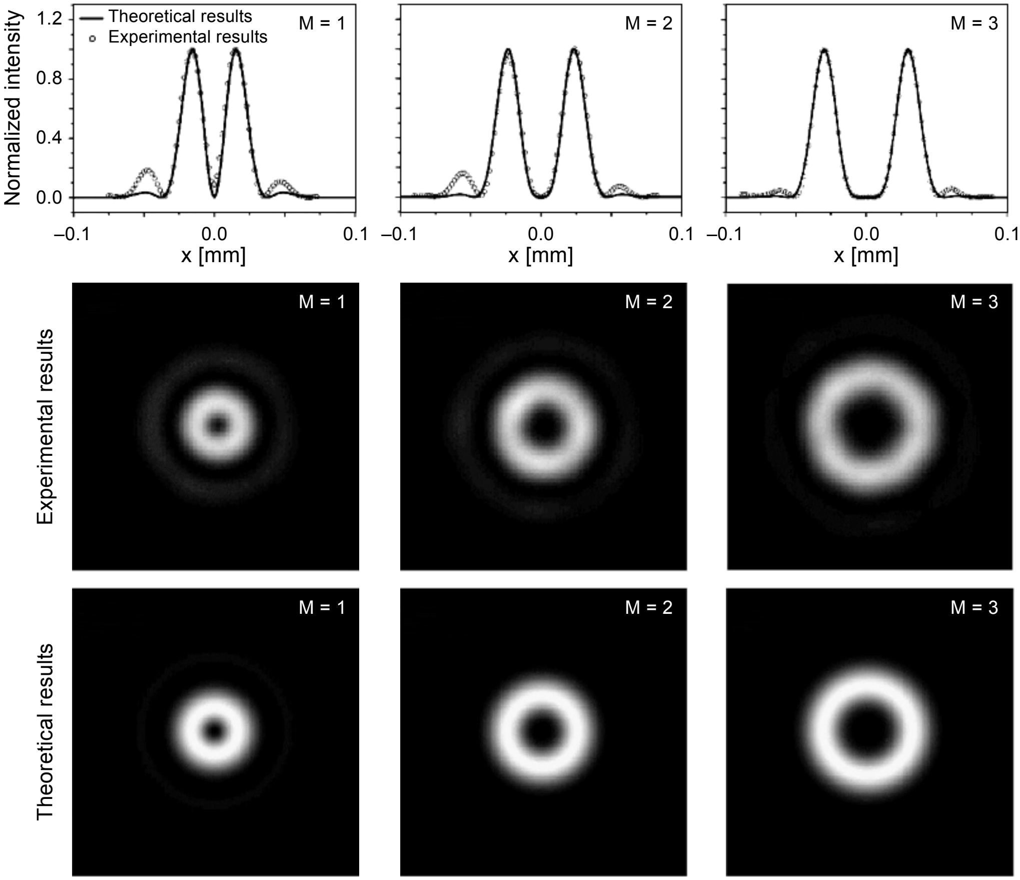

Fig. 4. Experimental results and the corresponding theoretical results of the intensity distribution of the generated AHB with OAM for different TCs at the focal plane $f = 400$ mm with $w_{0x} = w_{0y} = 2$ mm and $\lambda = 632.8$ nm.

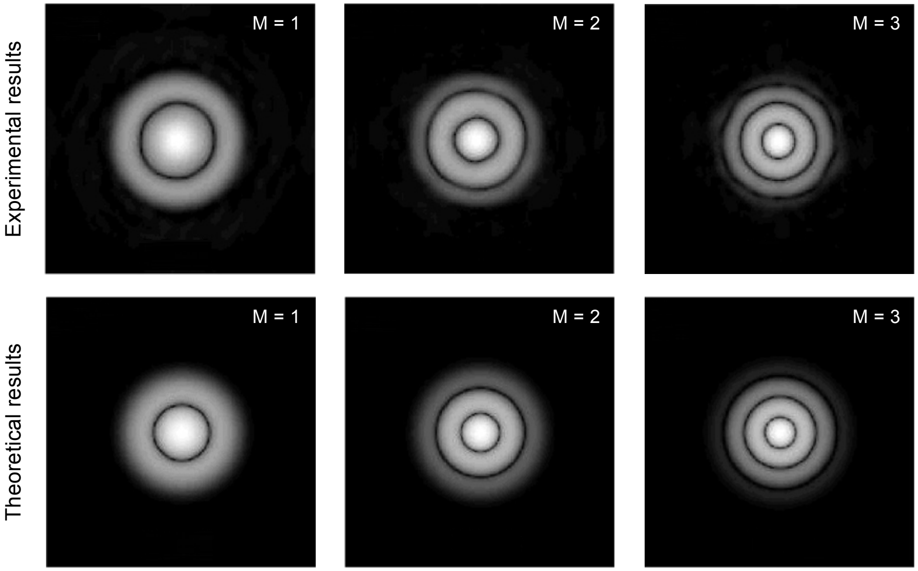

Fig. 5. Experimental results of the focused intensity and the corresponding FT pattern for three different TCs with $w_{0x} = w_{0y} = 2$ mm and $\lambda = 632.8$ nm.

Figure 4 shows our experimental results of the intensity distribution and the corresponding normalized intensity distribution at the crossline (circle dot) of the generated AHB with different values of the OAM at the focal plane $f = 400$ mm with $w_{0x} = w_{0y} = 2$ mm, and $\lambda = 632.8$ nm. From Fig. 4, one can see that the shape of AHB with OAM still has a doughnut profile at the focal plane. Moreover, the size of the beam spot increases with the increasing topological charge M. Figure 4 shows that the experimental results and the simulations coincide with each other very well.

Reference [20] revealed that the number of dark rings in the FT pattern of the intensity of a vortex beam is equal to its TC, thus we can measure the TC of the vortex beam through this convenient method. An experiment is carried out as well to verify if this method is still available to AHB with OAM. Figure 5 shows the experimental and theoretical FT pattern of the focal intensity for three different TCs. We can see that the number of dark rings in the FT is equal to the TC we set. Based on these properties, we can measure the TC of AHB with OAM easily.

4. Conclusion

In summary, we have studied an AHB with OAM both theoretically and experimentally. We found that the dark size of the beam spot increases with the increasing topological charge M from the near field to the far field. Our experimental results are consistent with the theoretical predictions very well. Furthermore, we have demonstrated that the number of dark rings in the FT of intensity patterns is equal to the TC, namely, this method for measuring OAM is still available for AHB.

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