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Effect of circular aperture diffraction on propagation properties of cylindrical vector partially coherent Laguerre–Gaussian beams in turbulent atmosphere

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On the basis of the extended Huygens–Fresnel integral formula, the analytical formulae for the cross -spectral density matrix of the arbitrary cylindrical vector partially coherent Laguerre–Gaussian beams diffracted by a circular aperture in turbulent atmosphere are derived. The average intensity, the degree of polarization, and the degree of coherence of the apertured cylindrical vector partially coherent beams propagating in turbulent atmosphere are investigated. The analyses indicate that both the beam diffraction effect by a circular aperture and the atmospheric turbulence have a great impact on the beam evolution, polarization and coherence properties of the apertured cylindrical vector partially coherent beams.

Keywords: cylindrical vector beams, diffraction, aperture, atmospheric propagation.

1. Introduction

As a typical vector beam with spatially variant state of polarization, cylindrical vector beams have exhibited wide applications in dark-field imaging [1], optical trapping [2], laser processing [3], and all-dielectric nanostructures [4]. The arbitrary cylindrical vector beam is a more interesting and broader beam model than the radially polarized beam. By varying the polarization order, the arbitrary cylindrical vector beam at the source plane can reduce to the radially, azimuthally, anti-vortex and linearly polarized beams [5, 6]. The cylindrical vector Laguerre–Gaussian beams (LGBs) can be obtained by using a spatial light modulator [7], or a digital micromirror device [8].

The effect of the atmospheric turbulence on the propagation properties of the electromagnetic beams has been extensively studied. The polarization properties of electromagnetic Gaussian Schell-model beams propagating through the atmospheric turbulence and the anisotropic non-Kolmogorov turbulence have been investigated [9, 10]. The evolution of electromagnetic spectral Gaussian Schell-model beams in atmosphere has been analyzed [11]. The changes in the states of polarization of random electromagnetic cosh-Gaussian beams in non-Kolmogorov atmospheric turbulence have been discussed [12]. The propagation properties of the radially polarized fully and partially coherent beams in turbulent atmosphere have been presented [13–15]. Previous studies dealt with the propagation of the cylindrical vector fully and partially coherent beams in the turbulent atmosphere [16, 17], but the results were restricted to the unapertured case. The effect of the circular aperture diffraction usually should be considered in the practical applications of laser beams. To the best of our knowledge, the circular aperture diffraction properties of the arbitrary cylindrical vector partially coherent beams propagating in turbulent atmosphere have not been reported elsewhere. In this paper, the analytical formulae for the cross-spectral density (CSD) matrix of the apertured cylindrical vector partially coherent LGBs with arbitrary polarization order in turbulent atmosphere are derived. The average intensity, the degree of polarization, and the degree of coherence of the apertured cylindrical vector partially coherent beams in turbulent atmosphere are discussed in detail.

2. Theoretical modeling

In the Cartesian coordinate system, the z-axis is taken to be the propagation axis. Assuming that a circular aperture with radius a is located at the source plane $z = 0$, the electric field of a cylindrical vector LGB with arbitrary polarization order just behind the aperture reads as [5, 7]

$$\mathbf{E}(x_0, y_0, 0) = E_0\left(\frac{\sqrt{2}\,r_0}{w_0}\right)^{|m|} L_n^{|m|}\left(\frac{2r_0^2}{w_0^2}\right)\exp\left(-\frac{r_0^2}{w_0^2}\right)t(x_0, y_0)$$

$$\times\Big[\cos(m\varphi_0+\phi)\mathbf{e}_x + \sin(m\varphi_0+\phi)\mathbf{e}_y\Big] \tag{1}$$

$$t(x_0, y_0) = \begin{cases} 1, & x_0^2 + y_0^2 \le a^2 \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

where $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unit vectors in the x and y directions, respectively; $r_0 = \sqrt{x_0^2 + y_0^2}$ and $\varphi_0 = \tan^{-1}(y_0/x_0)$ are the transverse coordinates in the cylindrical coordinate system; m is the polarization order; ϕ is the initial polarization angle; E_0 is an amplitude constant; w_0 is the waist width of the Gaussian term; $L_n^{|m|}(\cdot)$ is the Laguerre polynomial of radial order n and azimuthal order $|m|$ (the azimuthal order equals the absolute value of the polarization order here); $t(x_0, y_0)$ denotes the aperture function. When $m = 1$, Eq. (1) corresponds to the radially ($\phi = 0$) and azimuthally ($\phi = \pi/2$) polarized beams. For $m = 0$, Eq. (1) represents the linearly polarized beams. If $m < 0$, the polarization direction rotates in the opposite sense to the trajectory about the beam axis.

By using the relation between Laguerre–Gaussian and Hermite–Gaussian modes [18, 19]

$$\left(\frac{\sqrt{2}\,r_0}{w_0}\right)^m L_n^m\left(\frac{2r_0^2}{w_0^2}\right)\exp(\pm im\varphi_0)$$
$$= \frac{(-1)^n}{2^{2n+m}n!}\sum_{l=0}^{n}\sum_{s=0}^{m}(\pm i)^s\binom{n}{l}\binom{m}{s}H_{2l+m-s}\left(\frac{\sqrt{2}\,x_0}{w_0}\right)H_{2n-2l+s}\left(\frac{\sqrt{2}\,y_0}{w_0}\right) \tag{3}$$

where $H_n(\cdot)$ is the Hermite polynomial of order n, $\binom{n}{l}$ and $\binom{m}{s}$ are binomial coefficients, Eq. (1) can be expressed in the following form:

$$\begin{aligned}\mathbf{E}(x_0, y_0, 0) = {} & \frac{E_0(-1)^n t(x_0, y_0)}{2^{2n+|m|+1}n!}\exp\left(-\frac{r_0^2}{w_0^2}\right)\\ & \times\left\{\sum_{l=0}^{n}\sum_{s=0}^{|m|}\left(\frac{i|m|}{m}\right)^s\binom{n}{l}\binom{|m|}{s}\Big[\exp(i\phi)+(-1)^s\exp(-i\phi)\Big]\right.\\ & \times H_{2l+|m|-s}\left(\frac{\sqrt{2}\,x_0}{w_0}\right)H_{2n-2l+s}\left(\frac{\sqrt{2}\,y_0}{w_0}\right)\mathbf{e}_x\\ & +\sum_{l=0}^{n}\sum_{s=0}^{|m|}i^{s-1}\left(\frac{|m|}{m}\right)^s\binom{n}{l}\binom{|m|}{s}\Big[\exp(i\phi)-(-1)^s\exp(-i\phi)\Big]\\ & \left.\times H_{2l+|m|-s}\left(\frac{\sqrt{2}\,x_0}{w_0}\right)H_{2n-2l+s}\left(\frac{\sqrt{2}\,y_0}{w_0}\right)\mathbf{e}_y\right\}\end{aligned} \tag{4}$$

Based on the unified theory of polarization and coherence, the second-order correlation properties of the cylindrical vector partially coherent LGB can be characterized by the CSD matrix of the electric field [20]

$$\mathbf{W}(\mathbf{r}_1, \mathbf{r}_2, z) = \begin{bmatrix} W_{xx}(\mathbf{r}_1, \mathbf{r}_2, z) & W_{xy}(\mathbf{r}_1, \mathbf{r}_2, z)\\ W_{yx}(\mathbf{r}_1, \mathbf{r}_2, z) & W_{yy}(\mathbf{r}_1, \mathbf{r}_2, z)\end{bmatrix} \tag{5}$$

with elements

$$W_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2, z) = \langle E_\alpha(\mathbf{r}_1, z)E_\beta^*(\mathbf{r}_2, z)\rangle, \qquad (\alpha, \beta = x, y) \tag{6}$$

where $\mathbf{r}_1 = (x_1, y_1)$ and $\mathbf{r}_2 = (x_2, y_2)$ are the position vectors at the receiver plane. The asterisk stands for the complex conjugate and the angle bracket represents an ensemble average over the medium statistics. For a cylindrical vector partially coherent beam generated by a Schell-model source, the elements of the CSD matrix at the source plane can be expressed as [17]

$$W_{\alpha\beta}(\mathbf{r}_{01}, \mathbf{r}_{02}, 0) = E_\alpha(\mathbf{r}_{01}, 0) E_\beta^*(\mathbf{r}_{02}, 0) B_{\alpha\beta} \exp\left[-\frac{(\mathbf{r}_{01} - \mathbf{r}_{02})^2}{2\sigma_{\alpha\beta}^2}\right] \tag{7}$$

where $\mathbf{r}_{01} = (x_{01}, y_{01})$ and $\mathbf{r}_{02} = (x_{02}, y_{02})$ are the position vectors at the source plane; $B_{\alpha\beta}$ is the correlation coefficient between E_α and E_β field components; $\sigma_{\alpha\beta}$ is the mutual coherence length ($\alpha \neq \beta$) or auto-coherence length ($\alpha = \beta$). The realizability conditions for the cylindrical vector partially coherent LGB are $B_{\alpha\beta} = 1$ and $\sigma_{xx} = \sigma_{yy} = \sigma_{xy} = \sigma_{yx}$ [17].

In order to obtain the analytical expressions, the aperture function is expanded as the sum of complex Gaussian functions with finite terms [21]

$$t(x_0, y_0) = \sum_{j=1}^{N} A_j \exp\left[-\frac{B_j}{a^2}(x_0^2 + y_0^2)\right] \tag{8}$$

where the complex constants A_j and B_j are the expansion and Gaussian coefficients, respectively, which can be obtained by optimization computation. This method of the finite complex Gaussian expansion of the aperture function has been widely used [22–24].

Applying Eqs. (4)–(8), the elements of CSD matrix for the apertured cylindrical vector partially coherent LGB with arbitrary polarization order at the source plane have the form

$$\begin{aligned} W_{xx}(\mathbf{r}_{01}, \mathbf{r}_{02}, 0) = {} & \exp\left[-\frac{(x_{01} - x_{02})^2 + (y_{01} - y_{02})^2}{2\sigma_{xx}^2}\right] \\ & \times \sum_{l_1=0}^{n} \sum_{s_1=0}^{|m|} \sum_{l_2=0}^{n} \sum_{s_2=0}^{|m|} \sum_{j_1=1}^{N} \sum_{j_2=1}^{N} i^{s_1}(-i)^{s_2}\left[\exp(i\phi) + (-1)^{s_1}\exp(-i\phi)\right] \\ & \times \left[\exp(-i\phi) + (-1)^{s_2}\exp(i\phi)\right] F(\mathbf{r}_{01}, \mathbf{r}_{02}) \end{aligned} \tag{9}$$

$$\begin{aligned} W_{yy}(\mathbf{r}_{01}, \mathbf{r}_{02}, 0) = {} & \exp\left[-\frac{(x_{01} - x_{02})^2 + (y_{01} - y_{02})^2}{2\sigma_{yy}^2}\right] \\ & \times \sum_{l_1=0}^{n} \sum_{s_1=0}^{|m|} \sum_{l_2=0}^{n} \sum_{s_2=0}^{|m|} \sum_{j_1=1}^{N} \sum_{j_2=1}^{N} i^{s_1}(-i)^{s_2}\left[\exp(i\phi) - (-1)^{s_1}\exp(-i\phi)\right] \\ & \times \left[\exp(-i\phi) - (-1)^{s_2}\exp(i\phi)\right] F(\mathbf{r}_{01}, \mathbf{r}_{02}) \end{aligned} \tag{10}$$

$$W_{xy}(\mathbf{r}_{01}, \mathbf{r}_{02}, 0) = \exp\left[-\frac{(x_{01}-x_{02})^2+(y_{01}-y_{02})^2}{2\sigma_{xy}^2}\right]$$
$$\times \sum_{l_1=0}^{n}\sum_{s_1=0}^{|m|}\sum_{l_2=0}^{n}\sum_{s_2=0}^{|m|}\sum_{j_1=1}^{N}\sum_{j_2=1}^{N} i^{s_1}(-i)^{s_2-1}\left[\exp(i\phi)+(-1)^{s_1}\exp(-i\phi)\right]$$
$$\times\left[\exp(-i\phi)-(-1)^{s_2}\exp(i\phi)\right]F(\mathbf{r}_{01}, \mathbf{r}_{02}) \quad (11)$$

$$W_{yx}(\mathbf{r}_{01}, \mathbf{r}_{02}, 0) = \exp\left[-\frac{(x_{01}-x_{02})^2+(y_{01}-y_{02})^2}{2\sigma_{yx}^2}\right]$$
$$\times \sum_{l_1=0}^{n}\sum_{s_1=0}^{|m|}\sum_{l_2=0}^{n}\sum_{s_2=0}^{|m|}\sum_{j_1=1}^{N}\sum_{j_2=1}^{N} i^{s_1-1}(-i)^{s_2}\left[\exp(i\phi)-(-1)^{s_1}\exp(-i\phi)\right]$$
$$\times\left[\exp(-i\phi)+(-1)^{s_2}\exp(i\phi)\right]F(\mathbf{r}_{01}, \mathbf{r}_{02}) \quad (12)$$

with $F(\mathbf{r}_{01}, \mathbf{r}_{02})$ being given by

$$F(\mathbf{r}_{01}, \mathbf{r}_{02}) = \frac{E_0^2}{2^{4n+2|m|+2}(n!)^2}\exp\left(-\frac{r_{01}^2+r_{02}^2}{w_0^2}\right)A_{j_1}A_{j_2}^*\exp\left(-\frac{B_{j_1}r_{01}^2+B_{j_2}^*r_{02}^2}{a^2}\right)$$
$$\times\left(\frac{|m|}{m}\right)^{s_1+s_2}\binom{n}{l_1}\binom{|m|}{s_1}\binom{n}{l_2}\binom{|m|}{s_2}H_{2l_1+|m|-s_1}\left(\frac{\sqrt{2}\,x_{01}}{w_0}\right)H_{2n-2l_1+s_1}\left(\frac{\sqrt{2}\,y_{01}}{w_0}\right)$$
$$\times H_{2l_2+|m|-s_2}\left(\frac{\sqrt{2}\,x_{02}}{w_0}\right)H_{2n-2l_2+s_2}\left(\frac{\sqrt{2}\,y_{02}}{w_0}\right) \quad (13)$$

Within the framework of the paraxial approximation, the propagation of the apertured cylindrical vector partially coherent beam in turbulent atmosphere can be studied by using the extended Huygens–Fresnel integral formula [15, 25]

$$W_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2, z) = \left(\frac{k}{2\pi z}\right)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}W_{\alpha\beta}(\mathbf{r}_{01}, \mathbf{r}_{02}, 0)$$
$$\times\exp\left[-\frac{ik}{2z}(\mathbf{r}_1-\mathbf{r}_{01})^2+\frac{ik}{2z}(\mathbf{r}_2-\mathbf{r}_{02})^2\right]$$
$$\times\langle\exp[\psi(\mathbf{r}_{01}, \mathbf{r}_1, z)+\psi^*(\mathbf{r}_{02}, \mathbf{r}_2, z)]\rangle\,\mathrm{d}x_{01}\,\mathrm{d}y_{01}\,\mathrm{d}x_{02}\,\mathrm{d}y_{02} \quad (14)$$

where k is the wave number, and the ensemble average term is given by [25]

$$\langle \exp[\psi(\mathbf{r}_{01}, \mathbf{r}_1, z) + \psi^*(\mathbf{r}_{02}, \mathbf{r}_2, z)] \rangle = \exp\left[-\frac{(\mathbf{r}_{01} - \mathbf{r}_{02})^2 + (\mathbf{r}_{01} - \mathbf{r}_{02})(\mathbf{r}_1 - \mathbf{r}_2) + (\mathbf{r}_1 - \mathbf{r}_2)^2}{\rho_0^2}\right] \tag{15}$$

where $\rho_0 = (0.545 C_n^2 k^2 z)^{-3/5}$ is the coherence length of a spherical wave propagating in turbulent atmosphere, C_n^2 is the structure constant of the refractive index and describes the turbulence level.

Substituting Eqs. (9)–(12) into Eq. (14) and using the following formulae [26]

$$\int_{-\infty}^{\infty} \exp\left[-(x-y)^2\right] H_n(\alpha x) \mathrm{d}x = \sqrt{\pi}\,(1-\alpha^2)^{n/2} H_n\left(\frac{\alpha y}{(1-\alpha^2)^{1/2}}\right) \tag{16}$$

$$H_M(x) = \sum_{t=0}^{[M/2]} \frac{(-1)^t M!}{t!(M-2t)!}(2x)^{M-2t} \tag{17}$$

$$(\alpha + x)^n = \sum_{u=0}^{n} \binom{n}{u} x^u \alpha^{n-u} \tag{18}$$

$$\int_{-\infty}^{\infty} x^n \exp\left[-(x-\beta)^2\right] \mathrm{d}x = (2i)^{-n} \sqrt{\pi}\, H_n(i\beta) \tag{19}$$

where $[M/2]$ denotes the greatest integer less than or equal to $M/2$, we can obtain the analytical expressions for the elements of CSD matrix of the apertured cylindrical vector partially coherent LGB in turbulent atmosphere

$$W_{xx}(\mathbf{r}_1, \mathbf{r}_2, z) = \sum_{l_1=0}^{n} \sum_{s_1=0}^{|m|} \sum_{l_2=0}^{n} \sum_{s_2=0}^{|m|} \sum_{j_1=1}^{N} \sum_{j_2=1}^{N} i^{s_1} (-i)^{s_2} \left(\frac{|m|}{m}\right)^{s_1+s_2} \binom{n}{l_1} \binom{|m|}{s_1} \binom{n}{l_2} \binom{|m|}{s_2}$$
$$\times \left[\exp(i\phi) + (-1)^{s_1} \exp(-i\phi)\right]\left[\exp(-i\phi) + (-1)^{s_2} \exp(i\phi)\right] G_{xx}(\mathbf{r}_1, \mathbf{r}_2, z) \tag{20}$$

$$W_{yy}(\mathbf{r}_1, \mathbf{r}_2, z) = \sum_{l_1=0}^{n} \sum_{s_1=0}^{|m|} \sum_{l_2=0}^{n} \sum_{s_2=0}^{|m|} \sum_{j_1=1}^{N} \sum_{j_2=1}^{N} i^{s_1} (-i)^{s_2} \left(\frac{|m|}{m}\right)^{s_1+s_2} \binom{n}{l_1} \binom{|m|}{s_1} \binom{n}{l_2} \binom{|m|}{s_2}$$
$$\times \left[\exp(i\phi) - (-1)^{s_1} \exp(-i\phi)\right]\left[\exp(-i\phi) - (-1)^{s_2} \exp(i\phi)\right] G_{yy}(\mathbf{r}_1, \mathbf{r}_2, z) \tag{21}$$

$$W_{xy}(\mathbf{r}_1, \mathbf{r}_2, z) = \sum_{l_1=0}^{n}\sum_{s_1=0}^{|m|}\sum_{l_2=0}^{n}\sum_{s_2=0}^{|m|}\sum_{j_1=1}^{N}\sum_{j_2=1}^{N} i^{s_1}(-i)^{s_2-1}\left(\frac{|m|}{m}\right)^{s_1+s_2}\binom{n}{l_1}\binom{|m|}{s_1}\binom{n}{l_2}\binom{|m|}{s_2}$$
$$\times\left[\exp(i\phi)+(-1)^{s_1}\exp(-i\phi)\right]\left[\exp(-i\phi)-(-1)^{s_2}\exp(i\phi)\right]G_{xy}(\mathbf{r}_1, \mathbf{r}_2, z) \tag{22}$$

$$W_{yx}(\mathbf{r}_1, \mathbf{r}_2, z) = \sum_{l_1=0}^{n}\sum_{s_1=0}^{|m|}\sum_{l_2=0}^{n}\sum_{s_2=0}^{|m|}\sum_{j_1=1}^{N}\sum_{j_2=1}^{N} i^{s_1-1}(-i)^{s_2}\left(\frac{|m|}{m}\right)^{s_1+s_2}\binom{n}{l_1}\binom{|m|}{s_1}\binom{n}{l_2}\binom{|m|}{s_2}$$
$$\times\left[\exp(i\phi)-(-1)^{s_1}\exp(-i\phi)\right]\left[\exp(-i\phi)+(-1)^{s_2}\exp(i\phi)\right]G_{yx}(\mathbf{r}_1, \mathbf{r}_2, z) \tag{23}$$

with $G_{xx}(\mathbf{r}_1, \mathbf{r}_2, z)$, $G_{yy}(\mathbf{r}_1, \mathbf{r}_2, z)$, $G_{xy}(\mathbf{r}_1, \mathbf{r}_2, z)$ and $G_{yx}(\mathbf{r}_1, \mathbf{r}_2, z)$ being given by

$$G_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2, z) = \left(\frac{k}{z}\right)^2 \frac{E_0^2 A_{j_1} A_{j_2}^*}{2^{4n+2|m|+4}(n!)^2 p_{1\alpha\beta}}\left(1-\frac{2}{p_{1\alpha\beta}w_0^2}\right)^{(M_1+N_1)/2}$$
$$\times\exp\left[-\frac{ik(r_1^2-r_2^2)}{2z}-\frac{(x_1-x_2)^2+(y_1-y_2)^2}{\rho_0^2}+\frac{d_1^2+d_2^2}{4p_{1\alpha\beta}}+\frac{q_{2\alpha\beta}^2+q_{4\alpha\beta}^2}{4p_{2\alpha\beta}}\right]$$
$$\times\sum_{t_1=0}^{\left[\frac{M_1}{2}\right]}\sum_{t_2=0}^{\left[\frac{M_2}{2}\right]}\sum_{u=0}^{M_1-2t_1}\sum_{f_1=0}^{\left[\frac{N_1}{2}\right]}\sum_{f_2=0}^{\left[\frac{N_2}{2}\right]}\sum_{v=0}^{N_1-2f_1}(-1)^{t_1+t_2+f_1+f_2}(2i)^{-(M_2-2t_2+u+N_2-2f_2+v)}$$
$$\times\frac{M_1!M_2!N_1!N_2!\,c_{\alpha\beta}^{u+v}d_1^{M_1-2t_1-u}d_2^{N_1-2f_1-v}(\sqrt{p_{2\alpha\beta}})^{-(M_2-2t_2+u+N_2-2f_2+v+2)}}{t_1!(M_1-2t_1)!\,t_2!(M_2-2t_2)!\,f_1!(N_1-2f_1)!\,f_2!(N_2-2f_2)!}$$
$$\times\left[\frac{2}{\sqrt{2p_{1\alpha\beta}(p_{1\alpha\beta}w_0^2-2)}}\right]^{M_1-2t_1+N_1-2f_1}\left(\frac{2\sqrt{2}}{w_0}\right)^{M_2-2t_2+N_2-2f_2}$$
$$\times\binom{M_1-2t_1}{u}\binom{N_1-2f_1}{v}H_{M_2-2t_2+u}\left(\frac{iq_{2\alpha\beta}}{2\sqrt{p_{2\alpha\beta}}}\right)H_{N_2-2f_2+v}\left(\frac{iq_{4\alpha\beta}}{2\sqrt{p_{2\alpha\beta}}}\right) \tag{24}$$

where

$$M_1 = 2l_1 + |m| - s_1$$

$$M_2 = 2l_2 + |m| - s_2$$

$$N_1 = 2n - 2l_1 + s_1$$

$$N_2 = 2n - 2l_2 + s_2$$

$$c_{\alpha\beta} = \frac{1}{\sigma_{\alpha\beta}^2} + \frac{2}{\rho_0^2}$$

$$d_1 = \frac{ikx_1}{z} + \frac{x_2 - x_1}{\rho_0^2}$$

$$d_2 = \frac{iky_1}{z} + \frac{y_2 - y_1}{\rho_0^2}$$

$$p_{1\alpha\beta} = \frac{1}{w_0^2} + \frac{1}{2\sigma_{\alpha\beta}^2} + \frac{B_{j_1}}{a^2} + \frac{ik}{2z} + \frac{1}{\rho_0^2}$$

$$p_{2\alpha\beta} = \frac{1}{w_0^2} + \frac{1}{2\sigma_{\alpha\beta}^2} + \frac{B_{j_2}^*}{a^2} - \frac{ik}{2z} + \frac{1}{\rho_0^2} - \frac{c_{\alpha\beta}^2}{4p_{1\alpha\beta}}$$

$$q_{2\alpha\beta} = -\frac{ikx_2}{z} + \frac{x_1 - x_2}{\rho_0^2} + \frac{c_{\alpha\beta} d_1}{2p_{1\alpha\beta}}$$

$$q_{4\alpha\beta} = -\frac{iky_2}{z} + \frac{y_1 - y_2}{\rho_0^2} + \frac{c_{\alpha\beta} d_2}{2p_{1\alpha\beta}}$$

Equations (20)–(23) are the basic results obtained in this paper, which help us investigate the circular aperture diffraction properties of the arbitrary cylindrical vector partially coherent LGBs propagating in turbulent atmosphere.

The average intensity, the degree of polarization, and the degree of coherence of the apertured cylindrical vector partially coherent beam in turbulent atmosphere are separately expressed as [20]

$$I(\mathbf{r}, z) = \mathrm{Tr}\,\mathbf{W}(\mathbf{r}, \mathbf{r}, z) = W_{xx}(\mathbf{r}, \mathbf{r}, z) + W_{yy}(\mathbf{r}, \mathbf{r}, z) \tag{25}$$

$$P(\mathbf{r}, z) = \sqrt{1 - \frac{4\mathrm{Det}\,\mathbf{W}(\mathbf{r}, \mathbf{r}, z)}{[\mathrm{Tr}\,\mathbf{W}(\mathbf{r}, \mathbf{r}, z)]^2}} \tag{26}$$

$$\mu(\mathbf{r}_1, \mathbf{r}_2, z) = \frac{\mathrm{Tr}\,\mathbf{W}(\mathbf{r}_1, \mathbf{r}_2, z)}{\sqrt{\mathrm{Tr}\,\mathbf{W}(\mathbf{r}_1, \mathbf{r}_1, z)}\sqrt{\mathrm{Tr}\,\mathbf{W}(\mathbf{r}_2, \mathbf{r}_2, z)}} \tag{27}$$

where Tr and Det denote the trace and the determinant of the CSD matrix, respectively.

3. Numerical results

In order to illustrate the circular aperture diffraction properties of the arbitrary cylindrical vector partially coherent LGBs propagating in turbulent atmosphere, numerical calculations have been performed using the formulae derived in Section 2. Here $\delta = a/w_0$ is the truncation parameter and describes the beam diffraction effect by a circular aperture. The parameters are chosen as $\sigma_{\alpha\beta} = \sigma_0$, $w_0 = 2$ cm, $\lambda = 632.8$ nm, and $\phi = 0$.

Figure 1 presents the average intensity distributions of the apertured and unapertured (δ = inf) cylindrical vector partially coherent beams in turbulent atmosphere. It can be seen from Fig. 1**a** that the decreasing aperture radius confines the initial field into smaller area, and the outer ring part of the beam is blocked by the circular aperture. From Figs. 1**a** and 1**b**, one finds that the beam profile of the unapertured case changes from the doughnut shape to the peak-central shape on propagation in turbulent atmosphere, whereas the average intensity of the apertured case has still dark center. This phenomenon implies that the beam evolution properties are affected by the circular aperture diffraction and the atmospheric turbulence together. When the strength of the atmospheric turbulence is weak ($C_n^2 = 10^{-15}$ m$^{-2/3}$), the influence of the circular

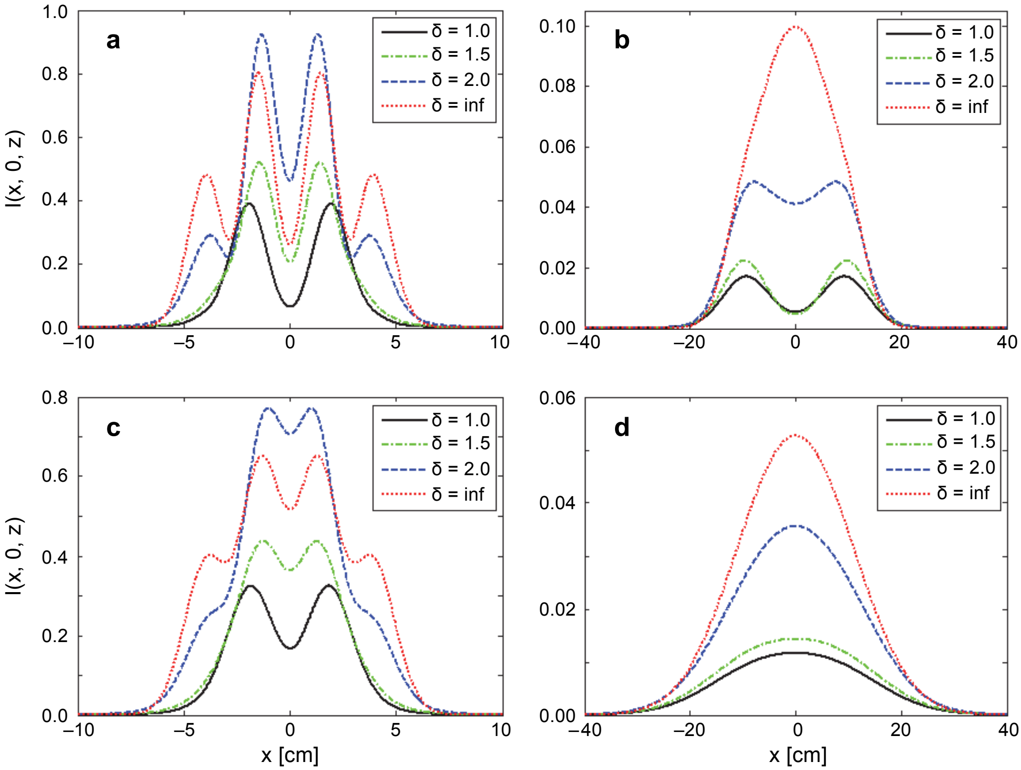

Fig. 1. Average intensity distributions of the apertured and unapertured cylindrical vector partially coherent beams in turbulent atmosphere with $n = 1$, $m = 2$, and $\sigma_0 = 2$ cm: $C_n^2 = 10^{-15}$ m$^{-2/3}$ and $z = 1$ km (**a**), $C_n^2 = 10^{-15}$ m$^{-2/3}$ and $z = 5$ km (**b**), $C_n^2 = 10^{-14}$ m$^{-2/3}$ and $z = 1$ km (**c**), and $C_n^2 = 10^{-14}$ m$^{-2/3}$ and $z = 5$ km (**d**).

aperture diffraction on the beam evolution behavior is larger than that of atmospheric turbulence. With the enhancement of the strength of the atmospheric turbulence ($C_n^2 = 10^{-14}$ m$^{-2/3}$), both the beam profiles of the apertured and unapertured cases convert into Gaussian shapes after propagating a long distance in turbulent atmosphere, as shown in Figs. 1**c** and 1**d**. It indicates that the effect of the atmospheric turbulence exceeds the effect of the circular aperture diffraction and plays a leading role in the beam evolution behavior.

The degree of polarization of the apertured and unapertured cylindrical vector partially coherent beams in turbulent atmosphere is plotted in Fig. 2. One finds that the degree of polarization equals to zero on the propagation axis and increases gradually with the spread of the off-axis position. It indicates that the polarization structure of the cylindrical vector partially coherent beam is destroyed due to the effect of the atmospheric turbulence. It can be seen from Figs. 2**a** and 2**c** that there is little difference among the degree of polarization for different values of the truncation parameter. This phenomenon implies that the circular aperture diffraction has little influence on the

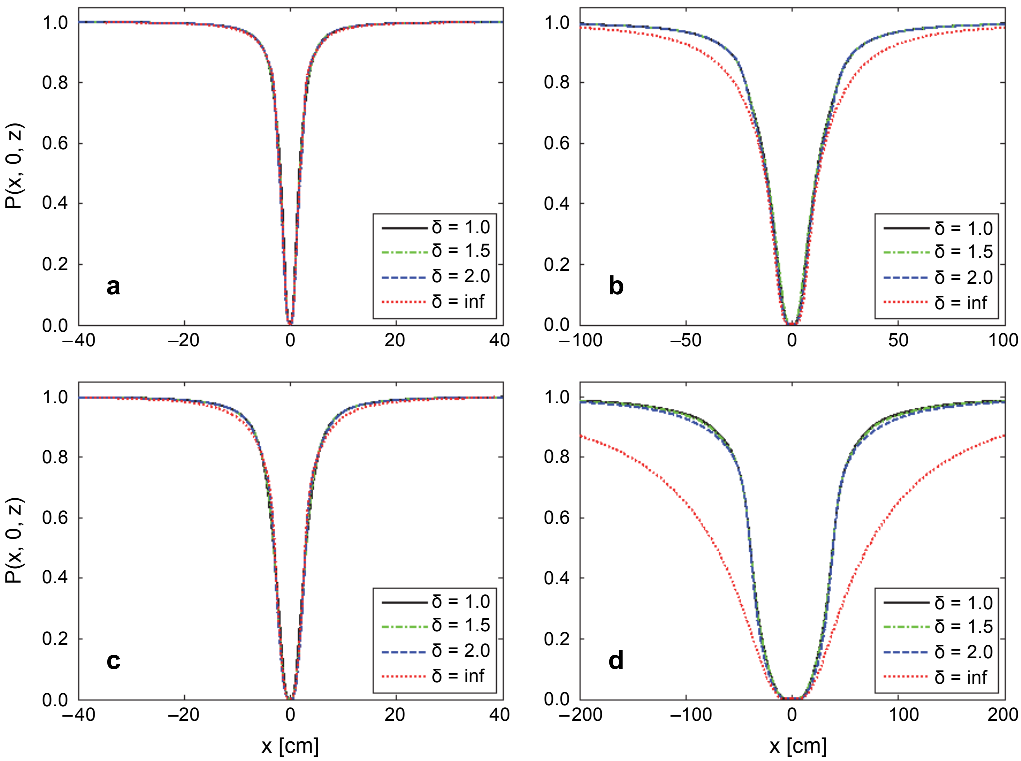

Fig. 2. Degree of polarization of the apertured and unapertured cylindrical vector partially coherent beams in turbulent atmosphere with $n = 1$, $m = 2$, and $\sigma_0 = 2$ cm: $C_n^2 = 10^{-15}$ m$^{-2/3}$ and $z = 1$ km (**a**), $C_n^2 = 10^{-15}$ m$^{-2/3}$ and $z = 5$ km (**b**), $C_n^2 = 10^{-14}$ m$^{-2/3}$ and $z = 1$ km (**c**), and $C_n^2 = 10^{-14}$ m$^{-2/3}$ and $z = 5$ km (**d**).

degree of polarization in the near field ($z = 1$ km). With the further increase of the propagation distance ($z = 5$ km), the effect of the circular aperture diffraction on the degree of polarization becomes more obvious, and the degree of polarization of the apertured case is larger than that of the unapertured case, as illustrated in Figs. 2**b** and 2**d**.

Figure 3 shows the degree of coherence of the apertured and unapertured cylindrical vector partially coherent beams in turbulent atmosphere. The degree of coherence of the apertured case exhibits a larger amplitude oscillation than that of the unapertured case in Fig. 3**a**. The oscillatory behavior of the degree of coherence becomes weak with the increase of the propagation distance, as shown in Fig. 3**b**. This phenomenon implies that the circular aperture diffraction compared with the atmospheric turbulence has a greater impact on the degree of coherence in the near field ($z = 1$ km), while the effect of the atmospheric turbulence accumulates on propagation. From Figs. 3**c** and 3**d**, one finds that the oscillatory distributions of the degree of coherence change into Gaussian distributions after propagating a long distance ($z = 5$ km) in turbulent atmosphere, additionally, the Gaussian distributions of the degree of coherence are almost same for

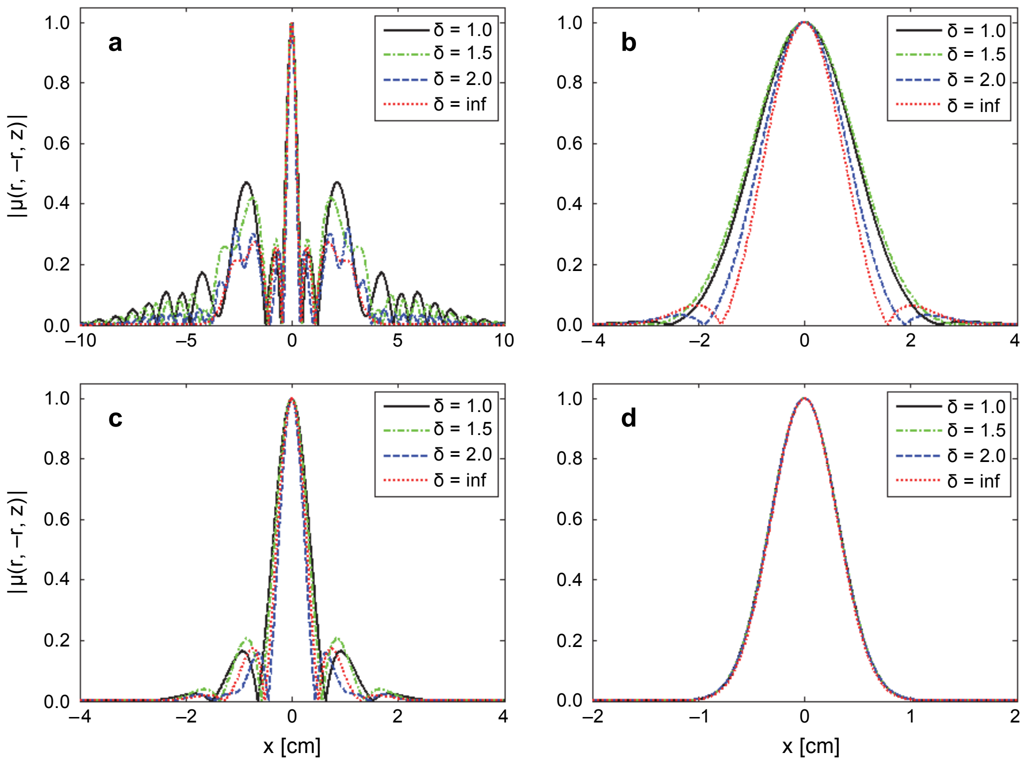

Fig. 3. Degree of coherence of the apertured and unapertured cylindrical vector partially coherent beams in turbulent atmosphere with $n = 1$, $m = 2$, $\sigma_0 = 2$ cm: $C_n^2 = 10^{-15}$ $m^{-2/3}$ and $z = 1$ km (**a**), $C_n^2 = 10^{-15}$ $m^{-2/3}$ and $z = 5$ km (**b**), $C_n^2 = 10^{-14}$ $m^{-2/3}$ and $z = 1$ km (**c**), and $C_n^2 = 10^{-14}$ $m^{-2/3}$ and $z = 5$ km (**d**).

different values of the truncation parameter. It indicates that the effect of the atmospheric turbulence overpasses the effect of the circular aperture diffraction and plays a dominant role in the degree of coherence in the far field.

4. Conclusion

In conclusion, the circular aperture diffraction properties of the arbitrary cylindrical vector partially coherent beams propagating in turbulent atmosphere have been studied. The results show that the beam evolution, polarization and coherence properties of the apertured cylindrical vector partially coherent beams are closely related to the beam diffraction effect by a circular aperture and the atmospheric turbulence. In the near field, the circular aperture diffraction compared with the atmospheric turbulence has a greater impact on the average intensity and the degree of coherence, while it has little influence on the degree of polarization. The effect of the atmospheric turbulence accumulates on propagation and exceeds the effect of the circular aperture diffraction in the far field.

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